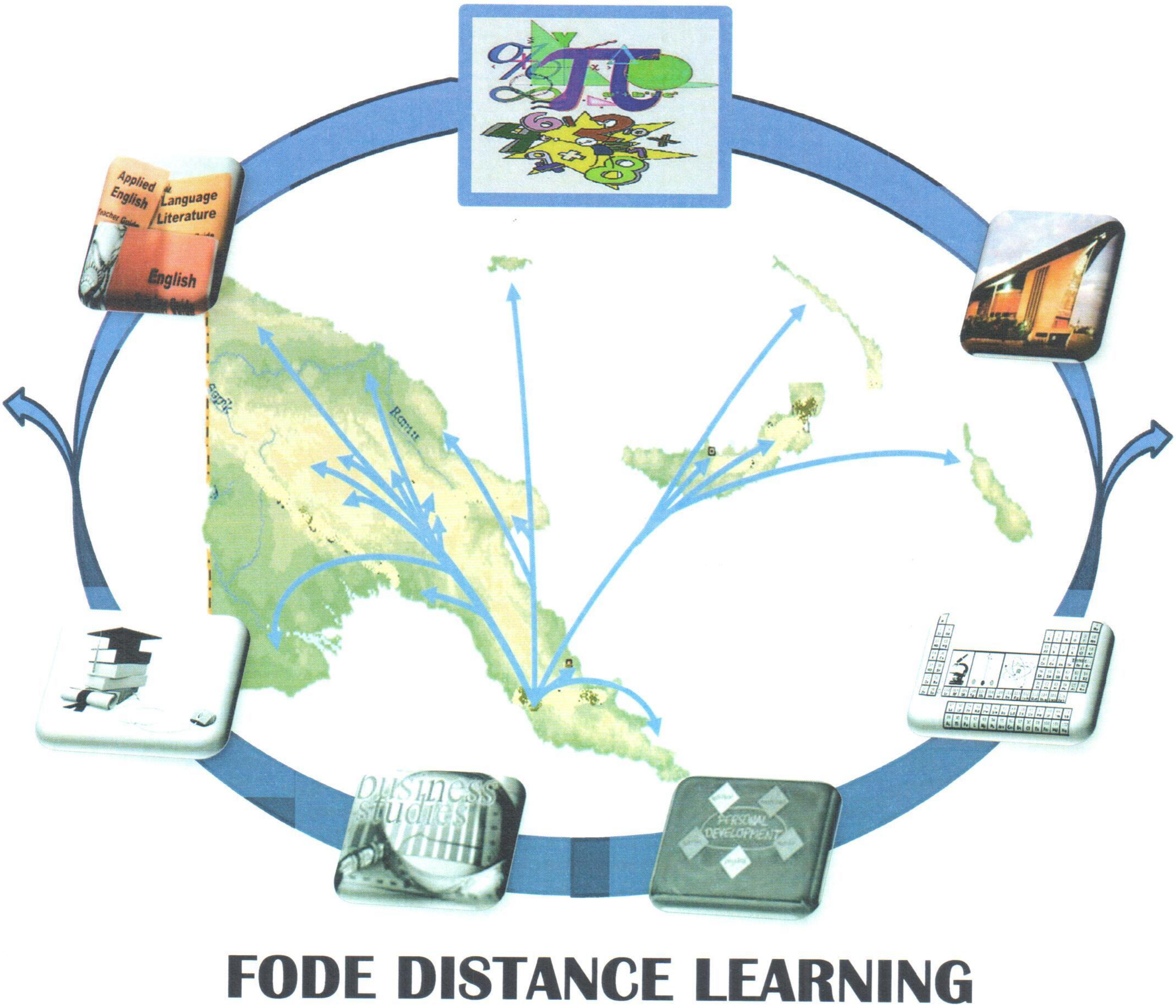
DEPARTMENT OF EDUCATION

GRADE 12 MATHEMATICS B

12.4: ALGEBRA AND GRAPHS



FODE DISTANCE LEARNING

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PAPUA NEW GUINEA

GRADE 12

MATHEMATICS B

UNIT MODULE 4

ALGEBRA AND GRAPH

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MR. DEMAS TONGOGO
Principal-FODE

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.

DR. UKE KOMBRA, PhD
Acting Secretary for Education

UNIT 4: ALGEBRA AND GRAPH

INTRODUCTION

The word "Algebra" was derived from the Arabic word "al-jabr" which appeared in the title of the second book of Mohammed ibn Musa al-Khwarizmi (c.780-850)

In this unit, we will discuss some of the techniques of solving linear equations and inequalities, system of equations, quadratic equations, logarithmic and exponential equations.

Some applications in the sciences, whether natural, physical or social, usually deal with exponential and logarithmic functions. Historically, logarithms were developed to help carry out complicated numerical computations. With the coming of the computers and hand calculators, however, computations using logarithms is no longer of much interest. Nevertheless, logarithmic and exponential equations and functions are still very important in mathematics today.

LEARNING OUTCOMES

On successful completion of this module, students will be able to:

- solve linear, quadratic, exponential and inequality equations
- solve word problems involving linear and quadratic expressions
- solve simultaneous equations graphically
- graph inequalities and shade regions
- graph quadratic functions
- sketch graph of hyperbolic, exponential, logarithmic functions
- derive equations of parabolic, hyperbolic and exponential functions
- discuss and calculate asymptote in given hyperbola, exponential, logarithmic functions
- solve problem applications of exponential and logarithmic functions

Time Frame

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not pass any question without solving it first.

12.4.1: LINEAR EQUATIONS AND INEQUALITIES

In this section, we shall study how to solve linear equations and inequalities. Real life applications involving these two concepts will also be considered.

We will begin by recalling basic definitions.

12.4.1.1: Solving Linear Equations

A statement that two quantities are equal is called an **equation**. The two quantities are written with an equal (=) sign between them. Some equations are true, and some are false, and some, the truth value cannot be determined. For example:

$4 + 5 = 9$	is true
$4 + 5 = 4 - 5$	is false
$x + 5 = 9$	is neither true nor false since the value of x is not known

In Algebra, equations involving letters or **variables** are important. A variable is defined as a symbol or letter that is used to represent one or more numbers. If the variable in an equation can be replaced by a single number or **constant** that makes the resulting equation true, that number is called the **solution** of the equation. The set of all solutions of an equation is called the **solution set** of the equation. The process of determining all solutions of a given equation is called **solving the equation.** Equations are classified as identity equation and conditional equation. An **identity or identical equation** is an equation which is true for all values of the variable involved. A **conditional equation** is an equation which is true only for a certain value (or values) of the variable. An equation which has no solution is called **contradiction**.

Some examples of conditional equations:
1. $4x + 1 = 9$
2. $x + 5 - 3(x - 6)=0$
3. $\frac{1}{2}x + \frac{3}{4} = x$

Some examples of identical equations:
1. $x + 3x - 8 = 5x - 8 - x$
2. $4x + 2 = x + 5 - 3 + 3x$

Some examples of contradiction:
1. $2x = 2x + 6$
2. $6x + 5 = 6x - 7$

In this section, we concentrate on **linear equations** in which the variable is raised to the first power only. For this, we often call linear equations as **first degree equations.** Every linear equation in one variable x is written in the standard form

$$\boxed{ax + by = c}$$

where **a** and **b** are known real numbers, **a** $\neq 0$.

Consider the two equations:

$$x - 6 = 3 \text{ and } x = 9$$

Both equations have 9 as a solution, and both can be solved by inspection. However, it is easier to recognize 9 as a solution to the second equation. Two equations which have exactly the same solution are called **equivalent equations.**

We apply the following properties of equality to solve linear equations in one variable.

> **Addition - Subtraction Property**
>
> If the same number or expression is added to or subtracted from both sides of an equation, the resulting equation is equivalent (or has the same solutions as) to the original.
>
> **Multiplication-Division Property**
>
> If each side of an equation is multiplied or divided by the same non-zero number or expression, the resulting equation is equivalent (or has the same solutions as) to the original.

Example 1 Solve $4x - 5 = 0$

Solution:

$4x - 5 = 0$	Given
$4x - 5 + 5 = 0 + 5$	Addition Property of Equality
$\left(\frac{1}{4}\right)4x = 5\left(\frac{1}{4}\right)$	Multiplication Property of Equality
$x = \frac{5}{4}$	Solution Set

Check: Substitute $x = \frac{5}{4}$ in the original equation $4x - 5 = 0$

$$4\left(\frac{5}{4}\right) - 5 = 0$$

$$\frac{20}{4} - 5 = 0$$

$$0 = 0$$

Example 2 Solve $3x + 7 = 13$

Solution:

$3x + 7 = 13$	Given
$3x + 7 - 7 = 13 - 7$	Subtraction Property of Equality
$\left(\frac{1}{3}\right)3x = 6\left(\frac{1}{3}\right)$	Multiplication Property of Equality
$x = 2$	Solution set

Check: Substitute $x = 2$ in the original equation.

$$3x + 7 = 13$$
$$3(2) + 7 = 13$$
$$6 + 7 = 13$$
$$13 = 13$$

Some equations may not be of the form $ax + by = c$ but by collecting like terms and simplifying, these equations may be expressed in standard form. In doing this, the convention is to put terms containing the variable on the left side of the equal sign and the constant on the right.

Note that the Addition-Subtraction Property of Equality allows us to move one term from one side of the equation to the other. This is called as **transposition**.

> **Transposition** is the process of moving one term of an equation from one side to the other.

Example 3 Solve the equation $2x + 3 = 2x + 8$

Solution:

$$2x + 3 = 2x + 8$$
$$2x + 3 - 3 = 2x + 8 - 3 \quad \text{Subtract 3 on both sides}$$
$$2x = 2x + 5$$
$$2x - 2x = 2x - 2x + 5 \quad \text{Subtract 2x on both sides}$$
$$0 = 5$$

Whenever a false statement, such as $0 = 5$ is obtained, the original equation has no solution. Equations like this are called **contradictions.**

Example 4 Solve the equation $5x + 1 = 1 + 5x$

Solution:

$$5x + 1 = 1 + 5x$$
$$5x + 1 - 1 = 1 - 1 + 5x \quad \text{Subtract 1}$$
$$5x = 5x$$

Whenever an equation, such as $5x = 5x$, is obtained, the original equation is true for all replacement of the variable and is called an **identity.**

When an equation contains one or more sets of grouping symbols, remove all grouping symbols using the distributive laws. Then collect like terms and proceed as before. (See examples on the next page).

Example 5 Solve the equation: $3(x-2)-6 = 5x + 4(3-2x)$.

Solution:

$3(x-2)-6 = 5x + 4(3-2x)$	Given
$3x - 6 - 6 = 5x + 12 - 8x$	Distributive Property
$3x - 12 = 12 - 3x$	Collect like terms
$6x = 24$	Addition Property of Equality (Transposition)
$x = 4$	Multiplication Property of Equality

Check: Substitute x = 4 to the original equation.

$$3(4-2)-6 = 5(4) + 4[(3-2(4)]$$
$$3(2) -6 = 20 + 4(3-8)$$
$$6-6 = 20 + 4(-5)$$
$$0 = 20 -20$$
$$0 = 0$$

Therefore, the solution set is {4}.

Example 7 Find the solution set of the equation $0.25x - 2.5 = 0.15x + 7.5$.

Solution: $0.25x - 2.5 = 0.15x + 7.5$

$0.25x - 2.5 + 2.5 = 0.15x + 7.5 + 2.5$	Add 2.5 on both sides
$0.25x = 0.15x + 10$	Collect like terms
$0.25x - 0.15x = 0.15 - 0.15x + 10$	Add 0.15 on both sides
$0.10x = 10$	Collect like terms
$x = \frac{10}{.10}$	Divide both sides by 0.10
$x = 100$	

Check: Substitute x = 100 in the original equation.

$$0.25(100) - 2.5 + 2.5 = 0.15(100) + 7.5 + 2.5$$
$$25 - 2.5 + 2.5 = 15 + 7.5 + 2.5$$
$$25 = 25$$

Therefore, the solution set is {100}.

It is sometimes wise to eliminate decimals by multiplying both sides of the equation by the appropriate power of 10. For example, in the above example, if we multiply both sides of the equation by 100, we obtain

$$25x - 250 = 15x + 750$$

There might be less chance of making an arithmetic error when solving this equation rather than the original.

$$25x - 250 = 15x + 750$$
$$25x - 15x = 750 + 250$$
$$10x = 1000$$
$$x = 100$$

If the equation has terms with fractional coefficients, it is helpful to remove the fraction by multiplying both sides of the equation by the lowest common denominator (LCD) of the fractions. (See Example 5)

Example 5 Solve the equation: $\frac{2}{x+2} + \frac{1}{2x-1} = \frac{5}{(x+2)(2x-1)}$

Solution: $\frac{2}{x+2} + \frac{1}{2x-1} = \frac{5}{(x+2)(2x-1)}$

$2(2x - 1) + 1(x + 2) = 5$ Multiply both sides by the LCD: $(x + 2)(2x - 1)$

$4x - 2 + x + 2 = 5$ Distributive property

$5x = 5$ Collect like terms

$x = 1$ Multiplication property

Check: Substitute $x = 1$ in the original equation.

$$\frac{2}{1+2} + \frac{1}{2(1)-1} = \frac{5}{(1+2)[2(1)-1]}$$
$$\frac{2}{3} + \frac{1}{2-1} = \frac{5}{(3)(2-1)}$$
$$\frac{2}{3} + 1 = \frac{5}{3(1)}$$
$$\frac{5}{3} = \frac{5}{3}$$

Therefore, the solution set is {1}.

We conclude this section by summarising the method to solve linear equations.

TO SOLVE A LINEAR EQUATION

1. Simplify both sides by removing grouping symbols and collecting like terms.
2. Eliminate fractions or decimals by multiplying both sides of the equation by an appropriate factor (the LCD for fractions or a power of 10 for the decimals).
3. Use the addition-subtraction rule to isolate all variable terms on one side and all constant terms on the other side. Collect like terms where possible.
4. Use multiplication-division rule to obtain a variable with coefficient of 1.
5. Check the solution by substituting in the original equation.
6. If an identity results, the original equation has every real numbers as a solution. If a contradiction results, there is n solution.

Now do the learning activity.

LEARNING ACTIVITY 12.4.1.1

20 minutes

A. Refer to the equation below and answer questions 1 to 8.

$$2x - 5 = 17$$

1. What is the variable? ______________
2. What is the right side of the equation? ______________
3. What is the left side f the equation? ______________
4. Is it an identity? ______________
5. Is it a contradiction? ______________
6. Is it a conditional equation? ______________
7. Is it equivalent to $x = 11$? ______________
8. What is the solution? ______________

B. Solve the following equations and check the solution set.

1. $6x + 4 = -20$

2. $3x + 5 = 2x + 11$

3. $\frac{3x+5}{12} - \frac{4x-1}{6} = \frac{x-2}{3}$

4. $6(x + 1) - 4(x - 3) = 0$

5. $2.1x + 45.2 = 3.2 - 8.4x$

12.4.1.2: Solving Linear Inequalities with One Variable

Statements such as

$$x + 2 < 2, \quad 3x \geq 9, \quad 3x - 1 > 11, \quad 2x + 1 \leq 5 - 7x,$$

are called **linear inequalities.** In general, a linear inequality can always be written as

$$ax + b < 0, \quad ax + b > 0, \quad ax + b \geq 0 \quad \text{or} \quad ax + b \leq 0,$$

where a and b are real numbers, $a \neq 0$.

A **solution** to an inequality is a number which, when substituted for a variable, makes the inequality true.

If the inequality symbols (<, >, ≤ and ≥) in the above statements were replaced with the equality (=), we would know how to solve the resulting equations. Fortunately, solving inequality is much like solving an equation since most of the basic rules apply. There is one exception which will be discussed shortly.

If we start with the true inequality

$$6 < 8$$

and add 4 on both sides, we obtain another inequality

$$6 + 4 < 8 + 4$$

$$10 < 12 \qquad \text{True}$$

Similarly, if we subtract 13 from both sides,

$$6 - 13 < 8 - 13$$

$$-7 < -5 \qquad \text{True}$$

We again obtain a true inequality. These observations lead us to the following rule.

Addition - Subtraction Property

If the same number or expression is added to or subtracted from both sides of an inequality, an equivalent inequality (or an inequality with exactly the same solutions) is obtained.

This rule is used to solve inequalities in the same way that the corresponding addition-subtraction rule is used to solve equations. The key is to isolate the variable on one side of the inequality.

Example 1 Solve: $x + 9 < 3$

Solution:

$$x + 9 < 3$$

$$x + 9 - 9 < 2 - 9 \quad \text{Subtract 9 from both sides}$$

$$x < -7$$

The solutions are all numbers less than 7.

Example 2 Solve: $x - 8 \geq 4$

Solution:

$$x - 8 \geq 4$$

$$x - 8 + 8 \geq 4 + 8 \quad \text{Add 8 to both sides}$$

$$x \geq 12$$

The solutions are all numbers greater than or equal to 12.

Generally, we will not make this statement but simply indicate the solution by writing $x \geq 12$.

If we start again with the true inequality

$$6 < 8$$

and multiply both sides by 4, we obtain another inequality

$$6\ (4) < 8\ (4)$$

$$24 < 32 \quad \text{True}$$

However, if we multiply both sides by -4, we obtain the false inequality

$$6\ (-4) < 8\ (-4)$$

$$-24 < -32 \quad \text{-32 is really less than -24}$$

In order to obtain a true inequality when we multiply by the negative number -4, we must reverse the symbol of inequality, that is, change from < to > or > to <.

These observations lead us to the next rule.

Multiplication-Division Property

1. If both sides of an inequality are multiplied or divided by the same positive number, the resulting inequality is equivalent to the original.
2. If both sides of an inequality are multiplied or divided by the same negative number, the resulting inequality is equivalent to the original if the symbol of inequality is reversed.

The only substantial difference between solving equation and solving inequality concerns multiplying or dividing both sides by a negative number.

Always reverse the symbol of inequality when multiplying or dividing by a negative number.

Example 3 Solve: $3x \geq 9$

Solution: $3x \geq 9$

$\frac{1}{3}(3x) \geq \frac{1}{3}(9)$ $\frac{1}{3}$ is positive so inequality remains the same.

$x \geq 3$

The solution is $x \geq 3$.

Example 4 Solve: $-\frac{1}{2}x < 8$

Solution: $-\frac{1}{2}x < 8$

$(-2)(-\frac{1}{2}x) > (-2)(8)$ -2 is negative so inequality is reversed.

$x > -16$

The solution is $x > -16$.

As with solving equations, many times we must use a combination of the addition-subtraction and multiplication-division rules.

Example 5 Solve: $5x - 2 > 8$

Solution: $5x - 2 > 8$

$5x - 2 + 2 > 8 + 2$ Add 2

$5x > 10$

$\frac{1}{5}(5x) > \frac{1}{5}(10)$ Multiply by positive $\frac{1}{5}$

$x > 2$

The solution is $x > 2$.

Example 6 Solve: $6 - 5x > 3 - 4x$

Solution:

$6 - 5x > 3 - 4x$	
$6 - 6 - 5x > 3 - 6 - 4x$	Subtract 6
$-5x > -3 - 4x$	
$-5x + 4x > -3 - 4x + 4x$	Add 4x
$-x > -3$	
$(-1)(x) < (-1)(-3)$	Multiply by -1 and reverse inequality
$x < 3$	

The solution is x < 3.

When an inequality involves grouping symbols, remove all grouping symbols, collect like terms (if such exist) on each side, and proceed as in previous examples.

Example 7 Solve: $x - 3(2 + x) > 2(3x - 1)$

Solution:

$x - 3(2 + x) > 2(3x - 1)$	
$x - 6 - 3x > 6x - 2$	Remove parentheses
$-2x - 6 > 6x - 2$	Collect like terms
$-2x - 6 + 6 > 6x - 2 + 6$	Add 6
$-2x > 6x + 4$	
$-2x - 6x > 6x - 6x + 4$	Subtract 6x.
$-4x > 4$	
$(-\frac{1}{4})(-4x) < (-\frac{1}{4})(4)$	Multiply by $-\frac{1}{4}$ and reverse the inequality.
$\mathbf{x < -1}$	

The solution is x < -1.

We conclude this section by summarising the method to solve linear inequalities.

TO SOLVE A LINEAR INEQUALITY

1. Simplify both sides by removing grouping symbols and collecting like terms. Fractions or decimals may be eliminated by multiplying both sides by an appropriate factor (the LCD for fractions or a power of 10 for the decimals).
2. Use the addition-subtraction rule to isolate all variable terms on one side and all constant terms on the other side. Collect like terms where possible.
3. Use multiplication–division rule to obtain a variable with coefficient of 1. Remember to reverse the symbol of inequality when multiplying or dividing by a negative number.

Now do the learning activity.

LEARNING ACTIVITY 12.4.1.2

20 minutes

A. Are the following statements true or false?

1. If $x > 9$ then $-3x > -27$. ________
2. If $x \geq 8$ then $x - 1 \geq 7$. ________
3. If $x < 4$ then $3x < 12$. ________
4. If $x < -2$ then $-x > 2$. ________
5. If $x < 3$ then $x + 5 < 8$. ________
6. If $x > -1$ then $3 - 2x < 5$. ________

B. Solve the following inequalities.

1. $x + 3 < 7$

2. $5 - 4x > 2 - 3x$

3. $2z + 3 > 5z - 3$

4. $5(3 - x) - 10 \leq 25$

5. $3(2x + 8) \leq 4(x - 3)$

C. Letting x represent the unknown number, translate the following into symbols and solve.

1. Twice a number is greater than 8.

2. If 3 is subtracted from a number, the result is more than 20.

3. If twice a number is diminished by 8, the result is at least 12.

12.4.1.3: Word Problems

The main reason for studying algebra is to equip yourself with the tools necessary to solve problems. Most problems are expressed in words. In this section, you will learn the methods for solving some traditional word problems. The skills learned in this section can be applied to solving mathematical problems encountered in many fields of learning as well as in real life situations.

Solving applied problems using algebra involves two steps. First, translate the words of the problem into algebraic equation. Second, solve the resulting equation. We learned how to solve several types of equations in Section 12.4.1 and now we concentrate on translating words into equations and solving word problems.

Some common terms and their symbolic translations are presented below.

Symbol	Stands for
+	and, sum, plus, sum of, added to, increased by, more than, greater than, larger than, expand
–	minus, less, subtracted from, less than, diminished by, difference between, difference, decreased by, smaller than, ago, reduced, take from, depreciate
•	times, product, product of, multiplied by, of, twice, multiply
÷	divided by, quotient of, ratio of
=	equals, is equal to, is as much as, is the same as, is

Any letter (we often use x) is used to stand for the unknown or desired quantity.

Here are some examples of translation.

a number	increased by	5	is	13
↓	↓	↓	↓	↓
x	+	5	=	13

two subtracted from a number is 5

$x - 2 = 5$

twice my age in two years will be 20

$2 \quad (x + 2) \quad = \quad 20$

four times a number diminished by 12 is as much as twice the number

$4 \quad \bullet \quad x \quad - \quad 12 \quad = \quad 2x$

Although **x** is often used as the variable in an applied problem, it is sometimes helpful to use a different letter more indicative of the quantity it represents. For example, we might use **V** for volume, **A** for area, **s** for salary, **t** for time or **p** for price.

Example 1

Select a variable to represent each quantity and translate the phrase into symbols.

Word phrase	Symbolic translation
(a) Twice a natural number	2x
(b) The time in six hours	t + 6
(c) My salary less K200 for taxes	s - 200
(d) One-third the volume	$\frac{1}{3}V$

Example 2

Use **x** for the variable and translate each word sentence into symbols.

Word expression	Symbolic translation
(a) The product of a number and 3 is 22.	3x = 22
(b) Four times a number is 8.	4x = 8
(c) Eight more than a number is 12.	x + 8 = 12
(d) One-tenth of a number is 6.	$\frac{1}{10}x = 6$

It is necessary to be able to read word problems with understanding and to translate them into algebraic symbols correctly. The following suggestions will help you to set up equations for solving word problems.

To solve word problems:

1. Read the problem very carefully to determine what is unknown. What is being asked for? **Do not** try to solve the problem at this time.
2. Represent one unknown number by a letter.
3. Reread the entire word problem, breaking it up into small pieces that can be represented by algebraic expressions.
4. After each of the pieces has been written as an algebraic expression, fit them together into an equation.
5. Solve the equation for the unknown letter by the methods learned in Unit 12.4.1.1.
6. Check the solution in the word problem not with the equation.

Following on the next pages are sample problems with their corresponding solutions.

Number Relation Problems

Problems that deal with relationships among consecutive integers, numerators and denominators of fractions and percentages of a number of objects or people are examples of number problems.

Consecutive integers are whole numbers which differ by one. Examples are 3, 4, 5 or 7, 8, 9. In general if x is an integer, the next consecutive integer is $x + 1$, and the next consecutive integer after it is $x + 2$. We can also consider consecutive even or odd integers such as 4, 6, 8 or 9, 11, 13. In these cases, if x is an integer, the next consecutive even or odd integer is $x + 2$, and the next consecutive even or odd after it is $x + 4$.

Example 1

The sum of two consecutive numbers is 47. Find the integers.

Solution: Let x = the first integer

$x + 1$ = the next consecutive integer

$x + x + 1$ = their sum

47 = given sum

Hence, $x + x + 1 = 47 \longleftarrow$ the equation

Solve $2x + 1 = 47$

$2x = 46$

$x = 23$ the first integer

$x + 1 = 24$ the second integer

The integers are 23 and 24.

Check: 23 and 24 are indeed consecutive integers which add up to 47

Example 2

The sum of two consecutive even integers is 94. Find the numbers.

Solution: Let x = the first even integer

x + 2 = the second even integer

x + x + 2 = their sum

94 = the given sum

Hence, x + x + 2 = 94 ← the equation

Solve 2x + 2 = 94

2x = 92

x = 46 the first even integer

x + 2 = 48 the second even integer

The integers are 46 and 48.

Check: 46 and 48 are indeed consecutive even integers which add up to 94.

Example 3

Three times an unknown number decreased by five is thirteen. What is the number?

Solution: Let x the unknown number

Three	times	an unknown number	decreased by	five	is	thirteen.
3	•	x	–	5	=	13

Form the equation: $3x - 5 = 13$

Solve the equation: $3x = 18$ add 5 to both sides

$x = 6$ divide both sides by 3

The number is 6.

We broke the word problem into small pieces, and then represented each pieces by an algebraic expression. This formed the equation.

Check:

Three	times	an unknown number	decreased by	five	is	thirteen
3	•	6	–	5	=	13

$3(6) - 5 = 13$ the unknown number was replaced by 6.

$18 - 5 = 13$

$13 = 13$

Age Problem

To find the age of any person n years ago we subtract n from his/her present age. To find his/her age n years from now we add n to his/her present age. Suppose a person is x years old now, then $x - n$ is his/her age n years ago. Also $x + n$ will be his/her age n years from now or n years hence.

Example 4

Ten years ago Baro was twice as old his sister Kira. In eight years the sum of their ages will be 63. Find the present age of each.

Note: It will be helpful to arrange the data of the problem in table form. Represent Kira's age ten years ago by x.

Solution:

	Age 10 years ago	Present age	Age in 8 years
Baro	2x	2x + 10	(2x + 10) + 8
Kira	x	X + 10	(x + 10) + 8

The equation is: $(2x + 10) + 8 + (x + 10) + 8 = 63$

$$2x + 10 + 8 + x + 10 + 8 = 63$$
$$2x + 18 + x + 18 = 63$$
$$3x + 36 = 63$$
$$3x = 63 - 36$$
$$3x = 27$$
$x = 9$ age of Kira 10 years ago

$2x = 2(9) = 18$ age of Baro 10 years ago

Therefore Kira's age at present is x + 10 = 9 + 10 = 19 years.

Baro's age at present is 2x + 10 = 2(9) + 10 = 18 + 10 = 28 years.

Check: In 8 years the sum of their ages is 63.

$$(19 + 8) + (28 + 8) = 63$$
$$27 + 36 = 63$$
$$63 = 63$$

Investment Problems

Investment problems are usually solved by the use of the formula $I = prt$ where I represents the number of Kina in the interest; **P** represents the number of kina in the Principal; **r** represents the rate of interest; and **t** represents the time or the number of years the principal is invested.

Example 5

How should K15 600 be invested so that the income from one investment at 10% will be twice the income from another investment at 12%?

Solution: Let x = amount to be invested at 10%

K15 600 – x = amount to be invested at 12%

Then, 0.10x = income from the 10% investment

0.12(K15 600 – x) = income from the 12% investment

Thus, the equation is: 0.10x = 2[0.12(K15 600 – x)]

0.10x = 2(K1 872 – 0.12x)

0.10x = K3 744 – 0.24x

0.10x + 0.24x = K3 744

0.34x = K3 744

x = K11 012 to the nearest kina

K15 600 – x = K15 600 – K11 012

= K4 588

Therefore, K11 012 should be invested at 10% and K4 588 at 12%.

Work Problems

These problems require us to solve the amount of time that a single piece of work can be done together by two or more people or machines if their individual rates to do the same work are given.

Example 6

Two pipes A and B operate independently at their respective constant rates. Pipe A alone takes 5 hours to fill a tank. When pipes A and B are used simultaneously, it takes them 2 hours to fill the tank. How long will it take pipe B alone to fill the tank?

Solution: Let x = number of hours it would take B working alone to fill the tank.

$\frac{1}{x}$ = part of the tank filled by B in 1 hour

$\frac{1}{5}$ = part of the tank filled by A in 1 hour

$\frac{1}{2}$ = part of the tank filled by A and B in 1 hour

Hence, the equation is $\quad \frac{1}{x} + \frac{1}{5} = \frac{1}{2}$

$$10x\left(\frac{1}{x} + \frac{1}{5}\right) = \left(\frac{1}{2}\right)10x$$ Multiply both sides by the LCD (10x).

$$10 + 2x = 5x$$

$$10 = 5x - 2x$$

$$10 = 3x$$

$$\frac{10}{3} = \frac{3x}{3}$$

$$3\frac{1}{3} = x \text{ or } x = 3\frac{1}{3}$$

Therefore, pipe B alone will take $3\frac{1}{3}$ hours to fill the tank.

Motion Problems

In motion problems we assume that the objects such as cars, trains, planes, or boats travel at uniform speeds. The basic formula for this type of problem is given by

$$D = rt, \quad \text{or } r = \frac{D}{t} \text{ or } t = \frac{D}{r}$$

Where D is the distance travelled, r is the rate or speed of the moving object and t is the time it takes for the object to cover the distance.

Below is a list of common situations where the formulas are applicable:

1. An object travels different distances at different rates in different times. Find the average rate in covering the total distance.
2. Two objects travel at different rates toward each other from different points of origin. Find the time when they meet.
3. Two objects travel at different rates away from each other from the same point of origin. Find the time when they are given distance apart or find how far apart they are after a given time.
4. An object starts to travel from a point. Then later, another object starts to travel from the same point along the same route. Find the distance covered between the point of origin and the point where they meet.

Example 7

Two cars A and B with average rate of 50 km/h and 60 km/h, respectively, leave two stations 770 km away at 8:00 a.m. and move toward each other on parallel tracks. At what time will they meet?

Solution: Let t = number of hours each car travels before they meet

Using the idea of the distance formula Distance = (rate)(time) or D = rt and in the table below, we get:

Car	Rate in km	Time in hours	Distance in km
Car A	50 km/h	t	50t
Car B	60 km/h	t	60t

Since the stations are 770 km apart we have:

Equation: Distance travelled by car A + Distance travelled by car B = 770 km

$$50t + 60t = 770$$

$$110t = 770$$

$$t = 7 \text{ hours}$$

Counting 7 hours from 8:00 a.m. is 3:00 p.m.

Therefore, the two cars will meet at 3:00 p.m.

Example 2

Two runners leave the Sir John Guise stadium at the same time and run in opposite directions. The speed of the faster runner is 90 metres per minute faster than the slower runner. At the end of 30 minutes they are 6 000 metres apart. Find the rate of each runner per minute.

Solution: Let x = rate of the slower runner
x + 90 = rate of the faster runner

Using the idea of the distance formula Distance = (rate)(time) or D = rt and in the table below, we get:

Runner	Rate in m/min	Time in minutes	Distance in m
Slower	x	30	30x
Faster	x + 90	30	30(x + 90)

Equation: Since they are 6 000 m apart, we have:

Rate of slower runner + rate of faster runner = 6 000

$$30x + 30(x + 90) = 6\,000$$
$$30x + 30x + 2\,700 = 6\,000$$
$$60x + 2\,700 = 6\,000$$
$$60x = 6\,000 - 2\,700$$
$$60x = 3\,300$$
$$x = 55$$

$x = 55$ m/min, rate of slower runner

$x + 90 = 55 + 90$

$= 145$ m/min, rate of faster runner

Check:

$$30x + 30(x + 90) = 6\,000$$
$$30(55) + 30(55 + 90) = 6\,000$$
$$1\,650 + 30(145) = 6\,000$$
$$1\,650 + 4\,350 = 6\,000$$
$$6\,000 = 6\,000 \quad \text{true}$$

Therefore, the rate of slower runner is 55 m/min and the rate of the faster runner is 145 m/min.

Now do the learning activity.

LEARNING ACTIVITY 12.4.1.3

20 minutes

1. Select a variable to represent each quantity and translate the phrase.

   a) The time 4 hours ago

   b) His wages plus 10%

   c) Twice the surface area

   d) K1 000 less than the cost

2. Problem Solving

a) If three times a number is increased by 2, the result is the same as 20 less than five times the number. Find the number.

b) Peter is 9 years older than John. If the sum of their ages is 87, how old is each?

c) Two cars leave the same town, one travelling east the other west. If one train is moving 10 km per hour faster than the other, and if after two hours they are 260 kilometers apart, how fast is each travelling?

d) What amount of money invested at 15% interest will increase to K1140 at the end of 6 years?

e) It takes 3 days for Mr. Kua and his son to plow a field. Mr. Kua can finish the work in 5 days. How long would it take the son to do the work alone?

f) Find three consecutive numbers such that one-fourth of the first, plus one-third of the second, and plus one-fifth the third is equal to 7.

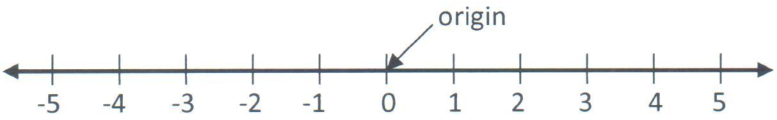
g) A steel rod is 17 meters long. It is to be cut into two pieces in such a way that one is seven meters longer than the other. How long is each piece?

12.4.1.4: Graphical Solution of Linear Equations and Inequalities with One Variable

Earlier in this topic, we solved linear equations and inequalities in one variable. In this section, we will plot the graphs of the solution of such equations and inequalities. We did this to some degree in your earlier study using the number line.

The solution set can only contain numbers from the **domain** of the variable. If the domain of the variable is not mentioned, we assume it to be the set of real numbers.

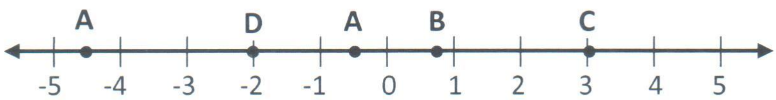
Recall that a **number line** is a line that has been marked off in unit lengths (any unit will do) with each point on the line associated with a real number, and vice versa. Below is a number line with the **origin** (point associated with the number zero) as identified.



The positive numbers correspond to points to the right of the origin, while the negative numbers correspond to points to the left of the origin. We think of this line as extending infinitely far in both directions and indicate this by the arrows. Thus, no matter what number we consider, we can always associate it with a point on the line and give any point on the line, a real number can be identified with it.

Example 1

In the number line below, **A** corresponds to the number $-\frac{1}{2}$ (A is halfway between 0 and -1), **B** to $\frac{3}{4}$, **C** to 3, **D** to -2 and **E** to $-\frac{9}{2}$. To find the point associated with the number 12, we extend the line to the right and continue marking unit length until the desired position is reached.



Now we consider graphing equations and inequalities.

1. To graph an equation, we first solve the equation, and then find the point (or points) on a number line which corresponds to the solution (or solutions) of the equation. We call this procedure **plotting points.** Thus, graphing an equation in one variable requires only a single number line.

Example 2

Graph: $3x - 1 = 8$

Solving, we obtain $3x = 9$

$x = 3$

The solution is $x = 3$. Plot the point corresponding to 3 on the number line, as in the figure below.

Example 3 Graph: $x + 1 = x - 1$

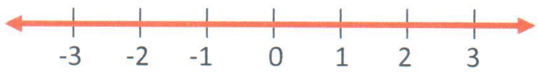
Solving, we obtain	$x - x + 1 = x - x - 1$	Subtract x
	$1 = -1$	A contradiction

There are no solutions and the graph is the plain number line below.

Example 4 Graph: $4x + 1 = l + 4x$

Solving, we obtain	$4x - 4x + 1 = 1 + 4x - 4x$	Subtract 4x
	$1 = 1$	An identity

Every real number is a solution and the graph is as shown below.

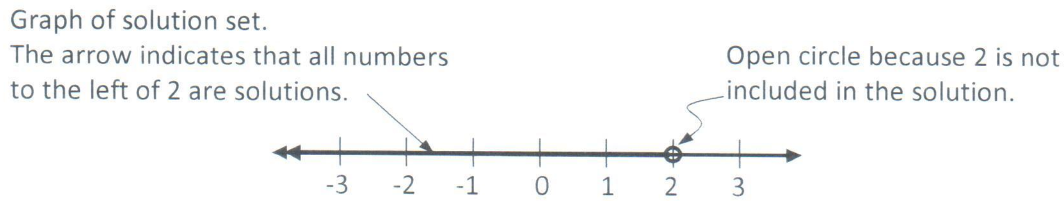


2. To graph an inequality, first we solve the inequality and then and plot the solution set on the number line. The graph of an inequality shows all the numbers that satisfy the inequality. When graphing inequalities on a number line, use solid circles (•) for ≤ and ≥ and open circles (⊙) for < and >.

Example 5 Graph: $5\,x + 3 < 13$

Solving, we obtain	$5x + 3 - 3 < 13 - 3$	Subtract 3
	$5x < 10$	Divide both sides by 5
	$x < 2$	

The solution is $x < 2$ and the graph is shown below.



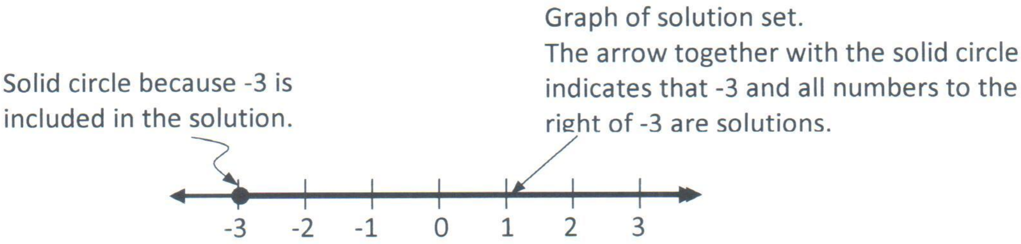
Example 6 Graph: $3x - 2 \geq -11$

Solving, we obtain $3x - 2 + 2 \geq -11 + 2$ Add 2 to both sides

$3x \geq -9$ Divide both sides by 3

$x \geq -3$

The solution is $x \geq -3$ and the graph is shown below.



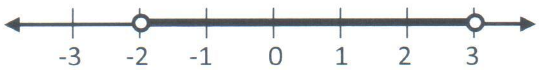
We sometimes have two inequality symbols in the same statement. For example, $-2 < x < 3$ In such combined inequalities, we actually have two inequalities.

$x > -2$ and $x < 3$

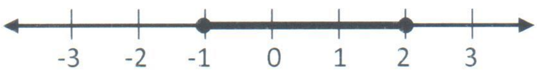
which could also be written as

$-2 < x$ and $x < 3$

The number that satisfy $-2 < x < 3$ are solutions to both those numbers which are both greater than -2 and less than 3. The graph is shown below.



Similarly the graph of $-1 \leq x \leq 2$ ($-1 \leq x$ and $x \leq 2$) is shown below.



Also the graph of $-3 \leq x < 0$ ($-3 \leq x$ and $x < 0$) is shown below.

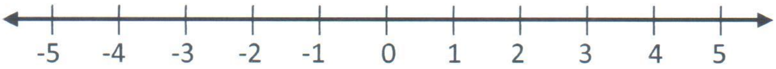
Now do the learning activity on the next page.

LEARNING ACTIVITY 12.4.1.4

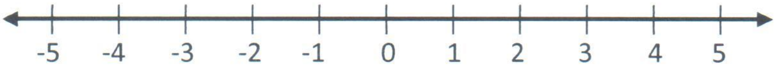
20 minutes

A. Graph the equation on the number line.

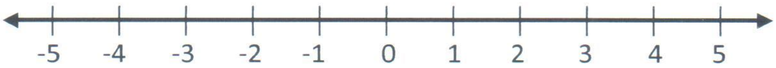
1. $3x - 1 = 5$



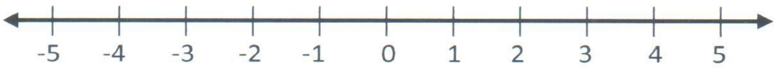
2. $2x + 5 = 5 + 2x$



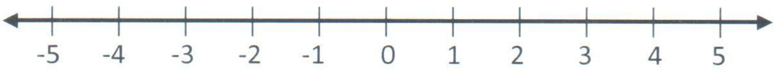
3. $2x + 1 = 5x + 1$



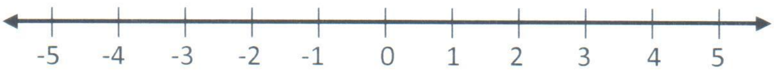
4. $(2x - 1) - (3x + 2) = 0$



5. $(1 - 2x) - (1 - x) = 2$

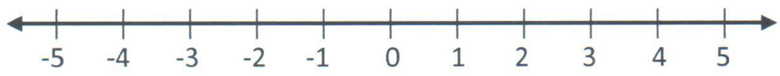


6. $4 - 2[x - (1 + x)] = 2x$



B. Graph the inequality or combined inequality on a number line.

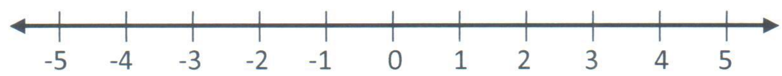
1. $x > 4$



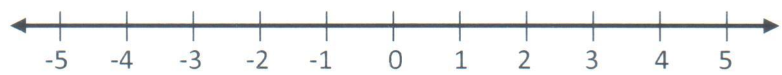
2. $x \leq -2$

3. $2x - 5 < -1$

4. $3 - 5x \geq -7$

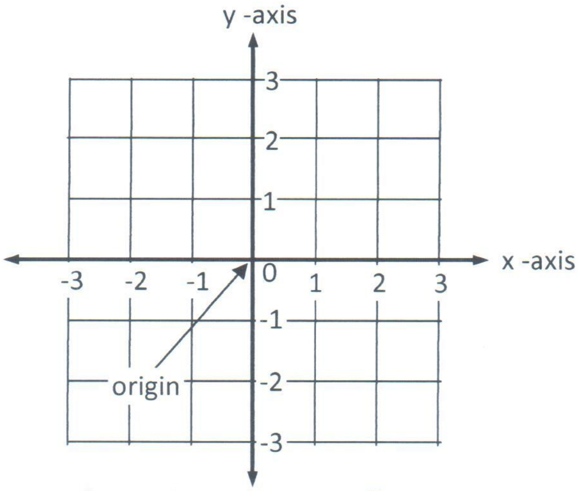


5. $x > -3$ and $x < 0$



12.4.1.5 Graphing Linear Equations with Two Variables

In Section 12.4.1.3 we graphed equations and inequalities in one variable by plotting the points corresponding to solutions on a number line. Now we will develop a system in which equations and inequalities in two variables can be graphed. When a horizontal number line and a vertical number line are placed together as in the figure below so that the two origins coincide and the lines are perpendicular, the resulting configuration is called a **rectangular** or **Cartesian coordinate system** (named after French mathematician Rene Descartes) or a **coordinate plane**.



Rectangular or Cartesian Coordinate System (Coordinate Plane)

The horizontal number line is called the **horizontal axis** or **x-axis** and the vertical number line is called the **vertical axis** or **y-axis**. The point of intersection of the axes is called the **origin**. The axes divide or separate the plane into four sections called **quadrants**. The first, second, third and fourth quadrants are identified by the Roman numerals I, II, III, and IV, respectively, in the coordinate plane. (See Figure 1 below)

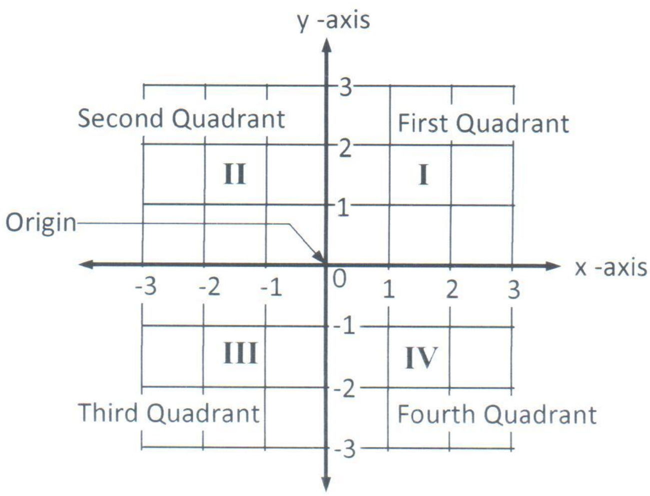


Figure 1

The graph of a point

A point in a plane is represented by an ordered pair of numbers. The point (5, 4) is shown in the Figure 2 below.

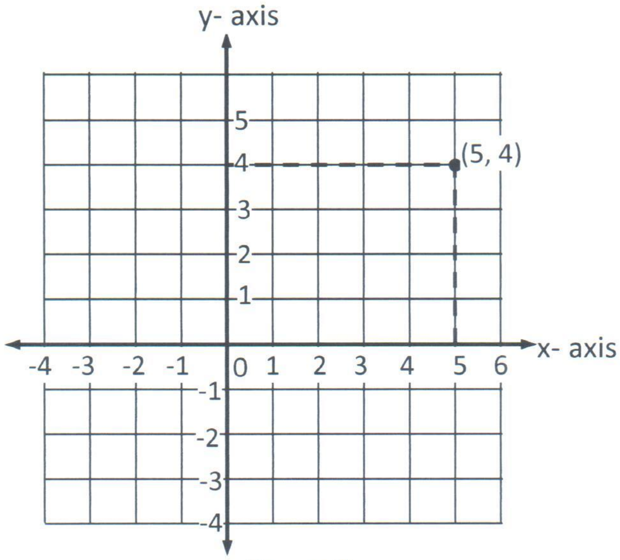


Figure 2

The first number (5) of the ordered pair is called the **horizontal coordinate** also called **abscissa** or **x-coordinate** of the point (5, 4). A positive horizontal coordinate indicates that the point is **to the right** of the vertical axis. A negative horizontal coordinate indicates that the point is **to the left** of the vertical axis.

The second number (4) of the ordered pair is called the **vertical coordinate** also called **ordinate** or **y-coordinate** of the point (5, 4). A positive vertical coordinate indicates that the point is **above** of the horizontal axis. A negative vertical coordinate indicates that the point is **below** the horizontal axis.

Note: When the order is changed in an ordered pair, we get a different point. For example (2, 5) and (5, 2) are two different points. (See Figure 3 below)

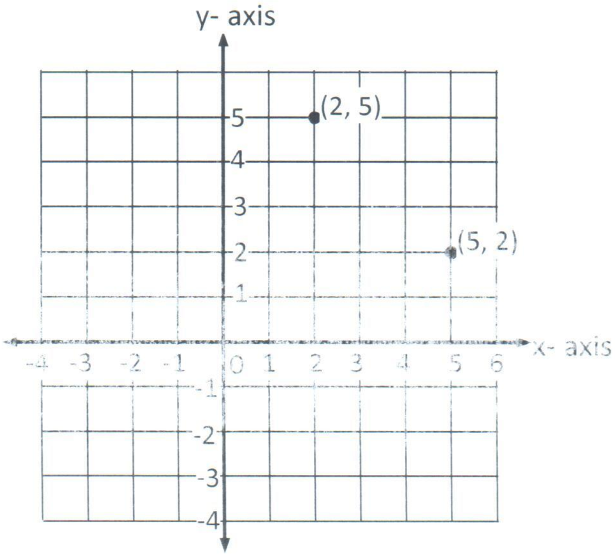


Figure 3

Graphing linear equations in two variables

Graphing linear equations that contain two variables requires a Cartesian coordinate system since the solutions to equations in two variables are ordered pair of numbers which when substituted for the variables result in a true equation.

For example, the equation $y = 3x - 2$

has infinitely many solutions. One solution is (2, 4) since if x is replaced with 2 and y is replaced with 4, the resulting equation is true. Carrying out this replacement we have

$y = 3x - 2$

$4 = 3(2) - 2$ X = 2 and y = 4

$4 = 6 - 2$

$4 = 4$

You can verify that (0, 2) and (-1, -5) are also solutions.

The graph of an equation with two variables x and y is the graph of the ordered pairs of numbers which make the equation true. Since there are usually infinitely many such ordered pairs, we cannot determine and plot each point. Generally we plot enough points to enable us to see a pattern, and then connect these points with a line or a curve to graph the equation.

One way to display a collection of solutions is to make a **table of values.** We assign several values for x, substitute these values into the equation and compute the corresponding values for y. We begin by making a table such as the one below. Each y-value we find is placed beside the x-value used to calculate it.

Example 1

Graph the equation $y = 3x - 2$

Solution:

Substitution	Result in y= 3x – 2
$x = 0$	$y = 3(0) - 2 = \mathbf{-2}$
$x = 1$	$y = 3(1) - 2 = 3 - 2 = \mathbf{1}$
$x = -1$	$y = 3(-1) - 2 = -3 - 2 = \mathbf{-5}$
$x = 2$	$y = 3(2) - 2 = 6 - 2 = \mathbf{4}$
$x = -2$	$y = 3(-2) - 2 = -6 - 2 = \mathbf{-8}$
$x = 3$	$y = 3(3) - 2 = 9 - 2 = \mathbf{7}$
$x = -3$	$y = 3(-3) - 2 = -9 - 2 = \mathbf{-11}$

x	y
0	-2
1	1
-1	-5
2	4
-2	-8
3	7
-3	-11

Now we plot the points that correspond to these ordered pairs in a Cartesian coordinate system. (See the figures 4 and 5 below).

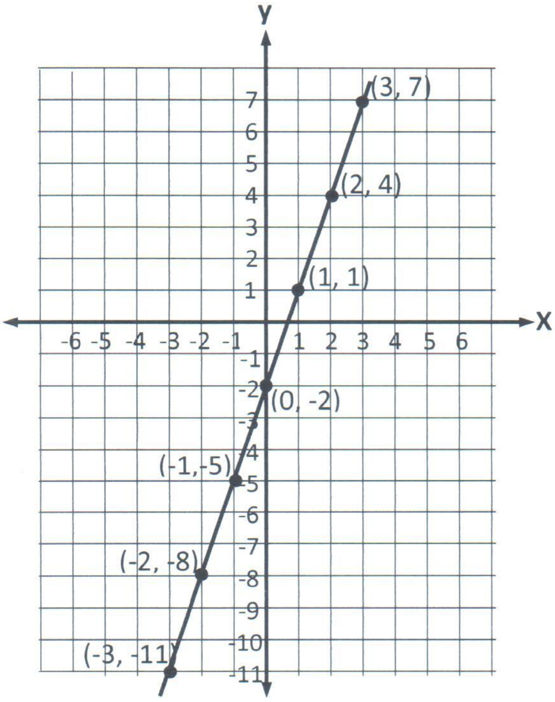


Figure 4

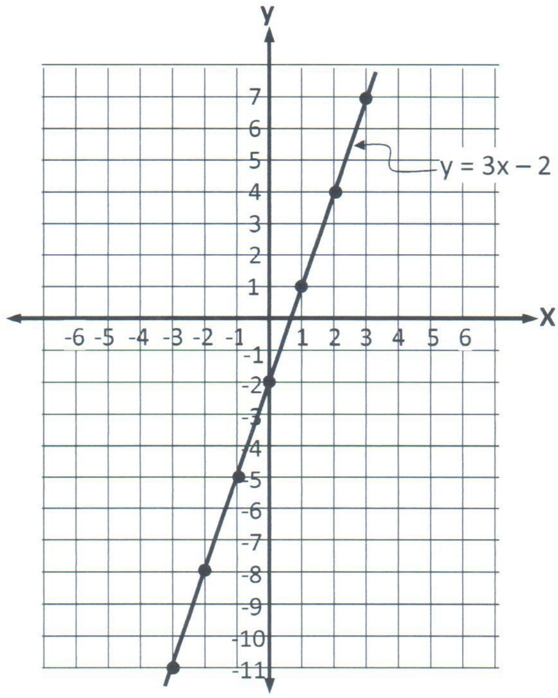


Figure 5

It appears that all the points lie on a straight line. Thus, it is reasonable to assume that the graph of the equation y = 3x – 2 is the straight line passing through these seven points in Figure 5.

TO GRAPH AN EQUATION IN THE TWO VARIABLES X AND Y
1. Make a table of values. These values represent the ordered pair solutions of the equation. 2. Plot the points which correspond to these solutions in a Cartesian coordinate system. 3. Connect the points to form a straight line.

Earlier, we graphed equations in one variable on a number line. Generally, for graphing an equation in one variable such as

$$y = 4 \qquad \text{or} \qquad x + 3 = 0$$

is thought of as an equation in two variables with the coefficient of the missing variable equal to zero. That is,

$$y = 4 \text{ is the same as } y = 0(x) + 4$$

and

$$x + 3 \text{ is the same as } x + y(0) + 3 = 0.$$

With this in mind, such equation can be graphed in a Cartesian coordinate system.

Example 2 Graph y = 4 in a Cartesian coordinate system.

Solutions to this equation always have a y-coordinate of 4 and can have any number as x-coordinate. For example: (5, 4), (0, 4) and (-1, 4) are all solutions since y =3 is the same as y = 0(x) + 4, and

$$0 \bullet (\text{any number}) + 4 = 4$$

Plot the seven points given in the table of value and draw the line through them. The graph of y = 4 in Figure 6 is a straight line parallel to the x-axis and 4 units above it.

Table of values for y = 4

x	y
0	4
1	4
-1	4
2	4
-2	4
3	4
-3	4

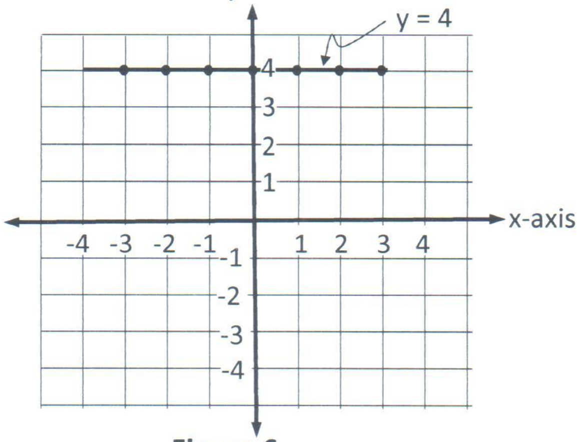


Figure 6

Example 3 Graph x + 2 = 0 in a Cartesian coordinate system

Solutions to this equation always have an x-coordinate of -2 and can have any number as y-coordinate. For example: (-2, 0), (-2, 1) and (-2, -1) are all solutions since x + 2 = 0 is the same as x + 0(y) + 2 = 0, and

$$-2 + 0 \bullet (\text{any number}) + 2 = -2 + 2 = 0$$

Plot the seven points given in the table of value and draw the line through them. The graph of x + 2 = 0 (x = -2) in Figure 7 is a straight line parallel to the y-axis and 2 units to its left.

x	y
-2	0
-2	1
-2	-1
-2	2
-2	-2
-2	3
-2	-3

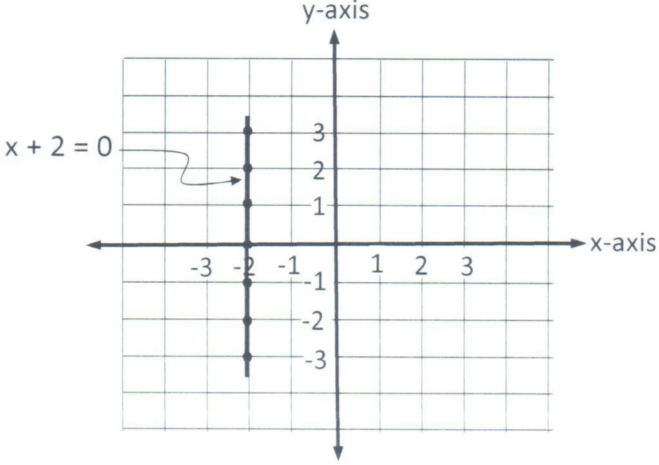


Figure 7

A linear equation in two variables has as its graph a straight line. It is easy to recognize a linear equation since the variables are raised to the first power only and it can always be written in the **general form**

$$\boxed{ax + by + c = 0}$$

where **a**, **b**, and **c** are real number constants, **a** and **b** both not zero. (If a = 0 and b = 0, the equation contains no variable, and it will be true if **c** is zero and false otherwise.)

Linear equations are also called **first degree equations** since the variables occur to the first power only.

Knowing that the graph of a linear equation is a **straight line**, we need only two solutions to graph it. In most cases, the two pairs which are easiest to determine are the **intercepts**, the points at which a line crosses the x and y axes. See Figure 8.

- The point at which the line crosses the x- axis is called **x-intercept**.
- The point at which the line crosses the y- axis is called **y-intercept**.

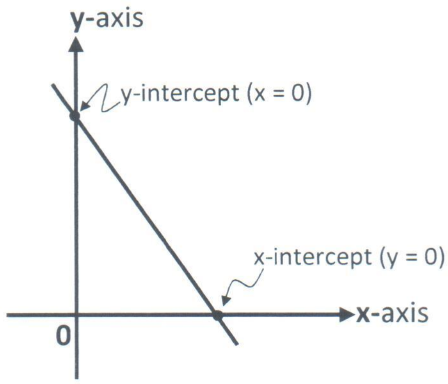


Figure 8

The intercept method of graphing a straight line

Example 1 Graph the equation 3x + 2y = 6.

Solution: Find the x- intercepts:

Set y = 0.

Then 3x + 2y = 6 becomes 3x + 2(0) = 6

$3x = 6$

$x = 2$

Therefore the x-intercept is (2, 0).

Table of values

x	y
2	0

Sometimes we say: "The x-intercept is 2" (the x-coordinate of the point where the line crosses the x-axis, instead of the point itself).

Find the x- intercept.

Set x = 0.

Then $3x + 2y = 6$ becomes $3(0) + 2y = 6$

$$2y = 6$$

$$y = 3$$

Therefore the x-intercept is (0, 3).

Table of values

x	y
2	0
0	3

Sometimes we say: "The y-intercept is 3" (the y-coordinate of the point where the line crosses the x-axis, instead of the point itself).

Plot the x- and y-intercepts (2, 0) and (0, 3). Then draw the straight line through them. See Figure 9.

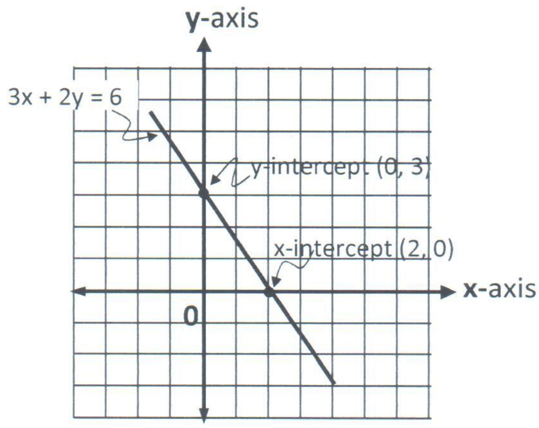


Figure 9

Example 2 Graph the equation $3x - 4y = 0$.

Solution: Find the x- and y- intercepts.

x-intercept: Set y = 0.

Then $3x - 4y = 0$ becomes $3x - 4(0) = 0$

$$3x = 0$$

$$x = 0$$

Therefore, the x-intercept is (0, 0) the origin.

y-intercept:

Since the line crosses the origin, the y-intercept is also (0, 0).

We have found only one point on the line: (0, 0).

Therefore, we must find another point on the line before we can draw the graph of $3x - 4y = 0$.

To find another point, we can set either of the letters equal to a number and then solve the equation for the other letter. For example:

Set y = 3.

Then $3x - 4y = 0$ becomes $3x - 4(3) = 0$

$$3x - 12 = 0$$
$$3x = 12$$
$$x = 4$$

This gives the point (4, 3) on the line. Plot the points (0, 0) and (4, 3). Then draw the straight line through them. (Figure 10)

Table of values

x	y
0	0
0	0
4	3

Both intercepts are the same point.

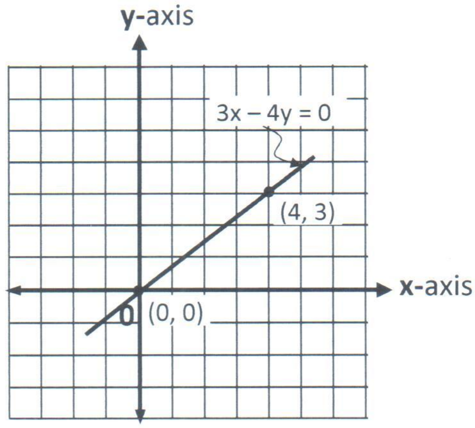


Figure 10

Some equations of a line have only one variable. Such equations have graphs that are either horizontal or vertical lines. (See Examples 2 and 3)

The method of graphing linear equations is summarized in the following box:

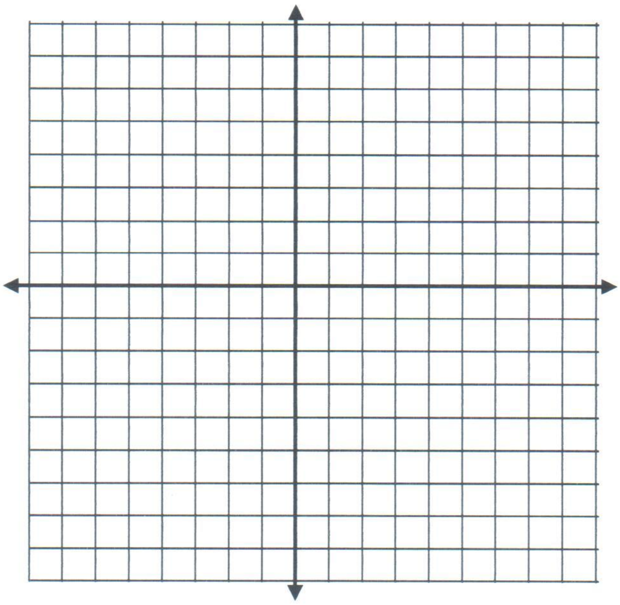
TO GRAPH A STRAIGHT LINE BY INTERCEPT METHOD

1. Find the x-intercept: Set y = 0, then solve for x.
2. Find the y-intercept: Set x = 0, then solve for y.
3. Plot the x- and y-intercepts.
4. Draw a straight line through the intercepts.
5. If the line goes through the origin (0, 0), another point must be chosen.
6. The graph of x = a is a vertical line (at x = a).
7. The graph of y = b is a horizontal line (at y = b).

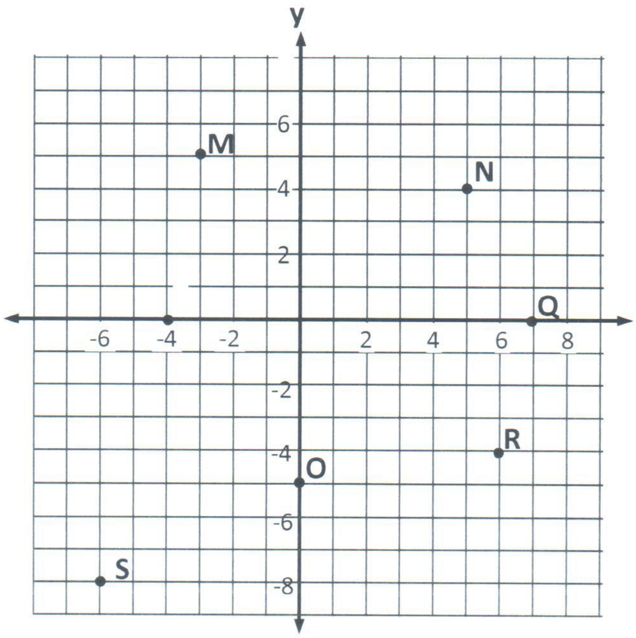
LEARNING ACTIVITY 12.4.1.5

20 minutes

1. Plot the points corresponding to the ordered pairs below.
   A(1, 5), B(-4, 3), C(5, 2), D(0, 1), E(-6, 0), F(0, -3), G(-6, -6), H(3, -4)

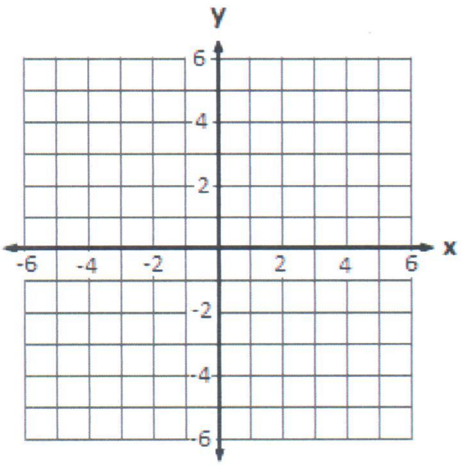


2. Identify the coordinates of the points M, N, O, P, Q, R, and S.

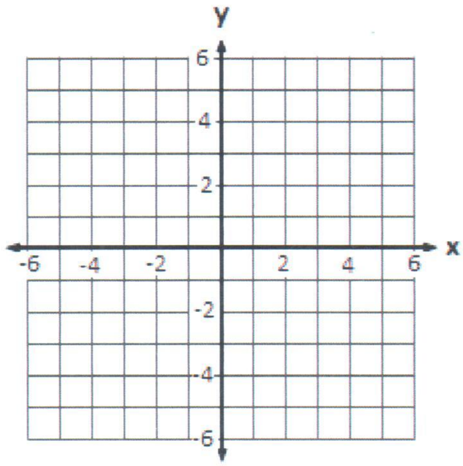


3. Determine the intercepts and graph the following.

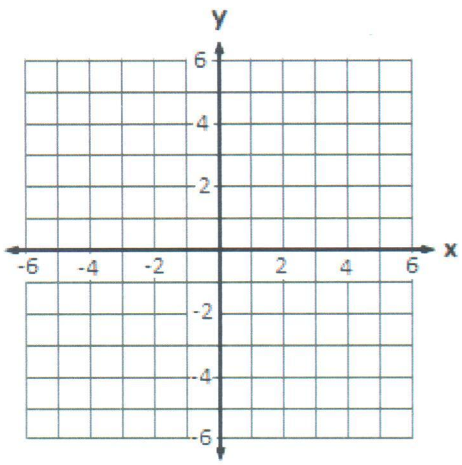
a. $x + y + 2 = 0$



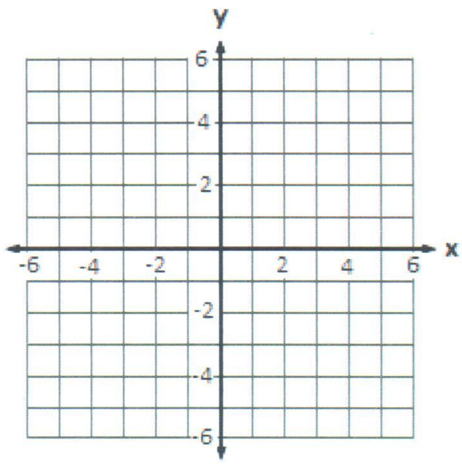
b. $3x + y = 0$



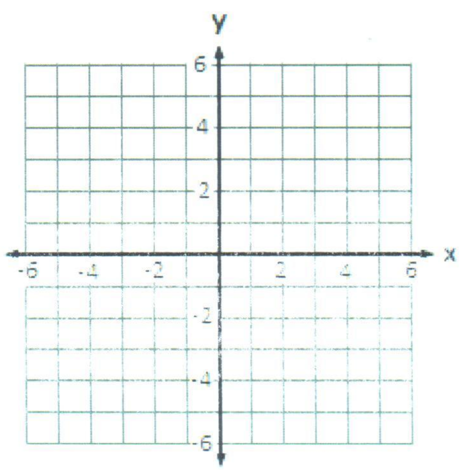
c. $x - y = 2$



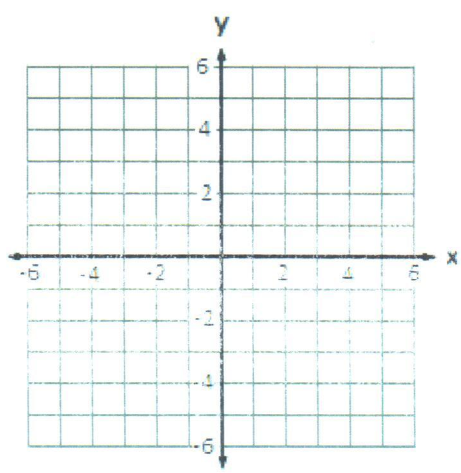
d. $3x - 2y = 0$



e. $y = -1$



f. $y = 0$



12.4.1.5 Graphing Linear Inequalities in Two Variables

We now consider inequalities in two variables in which the variables are raised only to the first power. Such inequalities, called **linear inequalities in two variables,** can always be written in one of the forms

$$ax + by > c, \qquad ax + by < c, \qquad ax + by \geq c, \text{ or } \qquad ax + by \leq c$$

where **a**, **b**. and **c** are real number constants, **a** and **b** not both zero. For example,

$$2x + 3y > 6$$

is a linear inequality in two variables x and y. A **solution** to such an inequality is an ordered pair (x, y) which makes the inequality true. Thus (-1, 4) is a solution to $2x + 3y > 6$ since

$$\begin{aligned} 2(-1) + 3(4) &> 6 \\ -2 + 12 &> 6 \\ 10 &> 6 \quad \text{is true.} \end{aligned}$$

However, (1, -4) is not a solution since

$$\begin{aligned} 2(1) + 3(-4) &> 6 \\ 2 - 12 &> 6 \\ -10 &> 6 \quad \text{is false.} \end{aligned}$$

Graphing First Degree Inequalities

Any first degree inequality (in no more than two variables) has a graph that is a **half-plane.** Any line in a plane divides that plane into two half-planes.

For example, in Figure 11, the line AB divides the plane into two half-planes shown.

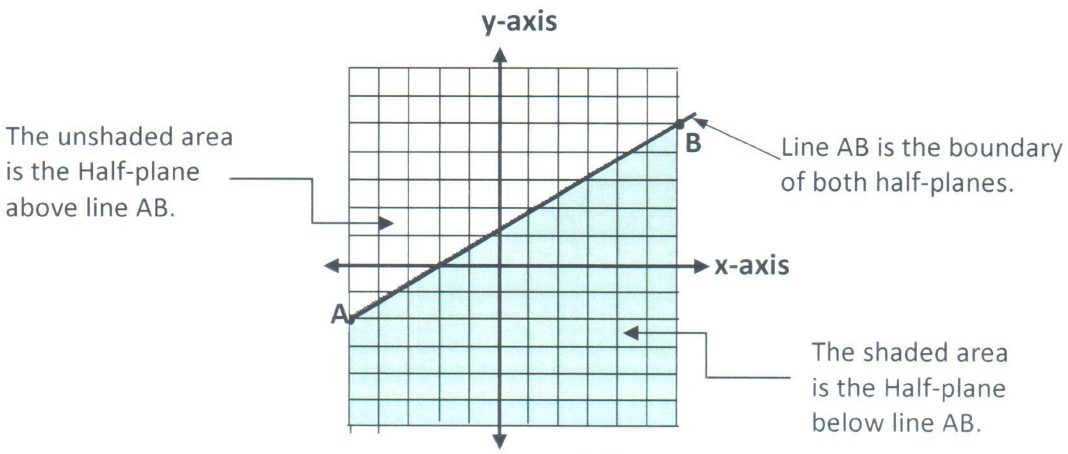
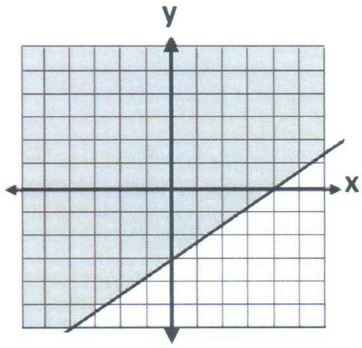


Figure 11

The equation of the boundary line is obtained by replacing the inequality sign by an equal sign.

How to determine when the boundary line is a dashed or a solid line.

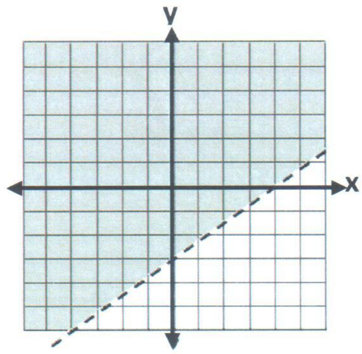
- The boundary line is a solid line when equality is included.
- The boundary line is a dashed line when equality is not included.



$3x - 4y \leq 12$

The boundary line is a solid line when equality sign is included. In this case, the boundary line is part of the solution.

Figure 12a



$3x - 4y < 12$

The boundary line is a dashed line when equality sign is not included. In this case, the boundary line is not part of the solution.

Figure 12b

Example 1 Graph the inequality $3x + 2y < 6$.

Solution: Change < to =.

Boundary line: $3x + 2y = 6$

1. The boundary line is a dashed line because the equality sign is not included.

$$3x + 2y < 6$$

2. Plot the graph of the boundary line by the intercept method.

x-intercept: Set $y = 0$. Then, $3x + 2(0) = 6$

$$3x = 6$$
$$x = 2$$

y-intercept: Set $x = 0$. Then, $3(0) + 2y = 6$

$$2y = 6$$
$$y = 3$$

Table of values

x	y
2	0
0	3

Therefore, the boundary line goes through (2, 0) and (0, 3).

3. Select the correct half-plane. The solution of the inequality is only one of the two half-planes determined by the boundary line. Substitute the coordinate of the origin into the inequality:

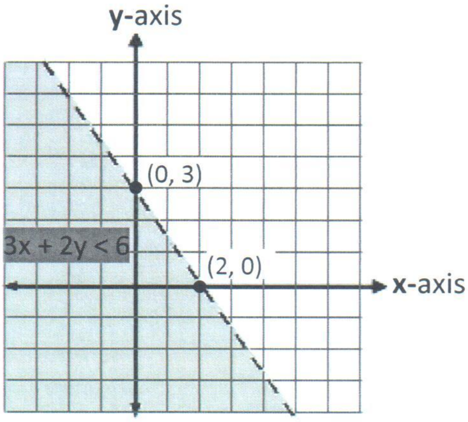
$$3x + 2y < 6$$

$$3(0) + 2(0) < 6$$

$$0 < 6 \textbf{ True}$$

Therefore the half-plane containing the origin is the solution. The solution is the shaded area in Figure 13.

Figure 13



How to determine the correct half-plane

1. **If the boundary line does not go through the origin,** substitute the coordinates of the origin (0, 0) into the inequality.
   - If the resulting inequality is true, the solution is the half-plane containing the origin.
   - If the resulting inequality is false, the solution is the half-plane not containing the origin.
2. **If the boundary line goes through the origin,** select a point not on the boundary. Substitute the coordinates of this point into the inequality.
   - If the resulting inequality is true, the solution is the half-plane containing the point selected.
   - If the resulting inequality is false, the solution is the half-plane not containing the point selected.

Example 2 Graph the inequality $3x + 4y \leq -12$.

Solution: Change $\leq$ to $=$.
Boundary line: $3x + 4y = -12$

1. The boundary line is a solid line because the equality sign is included.

$$3x + 4y \leq -12$$

2. Plot the graph of the boundary line intercepts method.

x-intercept: Set $y = 0$. Then $3x + 4(0) = -12$

$$3x = -12$$
$$x = -4$$

y-intercept: Set $x = 0$. Then $3(0) + 4y = -12$

$$4y = -12$$
$$y = -3$$

Table of values

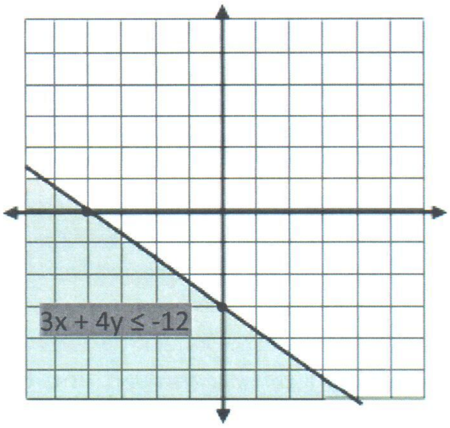
x	y
-4	0
0	-3

3. Select the correct half-plane. Substitute the coordinates of the origin to the inequality.

$$3x + 4y \leq -12$$
$$3(0) + 4(0) \leq -12$$
$$0 \leq -12 \quad \textbf{False}$$

Therefore, the solution is the half-plane not containing (0, 0), the origin.

Figure 14



Some inequalities have equations with only one variable. Such inequalities have graphs whose boundaries are either vertical or horizontal lines. See example 3.

Example 3 Graph the inequality $x + 4 < 0$.

Solution: Change < to =.
Boundary line: $x + 4 = 0$

1. The boundary line is a dashed line because the equality sign is not included.

$$x + 4 < 0$$

2. Plot the graph of the boundary line. $x + 4 = 0$
$x = -4$

(See Figure 15 on the next page.)

3. Select the correct half-plane. Substitute the coordinates of the origin (0, 0) to the inequality.

$$x + 4 < 0$$
$$0 + 4 < 0$$
$$4 < 0 \quad \textbf{False}$$

Therefore, the solution is the half-plane not containing (0, 0), the origin.

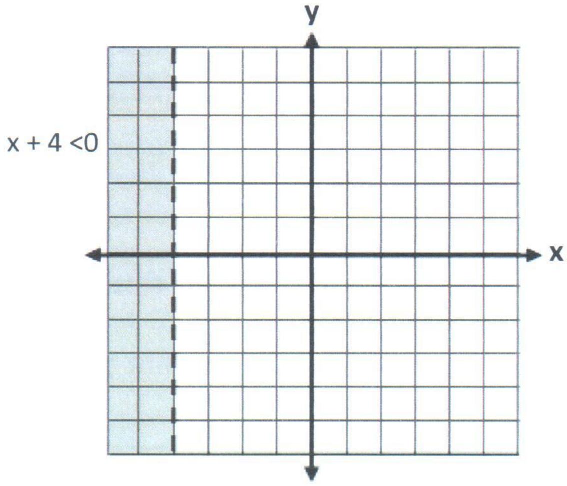


Figure 15

Example 4 Graph the inequality $y + 4 < 0$

Solution: Change < to =.

Boundary line: $y + 4 = 0$

1. The boundary line is a dashed line because the equality sign is not included.

$$y + 4 < 0$$

2. Plot the graph of the boundary line. $y + 4 = 0$
$y = -4$

(See Figure 16 on the next page.)

3. Select the correct half-plane. Substitute the coordinates of the origin (0, 0) to the inequality.

$$y + 4 < 0$$
$$0 + 4 < 0$$
$$4 < 0 \quad \textbf{False}$$

Therefore, the solution is the half-plane not containing (0, 0), the origin.

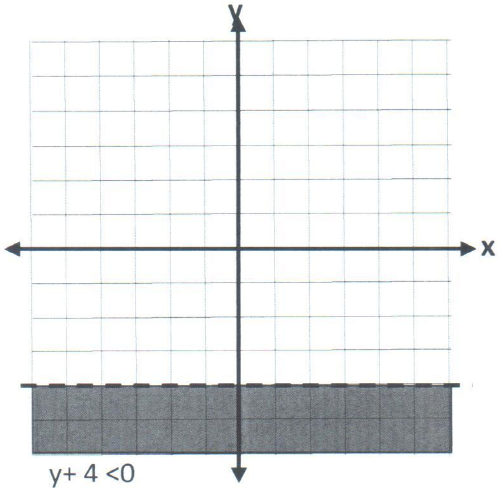


Figure 16

We summarize the techniques we have learnt.

To graph a linear inequality with two variables

1. Graph the boundary line using a dashed line if the inequality is < or > and a solid line if the inequality is ≤ or ≥.
2. Choose a test point which is not on the boundary line and substitute it into the inequality.
3. Shade the region which includes the test point if a true inequality is obtained, and shade the region which does not contain the test point if a false inequality results.

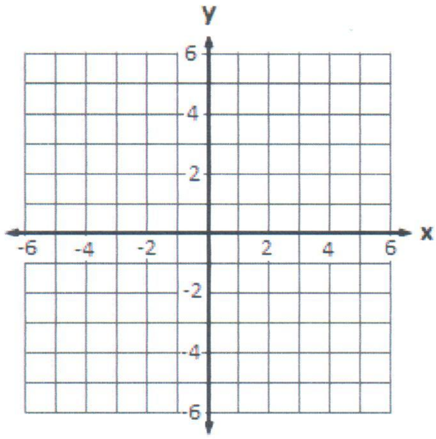
Now do the learning activity.

LEARNING ACTIVITY 12.4.1.6

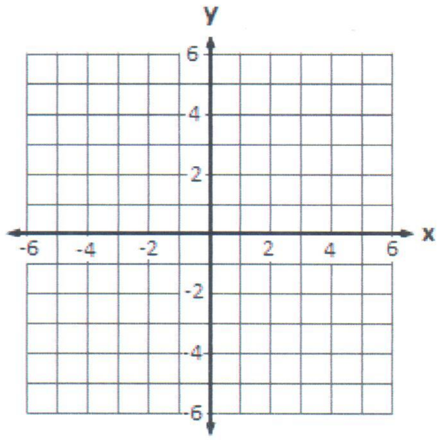
20 minutes

Graph the linear inequalities in two variables.

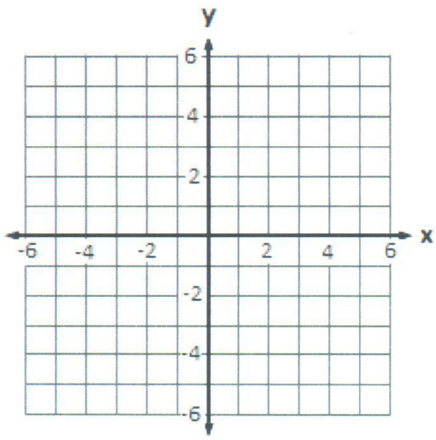
a. $x + y > 2$



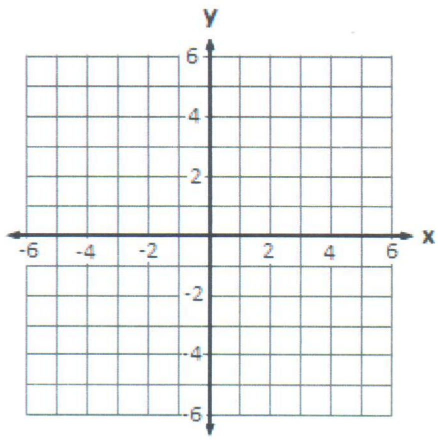
b. $x + y \leq 2$



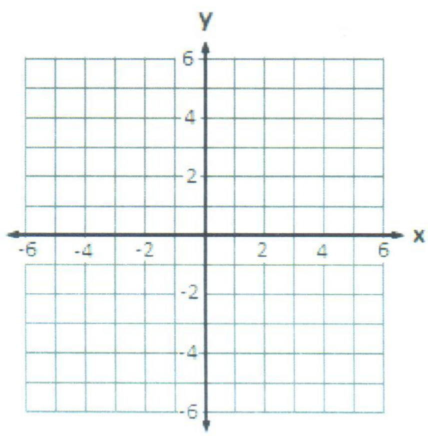
c. $4x + 8 > 0$



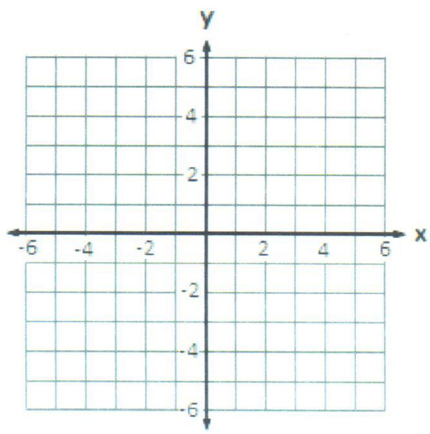
d. $4x + 8 \leq 0$



e. $3x + 4y < 12$



f. $3x + 4y \geq 12$



SUMMATIVE TASK 12.4.1

30 minutes

A. Choose the correct answer to each question from the alternatives A, B, C and D. Circle the letter of your answer.

1. Which equation has the same graph as that of 3x + 2y – 12 = 0?

A. $y = -\frac{3}{2}x + 6$ B. $y = \frac{3}{2}x - 6$

C. $y = -\frac{1}{3}x + 6$ D. $y = \frac{2}{3}x - 6$

2. What value of x will satisfy the equation 3(x – 7) + 1 = 6x + 13?

A. 11 B. -11

C. 3 D. -3

3. Which of the following is a solution to the equation 2x + 9y – 15 = 0?

A. (3, -1) B. (-3, 1)

C. (3, 1) D. (-3, -1)

4. Evaluate the expression $\frac{y + w}{x - w}$, when x = 2, y = -1 and w = 3.

A. -4 B. -2

C. 2 D. 4

5. If x is an integer and 3x > 40, then x cannot be

A. 12 B. 13

C. 14 D. 15

6. Which of the following has a graph which is a vertical line?

A. y + 3 = 2 B. 3x + 2y – 5 = 0

C. 2x = y D. x = -4

7. If 3x - 12 = 48, what is the value of x – 4?

A. 24 B. 18

C. 16 D. 14

8. On the number line shown below, what is the coordinate of the point halfway between B and L?

A. **F** B. **G**

C. **I** D. **J**

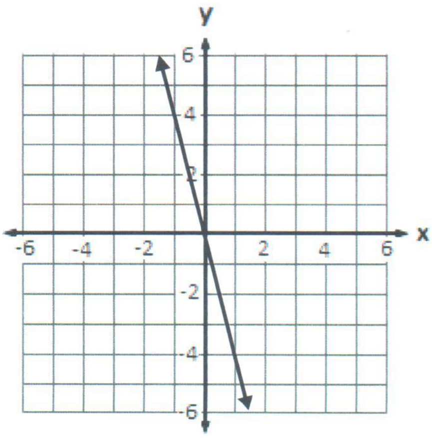
9. Two more than twice a number is 18. What is 9 less than five times the number?

A. 105 B. 31

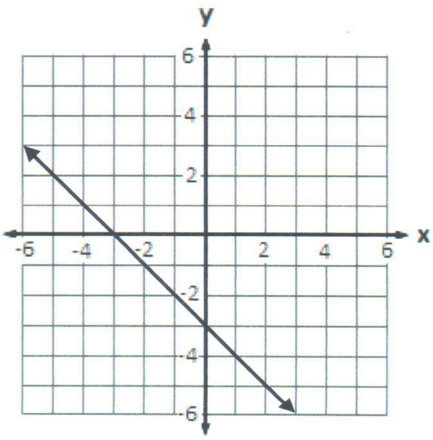
C. 23 D. 8

10. Which of the following graph below is the graph of $4x + y = 0$?

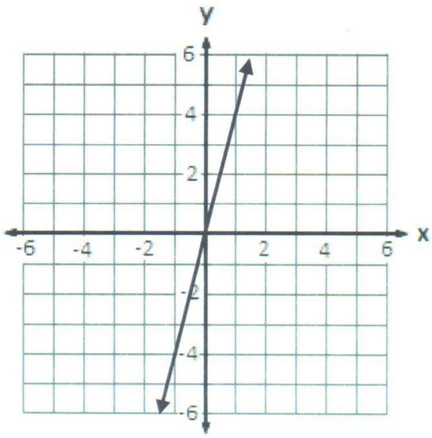
A.



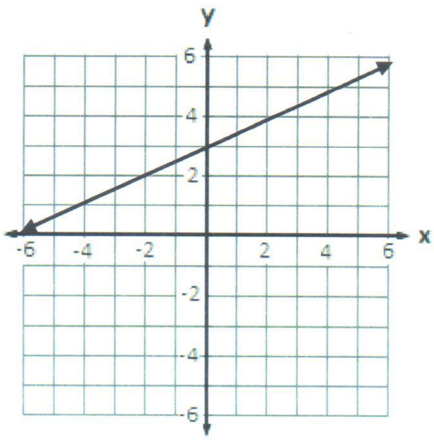
B.



C.



D.



B. Classify the following equations as identity, conditional equation or contradiction. Give the solution set.

1. $6x - 12 = 18$ ____________________

2. $4x - 9 = 2(2x + 7)$ ____________________

3. $-4(x + 3) + 5x = x - 12$ ____________________

4. $2x = 10$ ____________________

5. $6(x + 2) = 6x + 12$ ____________________

C. Problem solving

1. John Michael invested his K10 000 some amount at 4% and the rest at 8%. The annual income of his investment is K640. How much has he invested at 4%?

2. Find three consecutive integers whose sum is 54.

3. The perimeter of a rectangle is 276 cm. The length of the rectangle is 6 cm less than 2 times each width. Find the dimensions of the rectangle.

12.4.2 SYSTEM OF EQUATIONS

In the previous section, we showed how to solve a single equation for a single variable. In this section, we shall discuss some techniques of solving systems of linear equations in two variables.

A system of equations is a set of two or more equations that are considered together for a common solution.

Examples of systems of equations are:

(1) $\left.\begin{array}{r} x - 3y = 10 \\ 2x + 3y = 5 \end{array}\right\}$ is a system of linear equations

(2) $\left.\begin{array}{r} 3x - 2y > 6 \\ x - 2y < 4 \end{array}\right\}$ is a system of linear inequalities

(3) $\left.\begin{array}{r} x^2 + y^2 = 14 \\ x^2 - y^2 = 4 \end{array}\right\}$ is a system of quadratic equations

In this section we shall only discuss system of linear equations.

If each equation of a system is a first-degree equation, the system is called **linear system**.

When the system of equations is solved, there usually is only one set of values or an ordered pair which satisfies the system. We call this set of values a **solution** to the system. The solution set of the system is the set of all solutions. To solve the system means to find the solution set.

The system is said to be **inconsistent** if it has no solution. This means that no values of x and y can satisfy both equations.

Examples:

(1) $x + 2y = 6$
$3x + 6y = 10$

(2) $3x + 2y = 7$
$6x + 4y = 15$

Observe the above examples. Notice that in the first system, the left member of the second equation is obtained by multiplying the left member of the equation of the first by 3. Yet the right member of the second equation is not obtained by multiplying the right member of the first by 3 also. Therefore, there lies an inconsistency.

The system is **dependent** if it is infinitely many solutions. It is a system of equations in which one is related to the other so that it may be directly obtained from the other.

Examples:

(1) $x + 2y = 6$
$3x + 6y = 18$

(2) $2x + 5y = 1$
$6x + 15y = 3$

The second equation is obtained from the first by multiplying it by 3. In this system, any set of values which will satisfy one equation will also satisfy the other. Hence, it is called a **dependent system.**

A system of equations in the same number of unknowns where each equation is consistent with the other or others and independent of them is called a **simultaneous system**.

Examples: (1) $x - 2y = 10$
$2x + 3y = 5$

(2) $2x + y = 4$
$x - y = -1$

Such a system can be solved. The pair of values which will satisfy both equations of the first system are $x = 5$, $y = -\frac{5}{3}$. The pair of values which will satisfy the second system are $x = 1$ and $y = 2$.

We will consider three methods of solving the system of linear equations. These are **elimination method**, **substitution method** and the **graphical method**.

12.4.2.1 Solving Simultaneous Systems of Linear Equations by Elimination Method.

One method of solving a system of equations is by **elimination by addition or subtraction**.

Example 1 Solve the system: $x + y = 6$
$x - y = 2$

Solution: Equation (1): $x + y = 6$
Equation (2): $\underline{x - y = 2}$
$2x \quad = 8$ Add the equations vertically
$x = 4$ Divide by 2

Then, substitute $x = 4$ into equation (1) we have:

Equation (1): $x + y = 6$
$4 + y = 6$
$y = 2$

Check: Substitute $x = 4$ and $y = 2$ in equations (1) and (2).

Equation (1): $x + y = 6$
$4 + 2 = 6$
$6 = 6$ True

Equation (2): $x - y = 2$
$4 - 2 = 2$
$2 = 2$ True

Therefore, the solution for the system is x = 4 and y = 2, written (4, 2).

Example 2 Solve the system:
$$x - 3y = 10$$
$$2x + 3y = 5$$

Solution:
Equation (1): $x - 3y = 10$
Equation (2): $\underline{2x + 3y = 5}$
$3x = 15$ Add the equations vertically
$x = 5$ Divide by 3

Then, substitute x = 5 into equation (1) we have:

Equation (1):
$$x - 3y = 10$$
$$5 - 3y = 10$$
$$-3y = 5$$
$$y = -\frac{5}{3}$$

Check: Substitute $x = 5$ and $y = -\frac{5}{3}$ in equations (1) and (2).

Equation (1): $x - 3y = 10$
$$5 - 3(-\frac{5}{3}) = 10$$
$$5 + 5 = 10$$
$10 = 10$ True

Equation (2): $2x + 3y = 5$
$$2(5) + 3(-\frac{5}{3}) = 5$$
$$10 - 5 = 5$$
$5 = 5$ True

Therefore, the solution for the system is $x = 5$ and $y = -\frac{5}{3}$, written $(5, -\frac{5}{3})$.

Example 3 Solve the system:
$$3x + 2y = 13$$
$$3x - 4y = 1$$

Solution:
Equation (1): $3x + 2y = 13$
Equation (2): $\underline{3x - 4y = 1}$
$6y = 12$ Subtract the equation (2) from equation (1)
$y = 2$ Divide by 3

Then, substitute y = 2 into equation (2) we have:

Equation (2):
$$3x - 4y = 1$$
$$3x - 4(2) = 1$$
$$3x - 8 = 1$$
$$3x = 9$$
$$x = 3$$

Check: In Equation (1): $3(3) + 2(2) = 13$
$$9 + 4 = 13$$
$$13 = 13$$

Equation (2): $3(3) - 4(2) = 1$
$$9 - 8 = 1$$
$$1 = 1$$

Therefore, the solution for the system is x = 3 and y = 2, written (3, 2).

When we have found the value of one variable, that value may be substituted into either equation (1) or equation (2) to find the value of the other variable. Usually one equation is easier to work than the other.

Examples 1, 2 and 3 are simple because one letter can be eliminated by addition or subtraction. Sometimes we must multiply one or both equation by a number to make the coefficients of one variable the same in both equations.

Example 4

Solve the system: $x - 2y = -13$

$3x + y = -4$

Solution: Equation (1): $x - 2y = -13$

Equation (2): $3x + y = -4$

In this system, none of the variables have the same numerical coefficient. We can eliminate either variable x or y. If you want to eliminate y, multiply equation (2) by 2, then solve for x.

$$2(3x + y = -4) = 6x + 2y = -8$$

Thus we have

Equation (1): $x - 2y = -13$

Equation (2): $\underline{6x + 2y = -8}$

$7x = -21$ Add the equations vertically

$x = -3$ Divide by 7

Then, substitute x = -3 into equation (1) we have:

Equation (1): $x - 2y = -13$

$-3 - 2y = -13$

$-2y = -10$

$y = 5$

Check: In Equation (1): $-3 - 2(5) = -13$

$-3 - 10 = -13$

$-13 = -13$

Equation (2): $3(-3) + 5 = -4$

$-9 + 5 = -4$

$-4 = -4$

Therefore, the solution for the system is x = -3 and y = 5, written (-3, 5).

Example 5

Solve the system: $3x + 4y = 6$
$2x + 3y = 5$

Solution: Equation (1): $3x + 4y = 6$
Equation (2): $2x + 3y = 5$

If Equation (1) is multiplied by 2 and Equation (2) is multiplied by 3, the coefficient of x will be the same in both equations.

$$\begin{array}{lll} \text{Equation (1):} & 2(3x + 4y = 6) & \rightarrow 6x + 8y = 12 \\ \text{Equation (2):} & 3(2x + 3y = 5) & \rightarrow \underline{6x + 9y = 15} \\ & & \quad -y = -3 \\ & & \quad y = 3 \end{array}$$

Subtract the bottom equation from the top one.

Then, substituting $y = 3$ into Equation (2), we have:

$$\begin{aligned} \text{Equation (2):} \quad 2x + 3y &= 5 \\ 2x + 3(3) &= 5 \\ 2x + 9 &= 5 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

Check: In Equation (1): $3(-2) + 4(3) = 6$
$-6 + 12 = 6$
$6 = 6$

Equation (2): $2(-2) + 3(3) = 5$
$-4 + 9 = 5$
$5 = 5$

Therefore, the solution of the system $x = -2$ and $y = 3$, written $(-2, 3)$.

We summarize the techniques we have learnt.

To solve a system of equations by Elimination by Addition or Subtraction

1. If necessary multiply either or both of the given equations by numbers which will make the coefficients of one of the variables the same or alike in both equations.
2. If these same coefficients have like signs, subtract one equation from the other; if they have unlike signs, add the equations to eliminate the variable or letter whose coefficients were made the same.
3. The result will be an equation in one variable. Solve the equation.
4. Substitute the value thus obtained in one of the given equations to obtain the value of the other unknown.
5. Check in the original equations.

Now do the learning activity.

LEARNING ACTIVITY 12.4.2.1

20 minutes

Solve each of the following system of equations using elimination by addition or subtraction method.

1. $3x - y = 11$
   $3x + 2y = -4$

2. $6x + 5y = 2$
   $2x - 5y = -26$

3. $8x + 15y = 11$
   $4x - y = 31$

4. $x + 6y = 24$
   $5x - 3y = 21$

5. $2x + 3y = 13$
   $4x - y = 5$

12.4.2.2: Solving Simultaneous System of Equations by Substitution

The other method of solving system of linear equations is **substitution method**. It is important that you learn this method of solving relatively simple systems so that you can apply this method later to solve more complicated systems.

Example 1

Solve the system: $x - 2y = 11$
$3x + 5y = -11$

Solution: Equation (1): $x - 2y = 11$
Equation (2): $3x + 5y = -11$

Using Equation (1), solve for x in terms of y, we have:

$$x - 2y = 11$$
$$x = 11 + 2y$$

Substitute 11 + 2y in place of x in Equation (2)'

Equation (2): $3x + 5y = -11$

$3(11 + 2y) + 5y = -11$ Substitution

$33 + 6y + 5y = -11$ Distributive property

$33 + 11y = -11$ Collect like terms

$11y = -44$ Subtract 33 on both sides

$y = -4$ Divide both sides by 11

Solve for x, by using either of the given equations.

Using Equation (1), $x - 2y = 11$, substitute $y = -4$.

Hence, $x - 2(-4) = 11$

$$x + 8 = 11$$
$$x = 3$$

Check: In Equation (1): $x - 2y = 11$

$$3 - 2(-4) = 11$$
$$3 + 8 = 11$$
$$11 = 11$$

In Equation (2): $3x + 5y = -11$

$$3(3) + 5(-4) = -11$$
$$9 + (-20) = -11$$
$$-11 = -11$$

Therefore, the solution set of the system is x = 3 and y = -4, written (3, -4).

How to choose which letter to solve for?

a. One of the equations may already be solved for a letter.

For example: $2x - 5y = 4$
$y = 3x + 7$

Already solved for y

b. One of the letters may have a coefficient of 1.

For example: $2x + 6y = 3$
$x - 4y = 2$

x has a coefficient of 1

c. Choose the letter with the smallest coefficient.

For example: $11x - 7y = 10$
$14x + 2y = 9 \longrightarrow y = \dfrac{9 - 14x}{2}$

Smallest of the four coefficients

Smallest possible denominator in this case

Example 2

Solve the system: $6x + 5y = 2$
$5x + 3y = 4$

Smallest of the four coefficients

Solution: Equation (1): $6x + 5y = 2$
Equation (2): $5x + 3y = 4$

Smallest of the four coefficients

No letter has a coefficient of 1. Solve equation (2) for y because it has the smallest coefficient.

Solve equation (2) for y:

$$5x + 3y = 4$$
$$3y = 4 - 5x$$
$$y = \frac{4 - 5x}{3}$$

Substitute $\dfrac{4 - 5x}{3}$ in place of y in Equation (1) to solve for x.

Equation (1): $6x + 5y = 2$

$$6x + 5\left(\frac{4 - 5x}{3}\right) = 2$$

LCD =3 $\quad 3(6x) + (\cancel{3})5\left(\frac{4 - 5x}{\cancel{3}}\right) = 3(2)$

$$18x + 5(4 - 5x) = 6$$
$$18x + 20 - 25x = 6$$
$$-7x = -14$$
$$x = 2$$

Solve y, substitute x = 2 in Equation (2)

Equation (2): $5x + 3y = 4$

$$5(2) + 3y = 4$$
$$10 + 3y = 4$$
$$3y = -6$$
$$y = -2$$

Check: Equation (1): $6x + 5y = 2$

$$6(2) + 5(-2) = 2$$
$$12 - 10 = 2$$
$$2 = 2$$

Equation (2): $5x + 3y = 4$

$$5(2) + 3(-2) = 4$$
$$10 - 6 = 4$$
$$4 = 4$$

Therefore, the solution set of the system is x = 2 and y = -2, written (2, -2).

We summarize the techniques we have learnt.

To solve a system of equations by Substitution method

1. Solve one of the equations for one letter in terms of the other.
2. Substitute the expression obtained in Step 1 into the other equation.
3. Solve that equation.
4. Substitute the value thus obtained in one of the given equations or in the equation obtained in Step 1 to obtain the value of the other letter.
5. Check in the original equations.

Now do the learning activity.

LEARNING ACTIVITY 12.4.2.2

20 minutes

Solve each of the following system of equations using the Substitution method.

1. $4x + y = 10$
   $3x + 2y = -5$

2. $2x - 4y = 12$
   $4x + 3y = 2$

3. $2x + 5y = 10$
   $4x + 2y = 8$

4. $7x + 4y = 4$
   $y = 6 - 3x$

5. $5x - 4y = -1$
   $3x + y = -38$

12.4.2.3: Graphing Method for Solving System Two Equations in Two Variables

The third method of solving system of linear equations is the graphical method. There are many ways of graphing the equation of a line; we have point plotting, intercepts method and others.

In section 12.4.1.4 we showed how to graph a straight line. The graph of each equation in a system of linear equations in two variables is a straight line.

Example 1

Solve the system of equations $\left\{\begin{array}{rcr} x + y & = & 6 \\ x - y & = & 2 \end{array}\right\}$ graphically.

Solution: Equation (1): $x + y = 6$
Equation (2) $x - y = 2$

Solve the x and y-intercepts of Equation (1)

x-intercepts: Let y = 0, then $x + y = 6$

$x + 0 = 6$

$x = 6$

y-intercepts: Let x = 0, then $x + y = 6$

$x + 0 = 6$

$x = 6$

x	y
2	0
0	-2

Therefore, line (1) goes through the points (0, 6) and (6, 0).

Solve the x and y-intercepts of Equation (2).

x-intercepts: Let y = 0, then $x - y = 2$

$x - 0 = 2$

$x = 2$

y-intercepts: Let x = 0, then $x - y = 2$

$0 - y = 2$

$y = -2$

x	y
6	0
0	6

Therefore, line (2) goes through the points (2, 0) and (0, -2).

After solving the x- and y-intercepts of both equations, plot these points and draw the graph of each equation on the same set of axes and identify the coordinates of the points of intersection. The coordinates of the point of intersection represent the solution set of the given system of equations. (See Figure 16)

Here is the graph.

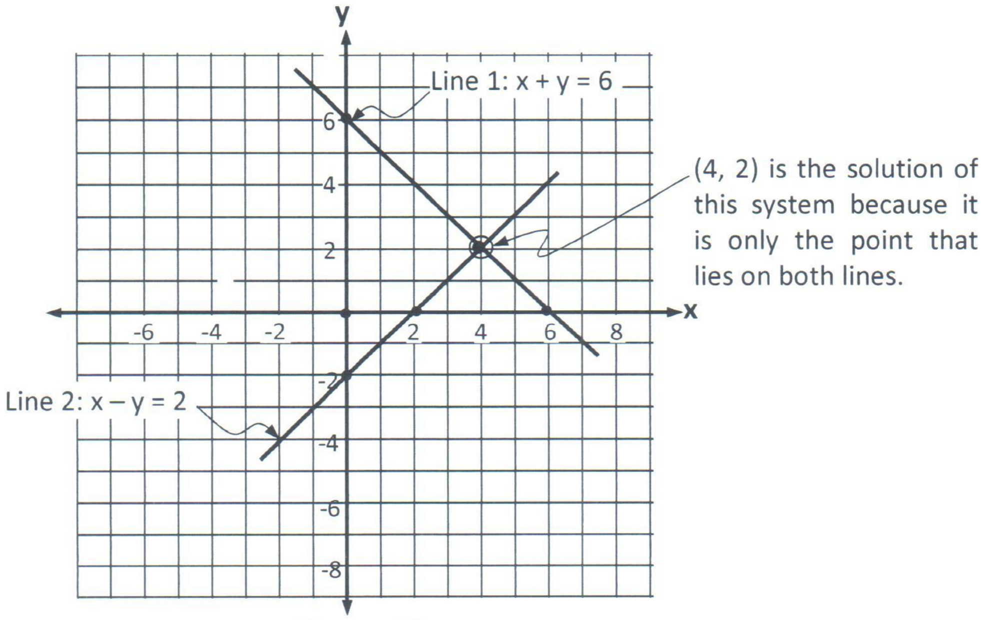


Figure 16

The coordinates of any point on Line (1) satisfy the equation of line (1). The coordinates of any point on Line (2) satisfy the equation of Line (2).The only point that lies on both lines is (4, 2). Therefore, it is the only point whose coordinates satisfy both equations.

When the system has only one solution set, it is called a **consistent system**. When each equation in the system has different graphs, they are called **independent equations.**

Example 2

Solve the system of equations: $\left\{\begin{array}{rcl} 2x - 3y &=& 6 \\ 6x - 9y &=& 36 \end{array}\right\}$ graphically.

Solution: Equation (1): $2x - 3y = 6$

Equation (2): $6x - 9y = 36$

Solve the x- and y-intercepts of Equation (1).

x-intercept: If $y = 0$, then $2x - 3y = 6$

$$2x - 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

x	y
3	0
0	-2

y-intercept: If $x = 0$, then $2x - 3y = 6$

$$2(0) - 3y = 6$$
$$-3y = 6$$
$$y = -2$$

Therefore Line (1) goes through (3, 0) and (0, -2).

Solve the x- and y-intercepts of Equation (2).

x-intercept: If $y = 0$, then $6x - 9y = 36$

$$6x - 9(0) = 36$$
$$6x = 36$$
$$x = 6$$

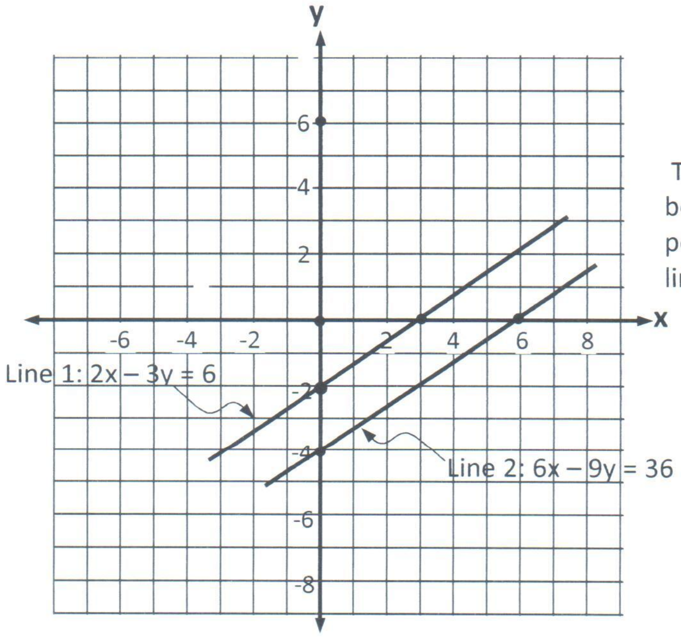
y-intercept: if $x = 0$, then $6x - 9y = 36$

$$6(0) - 9y = 36$$
$$-9y = 36$$
$$y = -4$$

x	y
6	0
0	-4

Therefore, Line (2) goes through (6, 0) and (0, -4).

Plot these points on the same axes and draw the graphs.



There is no solution because there is no point that lies on both lines.

Figure 17

There is no solution because the two lines never meet.

Lines that never meet, such as these, are called **parallel lines**.

When the system has no solution, it is called an **inconsistent system**.

Since each equation in this system has different graph, it is called **independent equation.**

Example 3

Solve the system of equations: $\begin{Bmatrix} 3x + 5y = 15 \\ 6x + 10y = 30 \end{Bmatrix}$ graphically.

Solution: Equation (1): $3x + 5y = 15$

Equation (2): $6x + 10y = 30$

Solve the x- and y-intercepts of Equation (1).

x-intercept: If $y = 0$, then $3x + 5y = 15$

$$3x + 5(0) = 15$$
$$3x = 15$$
$$x = 5$$

x	y
5	0
0	3

y-intercept: if $x = 0$, then $3x + 5y = 15$

$$3(0) + 5y = 15$$
$$5y = 15$$
$$y = 3$$

Therefore Line (1) goes through (5, 0) and (0, 3).

Solve the x- and y-intercepts of Equation (2).

x-intercept: If $y = 0$, then $6x + 10y = 30$

$$6x + 10(0) = 30$$
$$6x = 30$$
$$x = 5$$

x	y
5	0
0	3

y-intercept: if $x = 0$, then $6x + 10y = 30$

$$6(0) + 10y = 30$$
$$10y = 30$$
$$y = 3$$

Therefore Line (1) goes through (5, 0) and (0, 3).

Draw the graph of each line on the same set of axes.

Since each line goes through the same two points, they must be the same line.

See the graph on the next page.

Here is the graph.

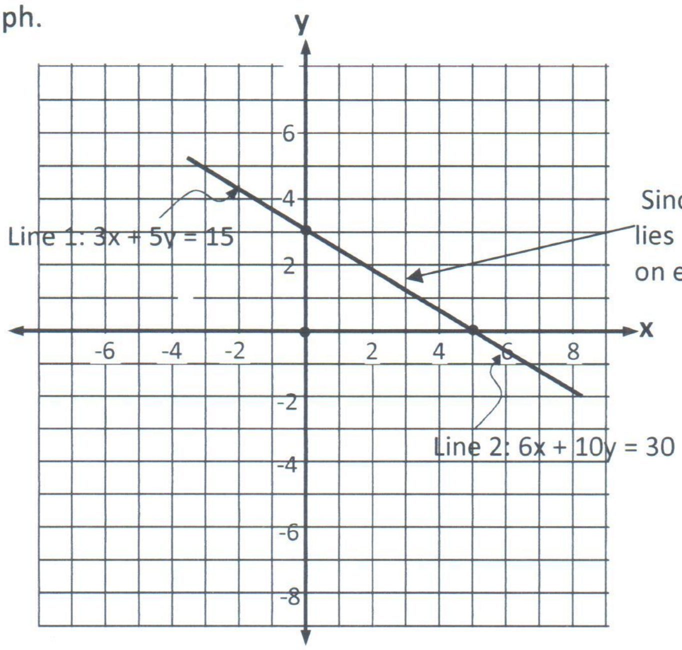


Figure 19

To find one of the many solutions, pick a value for one of the letters and substitute it into either equation. For example, let x = 1 in Equation (1);

$$3x + 5y = 15$$
$$3(1) + 5y = 15$$
$$3 + 5y = 15$$
$$5y = 12$$
$$Y = \frac{12}{5} \text{ or } 2\frac{2}{5}$$

Therefore, $(1, 2\frac{2}{5})$ is a solution of this system. The intercepts (5, 0) and (0, 3) are also solutions. Since this system has solutions, it is a **consistent system**. When each equation in the system has the same graph, they are called **dependent equations**.

To solve a linear system of two equations by graphical method:

1. Graph each equation of the system on the same set of axes.
2. There are three possible cases of the graph of the system:

Case 1: **The lines intersect at one point.** The solution is the ordered pair representing the point of intersection.

Case 2: **The lines never cross or meet**. They **are parallel**. There is no solution.

Case 3: **Both equations have the same line for their graph.** Any ordered pair that represents a point on the line is a solution.

Now do the learning activity.

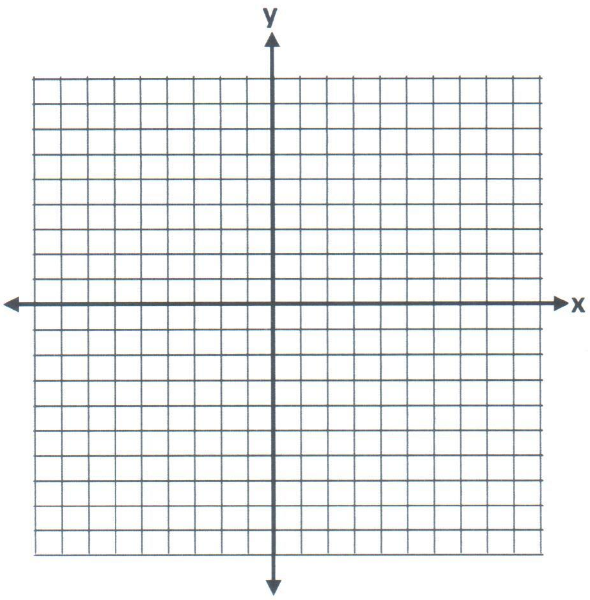
LEARNING ACTIVITY 12.4.2.3

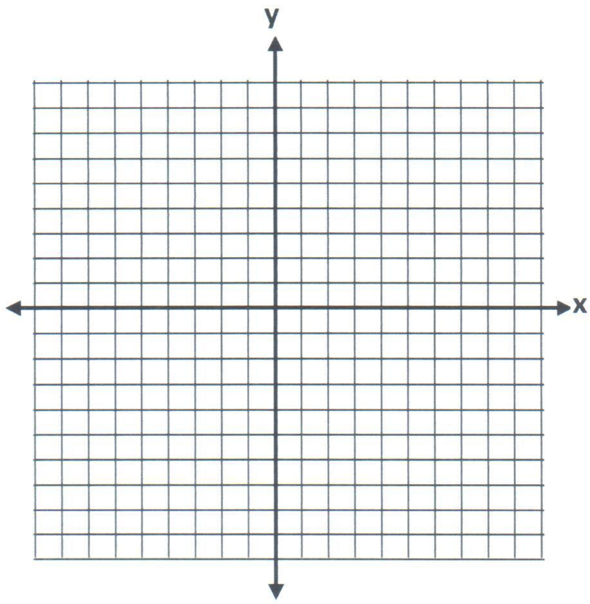
20 minutes

Find the solution of each system graphically.

1. $2x - y = 2$
   $8x + 6y = 48$

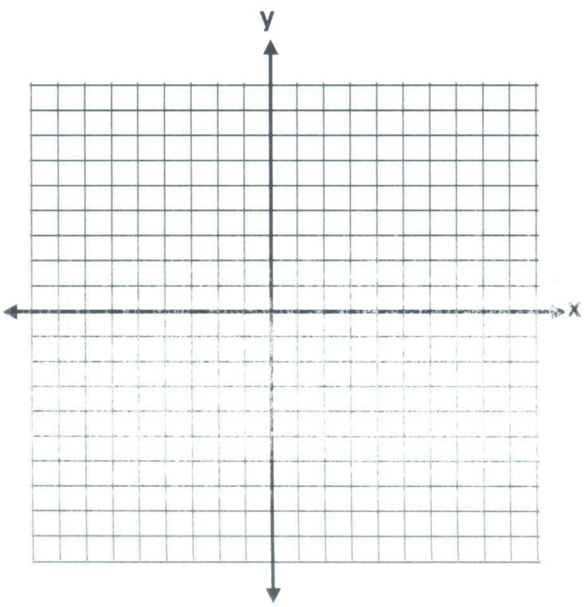
2. $2x + y = 6$
   $x - y = 0$

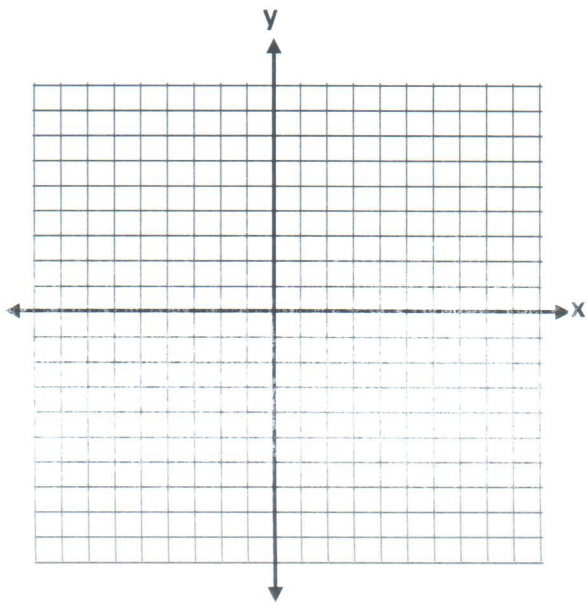




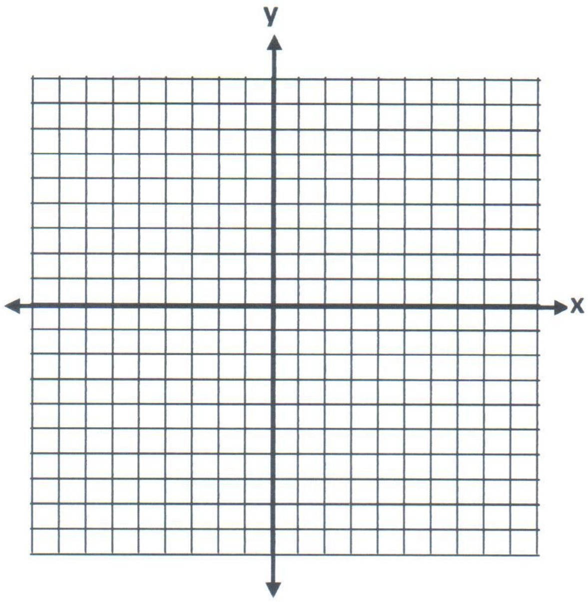
3. $-8x + 4y = 32$
   $3x + 6y = 18$

4. $3x + 4y = 12$
   $6x + 8y = 24$

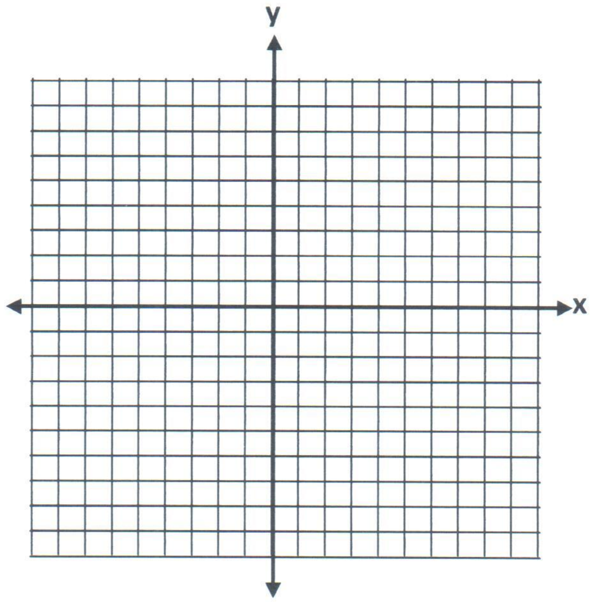




5. $-2x + y = 8$
   $-2x + y = 4$



6. $x + y = 6$
   $x + y = 2$



12.4.2.4 Graphing Method for Solving System of Two Linear Inequalities

So far we have discussed systems of equations. In this section, we discuss solving systems of linear inequalities. You may wish to refer back to section 12.4.1.5, where drawing the graph of a linear inequality in two variables was discussed in detail.

The solution of the system of inequalities in two variables will be the set of points satisfying both inequalities; hence the common solution is the intersection of the inequalities of a plane or half-plane.

Example 1

Solve the system of inequalities $2x - 3y < 6$

$3x + 4y \leq -12$

Solution: Inequality (1) $2x - 3y < 6$

Inequality (2) $3x + 4y \leq -12$

1. Graph inequality (1) $2x - 3y < 6$

   The boundary, $2x - 3y = 6$, is a dashed line because the equality is not included.

   $$2x - 3y < 6$$

   Find the x- and y-intercepts.

x	y
3	0
0	-2

x-intercept: Set y = 0. Then, $2x - 3(0) = 6$

$$2x = 6$$
$$x = 3$$

y-intercept: Set x = 0. Then, $2(0) - 3y = 6$

$$-3y = 6$$
$$y = -2$$

Therefore, the boundary line goes through (3, 0) and (0, -2).

Substitute the coordinates of the origin (0, 0) into inequality (1).

$$2x - 3y < 6$$
$$2(0) - 3(0) < 6$$
$$0 < 6 \quad \text{True}$$

Therefore, the half-plane containing (0, 0) is the solution. (See Figure 20)

2. Graph inequality (1) $3x + 4y \leq -12$

The boundary, $3x + 4y = -12$, is a solid line because the equality is included.

$$3x + 4y \leq -12$$

Find the x- and y-intercepts.

x-intercept: Set y = 0. Then, $3x + 4(0) = -12$

$$3x = -12$$
$$x = -4$$

y-intercept: Set x = 0. Then, $3(0) + 4y = -12$

$$4y = -12$$
$$y = -3$$

x	y
3	0
0	-2

Therefore, the boundary line goes through (-4, 0) and (0, -3).

Substitute the coordinates of the origin (0, 0) into inequality (1).

$$3x + 4y \leq -12$$
$$3(0) + 4(0) \leq -12$$
$$0 \leq -12 \quad \text{false}$$

Therefore, the half-plane not containing (0, 0) is the solution.(See Figure 20).

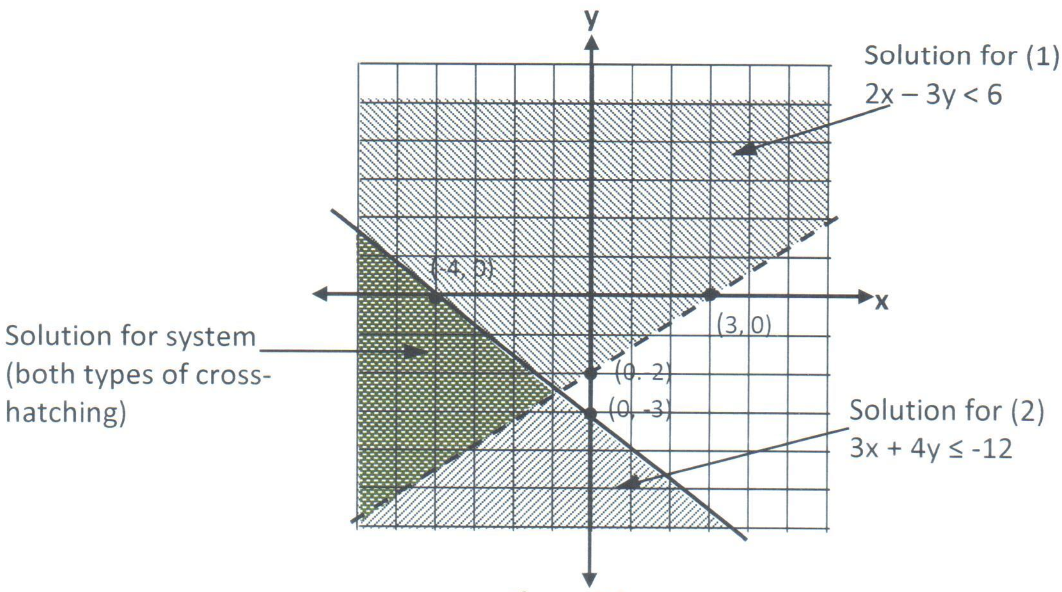


Figure 20

The solution sets of the given system of inequalities are all points lie within the intersection of boundary lines

Example 2

Solve the system: $x - y \geq 2$
$x - 2y - 8 \leq 0$

Solution: Inequality (1) $x - y \geq 2$
Inequality (2) $x - 2y - 8 \leq 0$

1. Graph inequality (1) $x - y \geq 2$

The boundary, $x - y = 2$, is a solid line because the equality is included.

$$x - y \geq 2$$

Find the x- and y-intercepts.

x	y
2	0
0	-2

x-intercept: Set y = 0. Then, x - 2(0) = 0
x = 0

y-intercept: Set x = 0. Then, (0) – 2y = 0
-y = 2
y = -2

Therefore, the boundary line goes through (2, 0) and (0, -2).

Substitute the coordinates of the origin (0, 0) into inequality (1).

$$x - y \geq 2$$
$$0 - 0 \geq 2$$
$$0 \geq 2 \quad \text{False}$$

Therefore, the half-plane not containing (0, 0) is the solution.

2. Graph Inequality (2) $x - 2y - 8 \leq 0$

The boundary, $x - 2y - 8 = 0$, is a solid line because the equality is included.

$$x - 2y - 8 \leq 0$$

Find the x- and y-intercepts.

x	y
8	0
0	-4

x-intercept: Set y = 0. Then, x - 2(0) – 8 = 0
x – 8 = 0
x = 8

y-intercept: Set $x = 0$. Then, $(0) - 2y - 8 = 0$

$$-2y - 8 = 0$$
$$-2y = 8$$
$$y = -4$$

Therefore, the boundary line goes through (8, 0) and (0, -4).

Substitute the coordinates of the origin (0, 0) into inequality (1).

$$x - 2y - 8 \leq 0$$
$$0 - 2(0) - 8 \leq 0$$
$$-8 \leq 0 \quad \text{True}$$

Therefore, the half-plane containing (0, 0) is the solution.

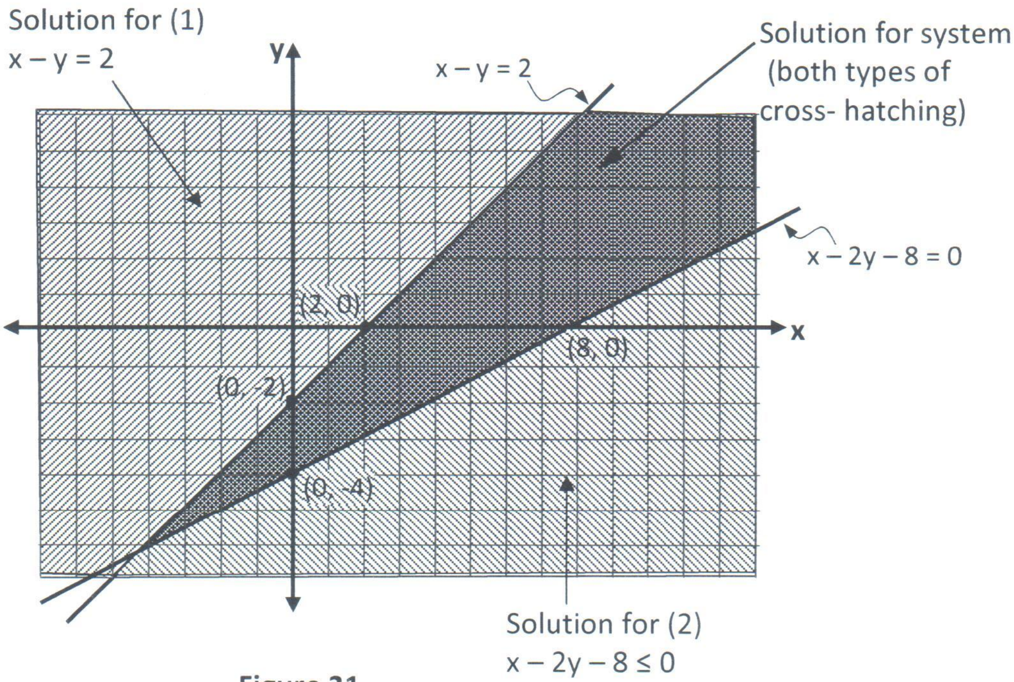


Figure 21

The solution sets of the given system of inequalities are all points lie within the intersection of boundary lines.

To solve a system of two linear inequalities with two variables graphically

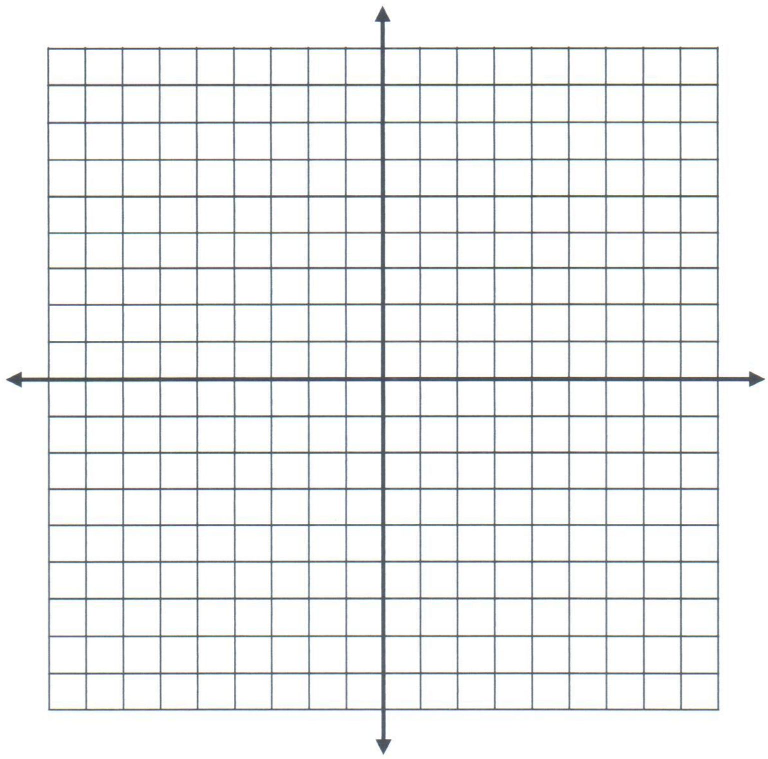
1. Graph the first inequality, shading the half-plane that represents its solution.
2. Graph the second inequality on the same set of axes. Use different type of shading for the half-plane that represents its solution.
3. The solution of the system is represented by the area with both types of shading.

LEARNING ACTIVITY 12.4.2.4

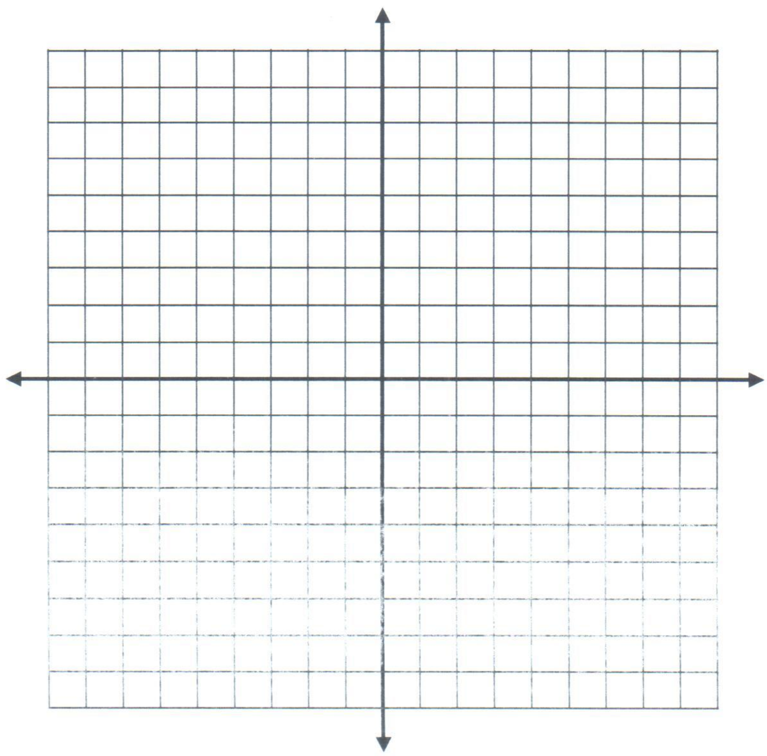
20 minutes

Draw the graph of each pair of inequalities and find the solution set.

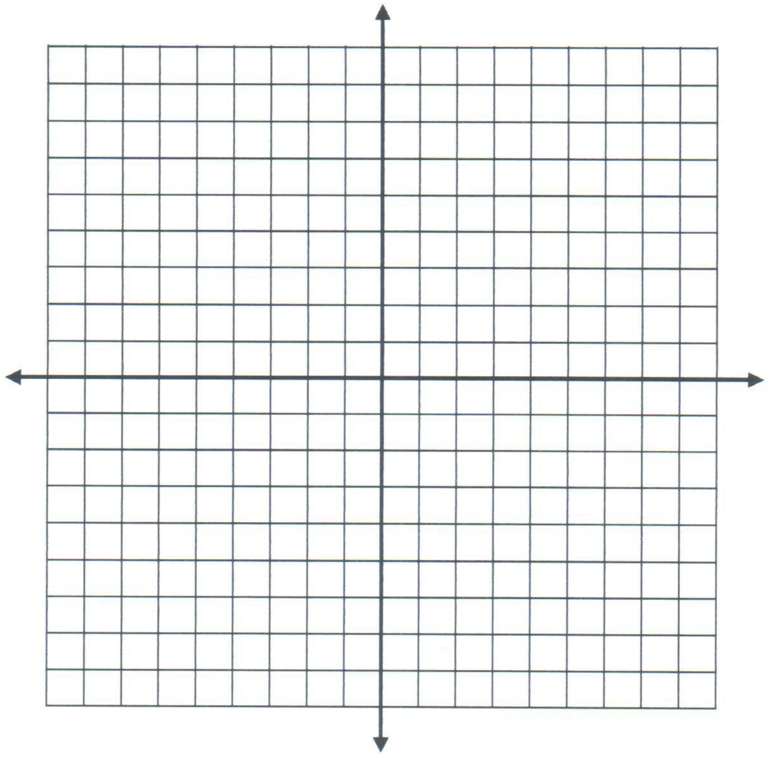
1. $4x + 5y \geq 20$
   $3x - 4y \leq 24$



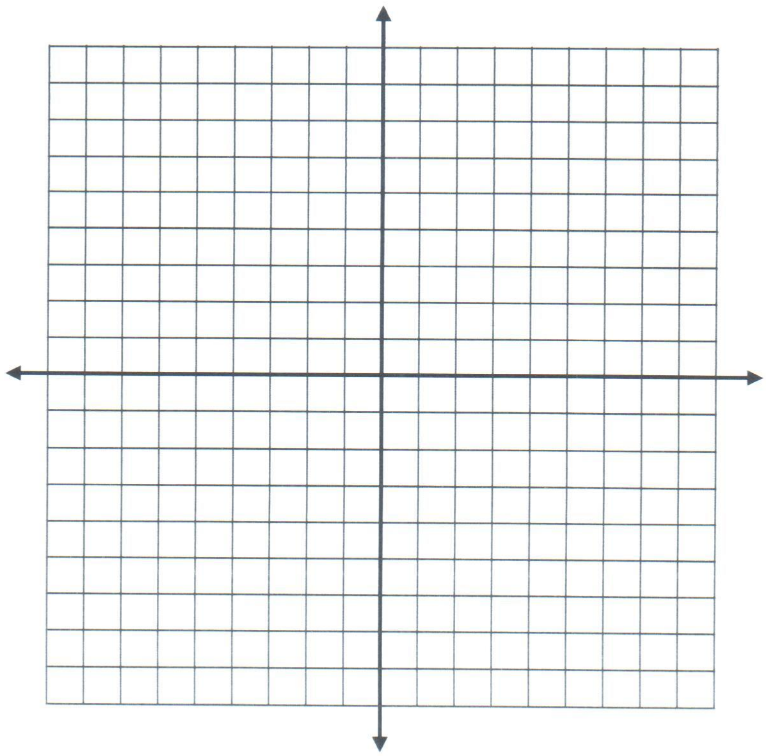
2. $4x + 2y \geq 8$
   $3x - 4y \leq 12$



3. $2x - y \leq 2$
   $x + y \geq 5$



4. $3x - 5y < 15$
   $x < 3$



SUMMATIVE TASK 12.4.2

30 minutes

1. Find the solution set of each system using the elimination by addition-subtraction method.

a. $4x - 8y = 4$
$3x - 6y = 3$

b. $3x - 2y = 10$
$5x + 4y = 24$

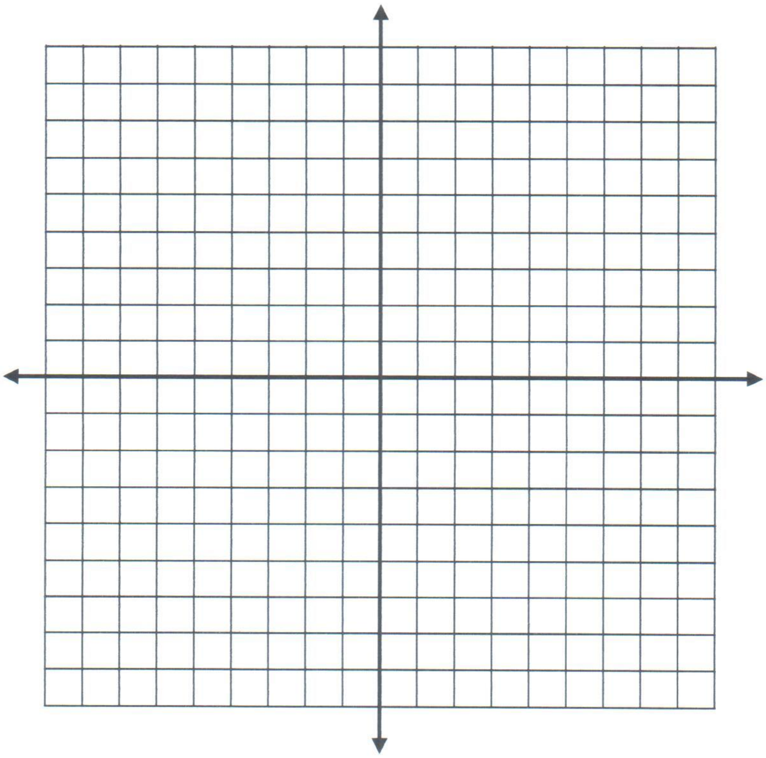
2. Solve each of the following using the substitution method. Write "inconsistent" if no solution exists and write "dependent" if an infinite number of solution exist.

a. $x = y + 2$
$4x - 5y = 3$

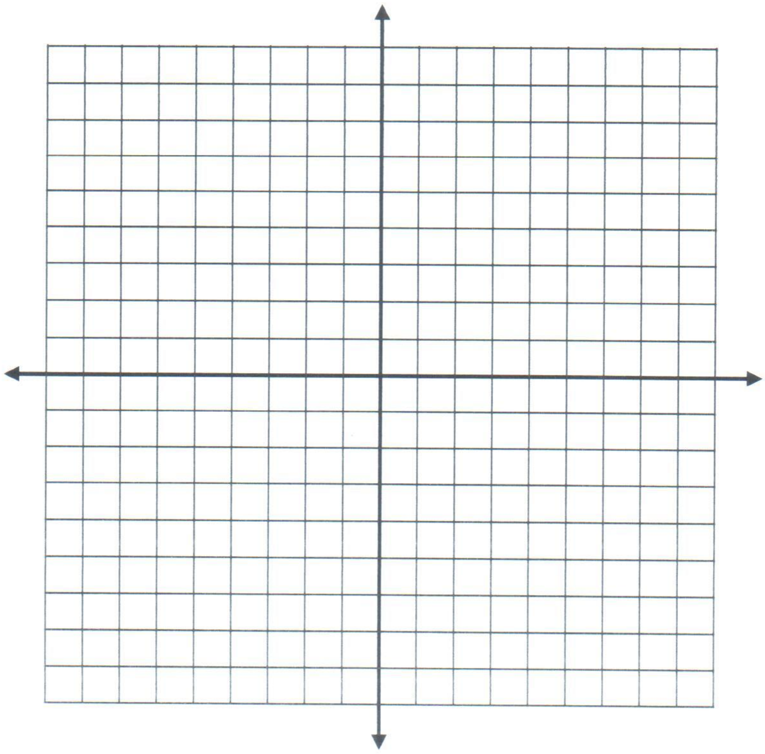
b. $7x - 3y = 1$
$y = x + 5$

3. Solve each system graphically.

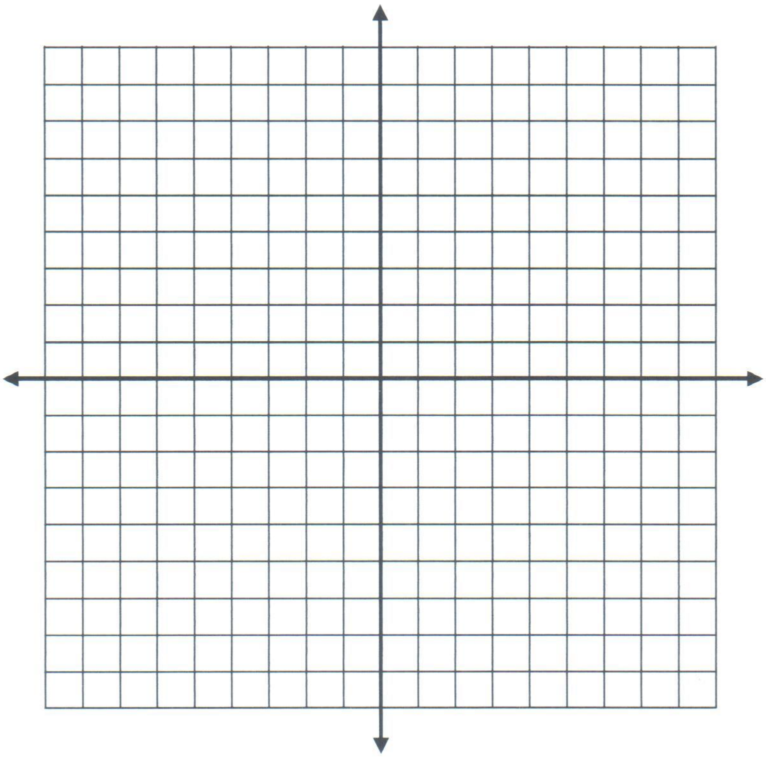
a. $4x + 5y = 22$
$3x - 2y = 5$



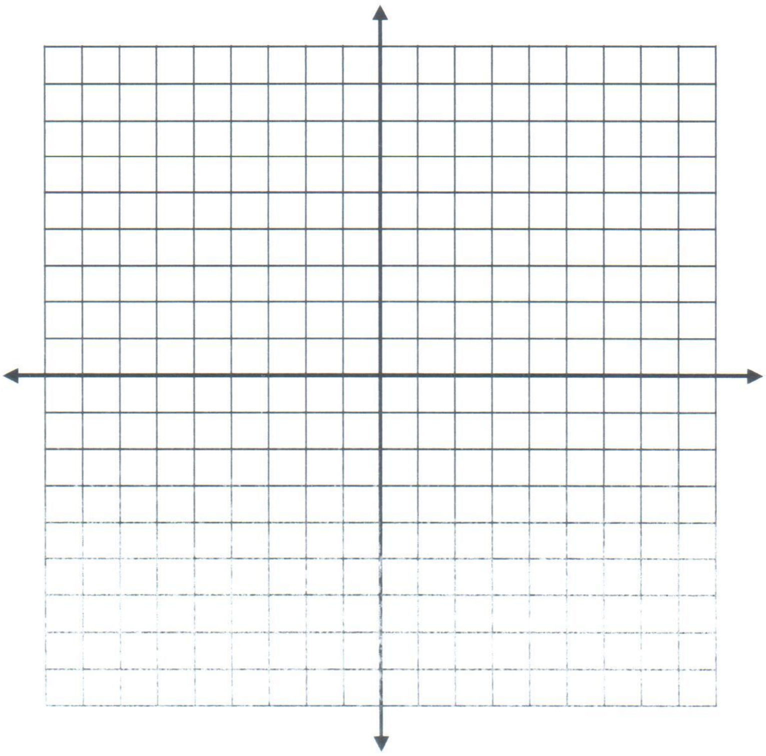
b. $2x - 3y = 3$
$3x - 2y = 6$



c. $x + 4y \leq 4$
$3x + 2y > 2$



d. $3x + 6 \geq 2$
$3x + y < 3$



12.4.3 SOLVING QUADRATIC EQUATIONS

In this topic, we will discuss several methods for solving quadratic equations as well as graphing of quadratic functions. You will also see that many real life problems can be modeled using quadratic equations.

12.4.3.1 Basic Concepts

A **quadratic equation** is a polynomial with a second degree term as its highest–degree term. Such equations are also called "**second degree equations**". The number of roots of a polynomial equation is at most equal to the degree of the polynomial. For this reason, we expect to get two roots when solving quadratic equations.

Examples of quadratic equations

1. $3x^2 + 7x + 2 = 0$
2. $x^2 - 4 = 0$
3. $\frac{1}{2}x^2 = \frac{2}{3}x - 4$
4. $5x^2 - 15x = 0$
5. $2 - 4x = 3x^2$
6. $8 = 2x^2$
7. $3x^2 = 4$

The general form of quadratic equation

Any quadratic equation can be arranged in the form:

$$\boxed{ax^2 + bx + c = 0}$$

where **a**, **b**, and **c** are real numbers except that **a** cannot be zero, is called a quadratic equation in **x**. This is known as the **general form** of quadratic equations.

Type 1 **Complete quadratic equation**

A quadratic equation is said to be a **complete quadratic** if it contains both the second power and the first power of the unknown quantity.

Examples

1. $7x^2 - 5x - 2 = 0$
2. $3x^2 + 5x = 2$
3. $2x^2 + 3 = 5x$

In example 1, a = 7, b = -5 and c = - 2. In example 2, a = 3, b = -7 and c = 0. In example 3, a = 2, b = -5 and c = 3.

Type 2 **Incomplete or pure quadratic equations**

An **incomplete or pure quadratic equation** is one in which **b** or **c** (or both) is zero. The only letter that cannot be zero is **a**. If **a** is zero, the equation would not be a quadratic.

Examples

1. $12x^2 + 5 = 0$ ------------------→(b = 0)
2. $7x^2 - 2x = 0$ ------------------→(c = 0)
3. $3x^2 = 0$ ------------------------→(b and c = 0)

An equation of the form

$$\boxed{ax^2 + bx = 0} \quad \textbf{or} \quad \boxed{ax^2 + c = 0}$$

is called **incomplete** or **pure quadratic.**

Changing quadratic equation into general form

To change quadratic equation into general form

1. Remove fractions by multiplying each term by the LCD.
2. Remove grouping symbols.
3. Combine like terms.
4. Get all terms to one side by adding the same expressions to both sides. Only zero must remain on the other side. Then arrange the terms in descending powers.

Example 1

Change the quadratic equation $7x = 5 - 2x^2$ into general form.

Solution: $7x = 5 - 2x^2$

$2x^2 + 7x = 5$ adding $2x^2$ to both sides

$\mathbf{2x^2 + 7x - 5 = 0}$ subtract 5 from both sides General form: a = 2
b = 7
c = -5

Example 2

Change the quadratic equation $5x^2 = 3$ into general form.

Solution: $5x^2 = 3$

$\mathbf{5x^2 + 0x - 3 = 0}$ Subtract 3 from both sides. General form: $a = 5$, $b = 0$, $c = -3$

Example 3

Change the quadratic equation $6x = 11x^2$ into general form.

Solution: $6x = 11x^2$

$0 = 11x^2 - 6x$ Subtract 6x from both sides. General form: $a = 11$, $b = -6$, $c = 0$

$\mathbf{11x^2 - 6x + 0 = 0}$

Example 4

Change the quadratic equation $\frac{2}{3}x^2 - 5x = \frac{1}{2}$ into general form.

Solution: LCD = 6

$6(\frac{2}{3}x^2) + 6(-5x) = 6(\frac{1}{2})$ Multiply both sides by the LCD (6).

$4x^2 - 30x = 3$

$\mathbf{4x^2 - 30x - 3 = 0}$ General form: $a = 4$, $b = -30$, $c = -3$

Example 5

Change quadratic equation $(x + 2)(2x - 3) = 3x - 7$ into general form.

Solution: $(x + 2)(2x - 3) = 3x - 7$

$2x^2 + x - 6 = 3x - 7$ Remove grouping symbols.

$\mathbf{2x^2 - 2x + 1 = 0}$ General form: $a = 2$, $b = -2$, $c = 1$

Now do the learning activity.

LEARNING ACTIVITY 12.4.3.1

20 minutes

Write each of the following quadratic equations in general form, then identify a, b and c.

1. $3x^2 + 5x = 2$

2. $3x + 5 = 2x^2$

3. $3x^2 = 4$

4. $2 - 4x = 3x^2$

5. $\frac{4}{3}x = 4 + x^2$

6. $x(x + 2) = 4$

12.4.3.2 Solving Quadratic Equations by Factoring

Solving quadratic equation means to find its roots or solution. We now present the first algebraic method of solving quadratic equations which is **factoring.**

Factoring is the simplest method of solving quadratic equation. In solving quadratic equation using factoring, we apply the two properties: (1) The Zero Product property or the Null Factor Law and (2) the Square Root property.

> The Zero-Product Property or the Null Factor Law states that if the product of two numbers is zero, then at least one of them is zero. That is,
>
> **If AB = 0, then A = 0 or B = 0**

This property helps in the solving of non-linear equations. In solving quadratic equations, first you rearrange so one side of the equation is zero, then, you factorize so that you have two linear factors.

Example 1 Solve the equation $x^2 + 5x = 24$.

Solution: We must first rewrite the equation in general form.

Hence, $x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$ Subtract 24

$(x - 3)(x + 8) = 0$ Factor the left side

$x - 3 = 0$ or $x + 8 = 0$ Zero-Product Property, equating the factors to zero

$x = 3$ $x = -8$ Solve for x in each equation

Checking:

For x = 3	For x = -8
$x^2 + 5x = 24$	$x^2 + 5x = 24$
$(3)^2 + 5(3) = 24$	$(-8)^2 + 5(-8) = 24$
$9 + 15 = 24$	$64 - 40 = 24$
$24 = 24$ true	$24 = 24$ true

If you substitute the values 3 and -8 to the original equation it makes the statement true.

Therefore, the solutions are x = 3 and x = -8.

Example 2 Solve $(x - 2)(x + 3) = 6$

Solution: It is very tempting to equate the factors on the left side to 6 and solving the resulting equations. That is not correct. It is not true that if the product of two numbers is 6 then at least one of them is 6.

What we do here is to rewrite the equation in general form by first expanding the left side.

Hence,	$(x - 2)(x + 3) = 6$	
	$x^2 + x - 6 = 6$	Expand bracket
	$x^2 + x - 12 = 0$	Rewrite the equation general form
	$(x + 4)(x - 3) = 0$	Factor the left side
	$x + 4 = 0$ or $x - 3 = 0$	Zero-Product Property
	$x = -4 \qquad x = 3$	Solve each equation.

Checking:	For $x = -4$	For $x = 3$
	$(x - 2)(x + 3) = 6$	$(x - 2)(x + 3) = 6$
	$(-4 - 2)(-4 + 3) = 6$	$(3 - 2)(3 + 3) = 6$
	$(-6)(-1) = 6$	$(1)(6) = 6$
	$6 = 6$ true	$6 = 6$ true

Therefore, the solutions are $x = -4$ and $x = 3$.

We summarize the steps we have learnt.

> To solve quadratic equation by factoring
> 1. Arrange the equation in general form $ax^2 + bx + c = 0$.
> 2. Factor the polynomial on the left side.
> 3. Set each factor equal to zero (Zero- product property).
> 4. Solve the unknown letter.
> 5. Check.

Now look at the following examples:

Example 4 Solve the quadratic equation $3x^2 + 5x = 0$.

Solution: To solve this we first find the GCF and then solve by factoring.

Thus we have,	$3x^2 + 5x = 0$	GCF is x
	$x(3x + 5) = 0$	Factor using GCF = x
	$x = 0$ or $3x + 5 = 0$	Zero-product property, equating the factors to zero
	$x = 0$ or $3x = -5$	Solve for x
	$x = -\frac{5}{3}$	

Checking:

When $x = 0$

$3x^2 + 5x = 0$

$3(0)^2 + 5(0) = 0$

$0 + 0 = 0$

$0 = 0$ true

When $x = -\frac{5}{3}$

$3x^2 + 5x = 0$

$3(-\frac{5}{3})^2 + 5(-\frac{5}{3}) = 0$

$3(\frac{25}{9}) - \frac{25}{3} = 0$

$\frac{25}{3} - \frac{25}{3} = 0$

$0 = 0$ true

Therefore, the solutions are $x = 0$ and $x = -\frac{5}{3}$.

Example 5 Solve the quadratic equation $\frac{2}{5}x = 3x^2$.

Solution: $\frac{2}{5}x = 3x^2$ LCD = 5

Hence, $5(\frac{2}{5}x) = 5(3x^2)$

$2x = 15x^2$

$15x^2 - 2x = 0$ write in general form

$x(15x - 2) = 0$ factor the left side GCF = x

$x = 0$ or $15x = 2$ Zero-product property

$x = 0$ or $x = \frac{2}{15}$

Checking:

When $x = 0$

$\frac{2}{5}x = 3x^2$

$\frac{2}{5}(0) = 3(0)^2$

$0 = 0$ true

When $x = \frac{2}{15}$

$\frac{2}{5}x = 3x^2$

$\frac{2}{5}\left(\frac{2}{15}\right) = 3\left(\frac{2}{15}\right)^2$

$\frac{4}{75} = 3\left(\frac{4}{225}\right)$

$\frac{4}{75} = \frac{4}{75}$ true

Therefore, the solutions are $x = 0$ and $x = \frac{2}{15}$.

Example 6 Solve the quadratic equation $12x^2 = 3x$.

Solution:

$12x^2 = 3x$	
$12x^2 - 3x = 0$	General form
$3x(4x - 1) = 0$	GCF = 3x
$3x = 0$ or $4x - 1 = 0$	Zero-product property
$x = 0$ $\quad 4x = 1$	Solve
$\quad x = \frac{1}{4}$	

Checking:

When $x = 0$	When $x = \frac{1}{4}$
$12x^2 = 3x$	$12x^2 = 3x$
$12(0)^2 = 3(0)$	$12\left(\frac{1}{4}\right)^2 = 3\left(\frac{1}{4}\right)$
$0 = 0$ true	$12\left(\frac{1}{16}\right) = \frac{3}{4}$
	$\frac{3}{4} = \frac{3}{4}$ true

Therefore, the solutions are $x = 0$ and $x = \frac{1}{4}$.

The equations in examples 4, 5 and 6 are quadratic equations of the form $ax^2 + bx = 0$.

We summarize the steps we have learnt.

To solve a quadratic equation of the form $\mathbf{ax^2 + bx = 0}$, $c = 0$

1. Find the greatest common factor (GCF) of the left side of the equation.
2. Then solve by factoring.

$$ax^2 + bx = 0$$
$$x(ax + b) = 0$$
$$x = 0 \text{ or } ax + b = 0$$
$$ax = -b$$
$$x = -\frac{b}{a}$$

Example 7 Solve $x^2 - 4 = 0$

Solution:

$x^2 - 4 = 0$	
$(x + 2)(x - 2) = 0$	Difference of two squares
$x + 2 = 0$ or $x - 2 = 0$	Zero product property
$x = -2$ or $x = 2$	Solve for x

Checking:

When $x = -2$	When $x = 2$
$x^2 - 4 = 0$	$x^2 - 4 = 0$
$(-2)^2 - 4 = 0$	$(2)^2 - 4 = 0$
$0 = 0$ true	$0 = 0$ true

Therefore, the solutions or roots are x = -2 and x = 2.

Here we have a quadratic equation of the form $ax^2 + c = 0$, where $c \geq 0$ and $b = 0$.

Equation such as this one can also be solved and done easily using the Square Root Property.

> **The square root property** states that
>
> **$x^2 = c$ is equivalent to $\pm\sqrt{c}$**

The symbol "±" is read "**plus or minus**" (positive or negative).

"±3" is read "plus or minus 3"

"x = ±3" is read "x equals plus or minus 3"

This means x = +3 or x = -3.

A positive real number **N** has two square roots. A positive root called the **principal square root**, written $\sqrt{N}$, and **a negative square root**, written $-\sqrt{N}$. We can represent these two square roots by the symbol $\pm\sqrt{N}$.

Now let us work out Example 7 using this method.

$x^2 - 4 = 0$	
$x^2 = 4$	
$x = \pm\sqrt{4}$	Square root property
$x = \pm 2$	Solutions or roots

This means x = +2 or x = -2.

Here is another example.

Example 8 Solve $3x^2 - 108 = 0$ using the square root property.

Solution:

$3x^2 - 108 = 0$

$3x^2 = 108$

$x^2 = 36$ Divide both sides by 3

$x = \pm\sqrt{36}$ Square root property

$x = \pm 6$ Roots or solutions

Checking:

When $x = 6$	When $x = -6$
$3(6)^2 - 108 = 0$	$3(-6)^2 - 108 = 0$
$3(36) - 108 = 0$	$3(36) - 108 = 0$
$108 - 108 = 0$	$108 - 108 = 0$
$0 = 0$ true	$0 = 0$ true

Therefore the solutions are $x = 6$ and $x = -6$.

Example 9 Solve $x^2 - 4x + 4 = 121$ using square root method.

Solution:

$x^2 - 4x + 4 = 121$

$(x-2)^2 = 121$ Factor

$x - 2 = \pm\sqrt{121}$ Square root property

$x - 2 = \pm 11$ Simplify

$x = 2 \pm 11$

$x = 2 + 11$ or $x = 2 - 11$

$x = 13$ $x = -9$ Roots

Checking:

When $x = 13$	When $x = -9$
$x^2 - 4x + 4 = 121$	$x^2 - 4x + 4 = 121$
$(13)^2 - 4(13) + 4 = 121$	$(-9)^2 - 4(-9) + 4 = 121$
$169 - 52 + 4 = 121$	$81 + 36 + 4 = 121$
$121 = 121$ true	$121 = 121$ true

Therefore the solutions are $x = 13$ and $x = -9$.

We summarize the steps we have learnt.

To solve a quadratic equation of the form $\mathbf{ax^2 + c = 0}$, where $c \geq 0$ and $b = 0$.

1. Arrange the equation so that the second-degree term is on the left side and the constant term is on the right side of the equal sign.
2. Divide both sides by the coefficient of x^2.
3. Take the square root of both sides and simplify the square root.
4. There are both positive and negative answers (written ±).

$$ax^2 + c = 0$$
$$ax^2 = -c$$
$$x^2 = -\frac{c}{a}$$
$$\sqrt{x^2} = \pm\sqrt{\frac{c}{a}}$$
$$x = \pm\sqrt{\frac{c}{a}}$$

5. When the radicand is $-\frac{c}{a}$ is:

   Positive: The square roots are real numbers.

   Negative: The square roots are complex numbers.

Now do the learning activities.

LEARNING ACTIVITY 12.4.3.2

20 minutes

Solve each quadratic equation by factoring and check the solutions.

1. $x^2 = x + 2$

2. $\frac{x}{2} + \frac{2}{x} = \frac{5}{2}$

3. $x(x - 2) = (2x + 3)x$

4. $12x = 8x^3$

5. $\frac{2x^2}{3} = 4x$

6. $x^2 = 144$

7. $x^2 + 5x + 6 = 0$

8. $(x + 2)(x - 3) = 50$

9. $2y^2 - 18 = 0$

10. $3x(2x + 3) = 0$

12.4.3.3 Solving Quadratic Equations by Completing the Square

Another method of solving quadratic equations is called **completing the square.**

Considering the equation $ax^2 + bx + c = 0$ when $a = 1$, the process of completing the square is done in the following steps:

Step 1: Place the unknown terms in the left member and the known term in the right member.
Step 2: Complete the square by adding to both sides of the equation the square of one-half of the coefficient of x. The left member of the resulting equation is now a perfect trinomial square.
Step 3: Factor the left- hand side.
Step 4: Extract the square root of both sides of the equation, prefixing the ± sign to the right member. Simplify the result.
Step 4: If **a ≠ 1**, divide both sides of $ax^2 + bx + c = 0$ by **a** and apply steps 1, 2, 3 and 4.
Step 5: Check the roots obtained.

Example 1 Solve the equations $x^2 - 8x + 12 = 0$.

Solution: $x^2 - 8x + 12 = 0$

$x^2 - 8x = -12$ Place known term to the right side.

Take $\frac{1}{2}(-8) = -4$. Then $(-4)^2 = 16$

$x^2 - 8x + 16 = -12 + 16$ Add 16 to both sides to make the left side a trinomial square.

$(x-4)^2 = 4$ Factor the left-hand side

$\sqrt{(x-4)^2} = \pm\sqrt{4}$ Take the square root of both sides

$x - 4 = \pm 2$ Simplify the radicals

$x = 4 \pm 2$ Add 4 to both sides

$x = 6$ or $x = 2$ Roots

Check: $x^2 - 8x + 12 = 0$

When x = 6,	$6^2 - 8(6) + 12 = 0$	When x = 2,	$2^2 - 8(2) + 12 = 0$
	$36 - 48 + 12 = 0$		$4 - 16 + 12 = 0$
	$-12 + 12 = 0$		$-12 + 12 = 0$
	$0 = 0$		$0 = 0$

Example 2 Solve the equation $x^2 = 21 + 4x$ by completing the square.

Solution: $x^2 = 21 + 4x$

$x^2 - 4x = 21$ Place unknown term to the left side.

Take $\frac{1}{2}(-4) = -2$. Then $(-2)^2 = 4$

$x^2 - 4x + 4 = 21 + 4$ Add 4 to both sides to make the left side a trinomial square.

$(x - 2)^2 = 25$ Factor the left side

$\sqrt{(x-2)^2} = \pm\sqrt{25}$ Take the square root of both sides.

$x - 2 = \pm 5$ Simplify the radicals.

$x = 2 \pm 5$ Add 4 to both sides.

$x = 7$ or $x = -3$ Roots

Check: $x^2 - 4x = 21$

When $x = 7$, $7^2 - 4(7) = 21$
$49 - 28 = 21$
$21 = 21$

When $x = -3$, $(-3)^2 - 4(-3) = 21$
$9 + 12 = 21$
$21 = 21$

Therefore, the solutions are $x = 7$ and $x = -3$.

When you have mastered the steps, we do not need to show the steps when solving the equation.

Example 3 Solve the equation $x^2 + 4x = 5$ by completing the square.

Solution:

$$x^2 + 4x = 5$$
$$x^2 + 4x + 4 = 5 + 4$$
$$(x + 2)^2 = 9$$
$$x + 2 = \pm\sqrt{9}$$
$$x = \pm 3 - 2$$
$$x = 3 - 2 \quad \text{or } x = -3 - 2$$
$$x = 1 \qquad x = -5$$

Check: $x^2 + 4x = 5$

When $x = 1$, $(1)^2 + 4(1) = 5$
$1 + 4 = 5$
$5 = 5$

when $x = -5$, $(-5)^2 + 4(-5) = 5$
$25 - 20 = 5$
$5 = 5$

Therefore, the solutions are $x = 1$ and $x = -5$.

We shall now consider the equation $ax^2 + bx + c = 0$ when $a \neq 1$. Since $a \neq 1$, we can make the coefficient of x^2 equal to 1 by dividing every term of the equation by the coefficient of x^2.

Example 4 Solve $2x^2 + 3x - 5 = 0$ by completing the square.

Solution:

$$2x^2 + 3x - 5 = 0$$

$$2x^2 + 3x = 5$$ Place unknown term to the left side.

$$x^2 + \frac{3}{2}x = \frac{5}{2}$$ Divide both sides by the coefficient of x^2

Take $\frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$. Then $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{5}{2} + \frac{9}{16}$$ Add $\frac{9}{16}$ to both sides to make the left side a trinomial square.

$$\left(x^2 + \frac{3}{4}\right)^2 = \frac{49}{16}$$ Factor the left side

$$x + \frac{3}{4} = \pm\frac{7}{4}$$ Extracting the square roots of both sides

$$x = \pm\frac{7}{4} - \frac{3}{4}$$ Solve for x

$$x = \frac{7}{4} - \frac{3}{4} = \frac{4}{4} \text{ or } \mathbf{1}$$

$$x = -\frac{7}{4} - \frac{3}{4} = -\frac{10}{4} \text{ or } -\frac{\mathbf{5}}{\mathbf{2}}$$

Solutions or roots

Check: $2x^2 + 3x - 5 = 0$

When $x = 1$, $2(1)^2 + 3(1) - 5 = 0$

$$2 + 3 - 5 = 0$$

$$0 = 0$$

when $x = -\frac{5}{2}$, $2(-\frac{5}{2})^2 + 3(-\frac{5}{2}) - 5 = 0$

$$2\left(\frac{25}{4}\right) - \frac{15}{2} - 5 = 0$$

$$\frac{25}{2} - \frac{15}{2} - 5 = 0$$

$$\frac{10}{2} - 5 = 0$$

$$5 - 5 = 0$$

$$0 = 0$$

Therefore, the solutions are x = 1 and x = $-\frac{5}{2}$.

Example 4 Solve the equation $3x^2 - 12x + 6 = 0$ by completing the square.

Solution: $3x^2 - 12x + 6 = 0$

$3x^2 - 12x = -6$ Place unknown term to the left side

Because a ≠ 1, we first divide both sides of the equation by 3

$x^2 - 4x = -2$

Take $\frac{1}{2}(4) = 2$. Then $2^2 = 4$

$x^2 - 4x + 4 = -2 + 4$ Add 4 on both sides to make the left side a trinomial square.

$x^2 - 4x + 4 = 2$ Addition

$(x - 2)^2 = 2$ Factor the left side

$x - 2 = \pm\sqrt{2}$

$x = 2 \pm \sqrt{2}$ Solutions or roots

Check: $3x^2 - 12x + 6 = 0$

When $x = 2 + \sqrt{2}$,

$$3(2 + \sqrt{2})^2 - 12(2 + \sqrt{2}) + 6 = 0$$
$$3(4 + 4\sqrt{2} + 2) - 24 - 12\sqrt{2} + 6 = 0$$
$$12 + 12\sqrt{2} + 6 - 24 - 12\sqrt{2} + 6 = 0$$
$$0 = 0$$

When $x = 2 - \sqrt{2}$,

$$3(2 - \sqrt{2})^2 - 12(2 - \sqrt{2}) + 6 = 0$$
$$3(4 - 4\sqrt{2} + 2) - 24 + 12\sqrt{2} + 6 = 0$$
$$12 - 12\sqrt{2} + 6 - 24 + 12\sqrt{2} + 6 = 0$$
$$0 = 0$$

Therefore, the solutions are $x = 2 + \sqrt{2}$, and $x = 2 - \sqrt{2}$,

Now do the learning activity.

LEARNING ACTIVITY 12.4.3.3

20 minutes

Solve each quadratic equation by completing the square.

1. $x^2 = 6x + 11$

2. $x^2 - 13 = 4x$

3. $x^2 + 6x + 4 = 0$

4. $x^2 + 6x = 16$

5. $x^2 - 3x = 4$

6. $x^2 - 10x = 39$

12.4.3.4 Solving Quadratic Equations by the Quadratic Formula

Another method of solving quadratic equation is by the **quadratic formula**. We observed in the previous section that we can solve any quadratic equation of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and when $a \neq 1$ using completing the square. Using the steps of completing the square, we can derive the quadratic formula.

The following shows how the quadratic formula may be derived using the general type of quadratic equation $ax^2 + bx + c = 0$.

$ax^2 + bx + c = 0$ General form

$ax^2 + bx = 0 - c$ subtract c from both sides

$x^2 + \frac{b}{a}x = \frac{-c}{a}$ divide both sides by a

Take $\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}$. Then $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$ add $\frac{b^2}{4a^2}$ to both sides to make the left side a trinomial square

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ Factor the left side and add fractions on the right side

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$ Take the square root of both sides

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ simplify radicals

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ add $-\frac{b}{2a}$ to both sides

Therefore, $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Quadratic Formula

The procedure for using the quadratic formula in solving quadratic equation can be summarized as follows:

To solve quadratic equation by Formula

1. Arrange the equation in general form.
2. Substitute the values of a, b, and c into the quadratic formula.
3. Simplify your answers.
4. Check your answers by substituting them in the original equation.

Example 1 Solve the equation $x^2 - 5x + 6 = 0$ by the quadratic formula.

Solution: Substitute a = 1, b = -5 and c = 6 in the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2} \quad \begin{cases} x = \frac{5+1}{2} = \frac{6}{2} = \mathbf{3} \\ x = \frac{5-1}{2} = \frac{4}{2} = \mathbf{2} \end{cases}$$

Checking: For x = 3

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (3)^2 - 5(3) + 6 &= 0 \\ 9 - 15 + 6 &= 0 \\ 15 - 15 &= 0 \\ 0 &= 0 \quad \text{true} \end{aligned}$$

For x = 2

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (2)^2 - 5(2) + 6 &= 0 \\ 4 - 10 + 6 &= 0 \\ 10 - 10 &= 0 \\ 0 &= 0 \quad \text{true} \end{aligned}$$

Therefore the solution set is {3, 2}.

Example 2 Find the solution set of the equation $x^2 + 3x - 10 = 0$ using the quadratic formula.

Solution: Identify the value of a, b, and c from the given equation.

a = 1, b = 3 and c = -10, substitute these values into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we have $$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

$$x = \frac{-3+7}{2} = \frac{4}{2} = \mathbf{2}$$

$$x = \frac{-3-7}{2} = \frac{-10}{2} = \mathbf{-5}$$

Checking: For x = 2

$$x^2 + 3x - 10 = 0$$
$$(2)^2 + 3(2) - 10 = 0$$
$$4 + 6 - 10 = 0$$
$$10 - 10 = 0$$
$$0 = 0 \quad \text{true}$$

For x = -5

$$x^2 + 3x - 10 = 0$$
$$(-5)^2 + 3(-5) - 10 = 0$$
$$25 - 15 - 10 = 0$$
$$10 - 10 = 0$$
$$0 = 0 \quad \text{true}$$

Therefore the solution set is {2, -5}.

Example 3 Find the solution set of the equation $x^2 + 4x - 5 = 0$ using the quadratic formula.

Solution: Identify the value of a, b, and c from the given equation.

a = 1, b = 4 and c = -5, substitute these values into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we have $$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 20}}{2}$$

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

$$x = \frac{-4 \pm 6}{2}$$

$$x = \frac{-4+6}{2} = \frac{2}{2} = \mathbf{1}$$

$$x = \frac{-4-6}{2} = \frac{-10}{2} = -5$$

Substituting these values of x = 1 and x = -5 to the original equation leads to a true statement.

Checking: For x = 1

$$x^2 + 4x - 5 = 0$$
$$(1)^2 + 4(1) - 5 = 0$$
$$1 + 4 - 5 = 0$$
$$5 - 5 = 0$$
$$0 = 0 \quad \text{true}$$

For x = -5

$$x^2 + 4x - 5 = 0$$
$$(-5)^2 + 4(-5) - 5 = 0$$
$$25 - 20 - 5 = 0$$
$$5 - 5 = 0$$
$$0 = 0 \quad \text{true}$$

Therefore the solution set is {1, -5}.

Example 4 Solve the equation $18x^2 - 12x + 2 = 0$ using the quadratic formula.

Solution: $18x^2 - 12x + 2 = 0$ ⟵ Observe that 2 is a common factor on the left member so divide both members of the equation by 2

Hence, we get $9x^2 - 6x + 1 = 0$ ⟵ Where the values of a, b and c became smaller and easy to handle.

Thus, $18x^2 - 12x + 2 = 0$ is equivalent to $9x^2 - 6x + 1 = 0$, where a = 9, b = -6 and c = 1.

Substituting these values to the quadratic formula, we have:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$
$$x = \frac{6 \pm \sqrt{36 - 36}}{18}$$
$$x = \frac{6 \pm \sqrt{0}}{18}$$
$$x = \frac{6}{18} = \mathbf{\frac{1}{3}}$$

Checking:

$$18x^2 - 12x + 2 = 0$$
$$18\left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 2 = 0$$
$$18\left(\frac{1}{9}\right) - 4 + 2 = 0$$
$$2 - 4 + 2 = 0$$
$$4 - 4 = 0$$
$$0 = 0$$

Therefore the solution set is $\left\{\mathbf{\frac{1}{3}}, \mathbf{\frac{1}{3}}\right\}$. We say that $\mathbf{\frac{1}{3}}$ is a root of multiplicity 2.

Example 5 Solve the equation $2x^2 + 5x - 5 = 0$, using quadratic formula.

Solution: Identify the value of a, b, and c from the given equation.

a = 2, b = 5 and c = -5, substitute these values into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we have $$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 40}}{4}$$

$$x = \frac{-5 \pm \sqrt{65}}{4}$$

$$x = \frac{-5 \pm \sqrt{65}}{4} \longleftarrow \begin{cases} x = \dfrac{-5 + \sqrt{65}}{4} \\[2ex] x = \dfrac{-5 - \sqrt{65}}{4} \end{cases}$$

Substituting these values of $x = \frac{-5 + \sqrt{65}}{4}$ and $x = \frac{-5 - \sqrt{65}}{4}$ to the original equation makes a true statements.

Therefore, the solution set is $\left\{\frac{-5 + \sqrt{65}}{4}, \frac{-5 - \sqrt{65}}{4}\right\}$.

Now do the learning activity.

LEARNING ACTIVITY 12.4.3.4

20 minutes

Solve each of the following using the quadratic formula.

1. $x^2 - 8x + 12 = 0$

2. $x^2 - 4x + 2 = 0$

3. $2x^2 - 3x = 9$

4. $3x^2 - x - 2 = 0$

5. $\frac{3x}{2} - \frac{1}{3x} = -\frac{1}{2}$

12.4.3.5: Graphing of Quadratic Functions

We have learnt in the preceding topics that the graph of a first degree equation in x and y is a straight line. In this section, we will learn how to graph quadratic functions.

Quadratic function is a function whose equation can be written in the form $\mathbf{y = ax^2 + bx + c}$ where a, b and c are constants and $a \neq 0$.

Quadratic function is also known as **second degree function** because the highest power is 2.

The graph of a quadratic function is a U-shaped curve called a **parabola** which opens up, such as the graph of $f(x) = x^2$ in Figure A, or opens down, such as the graph of $f(x) = -x^2$ in figure B.

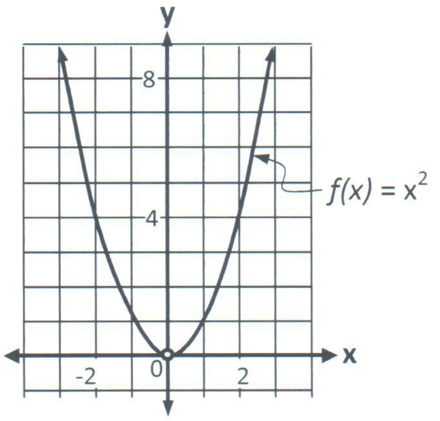


Figure A

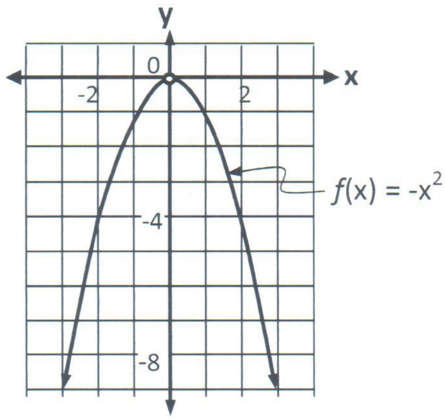


Figure B

Although every quadratic function can be graphed by making a table of values and plotting, it is better to look for ways to simplify the process. In order to help us determine such method, consider the graph of $f(x) = x^2 - 2x - 3$, given in Figure B. Notice that the graph crosses x-axis (y = 0) at – 1 and 3. The points (-1, 0) and (3, 0) are called **x – intercepts.** The graph has a lowest point or minimum value at (1, -4), called the **vertex**. Also, for each point on the graph there is a corresponding point on the other side of the line x = 1. This is called the **axis or line of symmetry.**

x	$f(x)$
0	-3
1	-4
-1	0
2	-3
-2	5
3	0
4	5

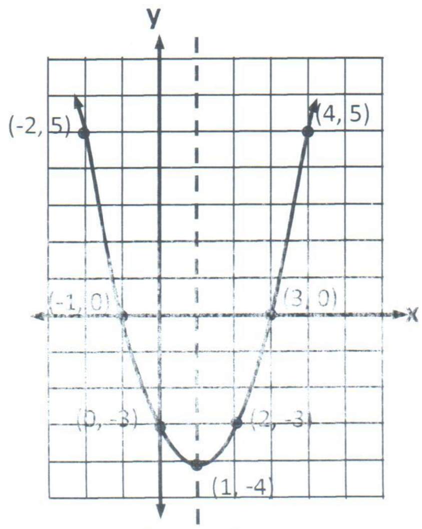


Figure B

If we know the x-intercepts, the vertex, and the axis of symmetry of a parabola, the rest of the graph is easy to determine. In addition, knowing that **a** is positive, tells us the parabola opens upward and if **a** is negative, the parabola opens downward. This is summarized in Figure C.

The graph of a parabola $f(x) = ax^2 + bx + c$

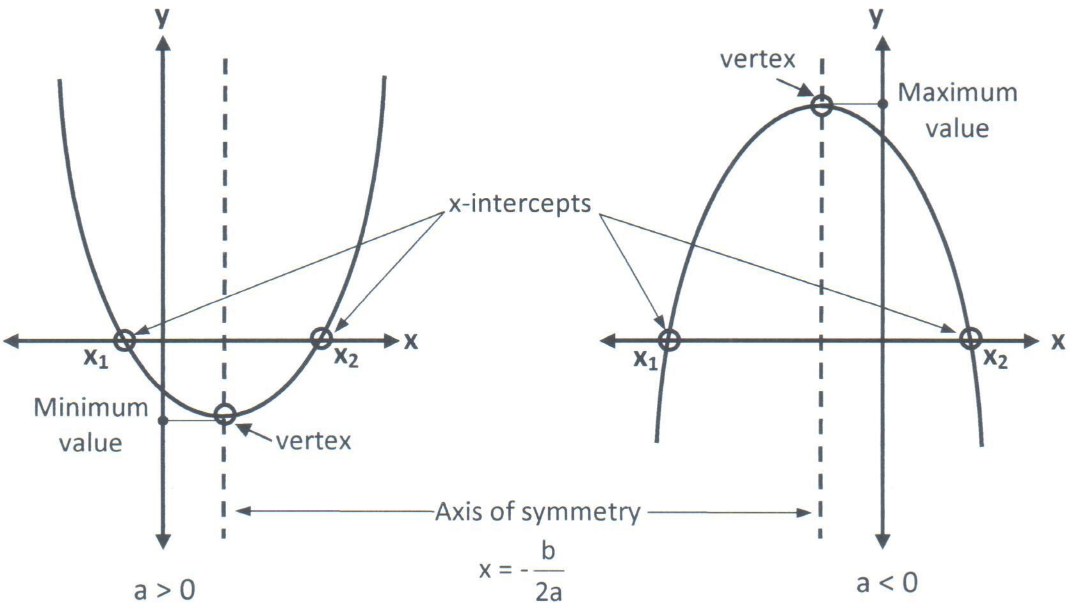


Figure C

The x and y-intercept of a quadratic function

- The y-intercept of the quadratic function is found by setting x = 0.

$$y = ax^2 + bx + c$$
$$y = a(0)^2 + b(0) + c$$
$$y = c$$

The y- intercept of a quadratic function is always equal to the constant term.

- The x–intercepts of a quadratic function are found by setting y = 0 and then solving the resulting quadratic equation.

$$y = ax^2 + bx + c = 0$$

Example 1 Find the x-intercepts of $y = x^2 + x - 6$

Solution: Set y = 0 $y = x^2 + 2x - 6 = 0$

$$(x - 2)(x + 3) = 0$$
$$x - 2 = 0; \quad x + 3 = 0$$
$$x = 2; \quad x = -3$$

The x-intercepts are 2 and -3.

Example 2 Find the x-intercepts of $y = f(x) = 9x^2 - 6x + 1$.

Solution: Set $y = 0$

$$y = 9x^2 - 6x + 1 = 0$$
$$(3x - 1)(3x - 1) = 0$$
$$3x - 1 = 0;\ 3x - 1 = 0$$
$$x = \frac{1}{3}\ ;\ x = \frac{1}{3}$$

The x-intercepts are both the same, which means that the parabola touches the x-axis in only one point, $x = \frac{1}{3}$.

Axis of Symmetry of a Parabola

Symmetrical points of the parabola are shown in Figure D.

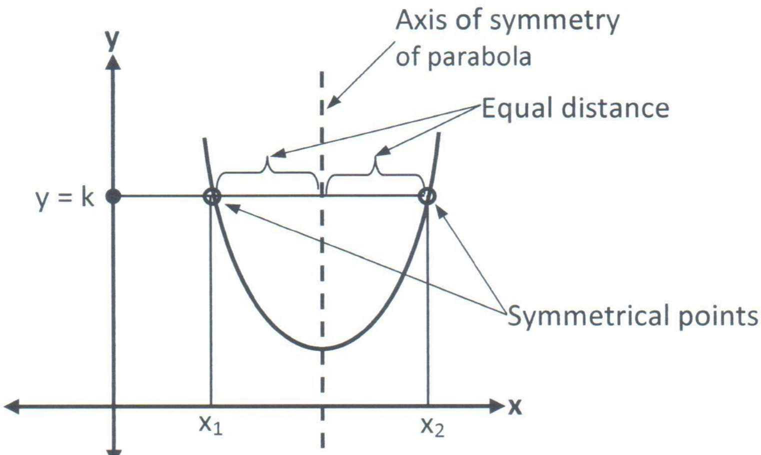


Figure D

To find the axis of symmetry of the parabola

$y = ax^2 + bx + c$ General equation of a parabola

when $y = k$, then $y = ax^2 + bx + c = k$

$ax^2 + bx + c - k = 0$ Quadratic equation

Using the Quadratic formula:

$$x_1 = \frac{-b - \sqrt{b^2 - 4a(c-k)}}{2a} = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4a(c-k)}}{2a}$$

$$x_2 = \frac{-b + \sqrt{b^2 - 4a(c-k)}}{2a} = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4a(c-k)}}{2a}$$

$$x_1 + x_2 = 2\left(\frac{-b}{2a}\right)$$ The sum of the two preceding equations

$$\frac{x_1 + x_2}{2} = \frac{\cancel{2}\left(\frac{-b}{2a}\right)}{\cancel{2}} = -\frac{b}{2a}$$ This is the average of x_1 and x_2.

The **axis of symmetry** of a parabola is a vertical line midway between any pair of symmetrical points on that parabola. Therefore, the axis of symmetry is the average of x_1 and x_2.

The equation of the axis of symmetry
$\mathbf{x = -\frac{b}{2a}}$

Example 3 Find the equation of the axis of symmetry for the graph of the quadratic function $y = f(x) = 2x^2 + 7x - 4$.

Solution: $x = -\frac{b}{2a} = -\frac{7}{2(2)} = -\frac{7}{4}$

Therefore, $x = -\frac{7}{4}$ is the equation of the axis of symmetry.

Minimum and maximum values of quadratic function

The **minimum value** of a quadratic function is the smallest value that it can have. A quadratic function of one variable has a minimum value when **a** is positive. The minimum value is obtained by evaluating the function at $x = -\frac{b}{2a}$, the axis of symmetry. (Figure E).

The **maximum value** of a quadratic function is the largest value that it can have. A quadratic function of one variable has a maximum value when **a** is negative. The minimum value is obtained by evaluating the function at $x = -\frac{b}{2a}$, the axis of symmetry. The maximum value is obtained by evaluating the function at $x = -\frac{b}{2a}$, the axis of symmetry. (Figure F).

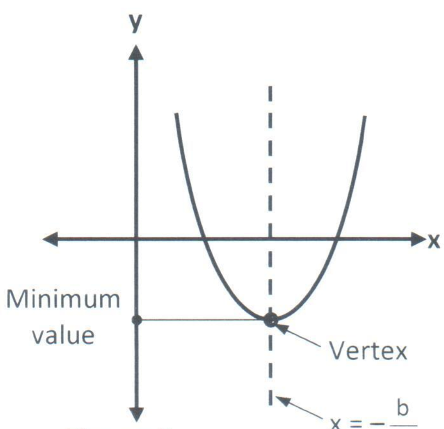


Figure E

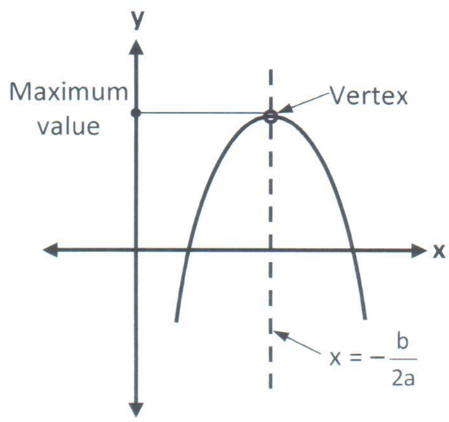


Figure F

Turning Point or Vertex

The **turning point** or **vertex** of any parabola is that point of the parabola where the maximum and minimum value of the function is reached. (See Figures E and F on page 107).

Example 4 Find the maximum or minimum value of $f(x) = 3x^2 - 6x + 2$ and find its vertex.

Solution: **a** is positive, therefore, $f(x)$ has a minimum value.

$$\text{Minimum value} = f\left(-\frac{b}{2a}\right) = f\left(-\frac{-6}{2(3)}\right) = f\left(\frac{6}{6}\right) = f(1)$$

$$\text{Minimum value} = f(1) = 3(1)^2 - 6(1) + 2 = 3 - 6 + 2 = -1$$

Vertex = (1, -1)

Example 5 Find the maximum or minimum value of $f(x) = 5 - 2x - 4x^2$ and find its vertex.

Solution: $f(x) = -4x^2 - 2x + 5$ in General Form

a is negative, therefore, $f(x)$ has a maximum value.

$$\text{Maximum value} = f\left(-\frac{b}{2a}\right) = f\left(-\frac{-2}{2(-4)}\right) = f\left(\frac{2}{-8}\right) = f\left(-\frac{1}{4}\right)$$

$$\text{Maximum value} = f\left(-\frac{1}{4}\right) = -4\left(-\frac{1}{4}\right)^2 - 2\left(-\frac{1}{4}\right) + 5 = -\frac{1}{4} + \frac{2}{4} + 5$$

$$= 5\frac{1}{4} \text{ or } \frac{21}{5}$$

Vertex = $(-\frac{1}{4}, 5\frac{1}{4})$

Steps in Graphing a Quadratic Function

To graph a quadratic function $f(x) = ax^2 + bx + c$

1. Determine whether the graph opens up ($a > 0$) or down ($a < 0$).
2. Find the vertex and axis of symmetry using the fact that the x-coordinate of the vertex is $-\frac{b}{2a}$ and the axis of symmetry is $x = -\frac{b}{2a}$.

   The y-coordinate of the vertex is found by evaluating $f\left(-\frac{b}{2a}\right)$.
3. If the vertex is located above the x-axis and the graph opens down, or if the vertex is located below the x-axis and opens up, find the x-intercepts by solving the equation $ax^2 + bx + c = 0$.
4. Plot the vertex and the x-intercepts (or two other points on either side of the line of symmetry if there are no x-intercepts) and sketch the parabola.

Example 6 Graph $f(x) = x^2 + 5x + 4$

Solution: Since a = 1 > 0, the parabola opens upward.

With a = 1 and b = 5, $-\frac{b}{2a} = -\frac{5}{2(1)} = -\frac{5}{2}$

So, the axis of symmetry is $x = -\frac{5}{2}$.

Also, $f\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + 4$

$$= \frac{25}{4} - \frac{25}{2} + 4$$

$$= \frac{50 - 100 + 32}{8}$$

$$= -\frac{18}{8}$$

$$= -\frac{9}{4}$$

So the vertex is $\left(-\frac{5}{2}, \ -\frac{9}{4}\right)$. With the vertex located below the x-axis, and the parabola opens up, we look for the x-intercepts by evaluating the equation:

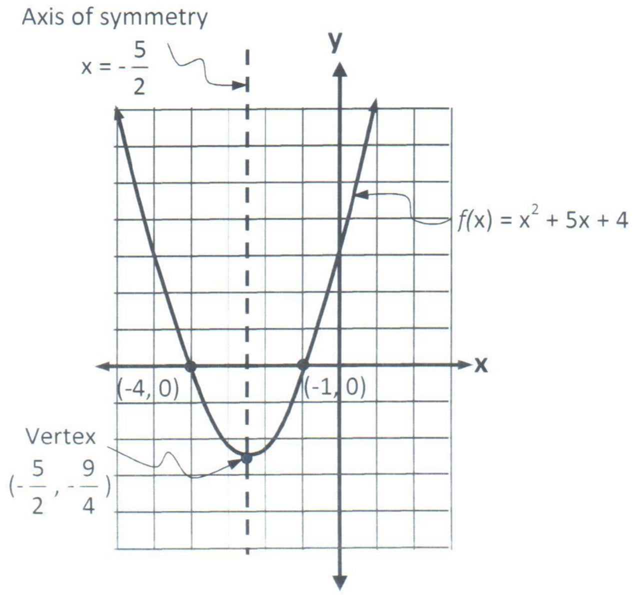
$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$x + 4 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = -4 \text{ or} \qquad x = -1$$

Thus, the x- intercepts are (-4, 0) and (-1, 0) and the graph is given below.



Example 7 Graph $f(x) = 3x^2 + 6x + 4$

Solution: Since a = 3 > 0, the parabola opens upward.

With a = 3 and b = 6, $-\frac{b}{2a} = -\frac{6}{2(3)} = -\frac{6}{6} = -1$

So, the axis of symmetry is x = –1 and the vertex is [-1, f(-1)] = (-1, 1)

With the vertex above the x-axis and the parabola opening upward, there will be no x-intercepts. If we were to solve $3x^2 + 6x + 4 = 0$, the solutions we obtained would be

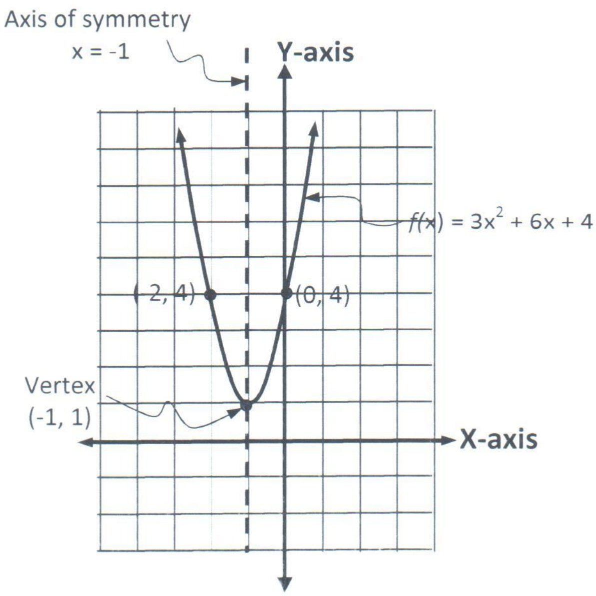
$$\frac{-3 \pm i\sqrt{3}}{3}$$

which are not real numbers. (Remember that coordinate axes are only used to plot real numbers.)

Since there are no x-intercepts, we have to determine two additional points on either side of the axis of symmetry x = -1.

For example, when x = 0, $f(0) = 4$, and when x = -2, $f(-2) = 4$. This gives us the points (0, 4) and (-2, 4).

The graph is given below.



Now do the learning activity.

LEARNING ACTIVITY 12.4.3.5

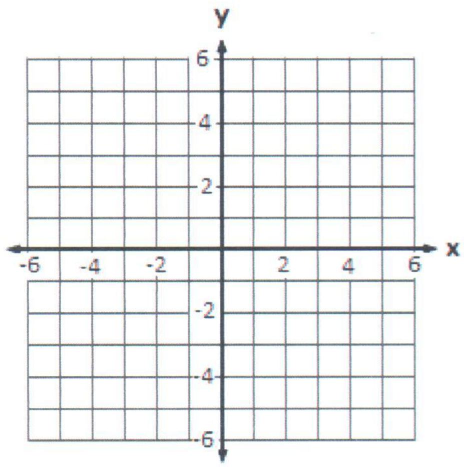
20 minutes

Without graphing, determine the x-intercepts (if they exist), the vertex and whether the graphs of each quadratic function open upward or downward.

1. $f(x) = x^2 - 4x + 3$
2. $f(x) = x^2 + 4x + 3$
3. $f(x) = x^2 + 2x + 2$
4. $f(x) = x^2 + 8x$
5. $f(x) = -x^2 + 5x - 6$
6. $f(x) = -2x^2 + 4x - 3$

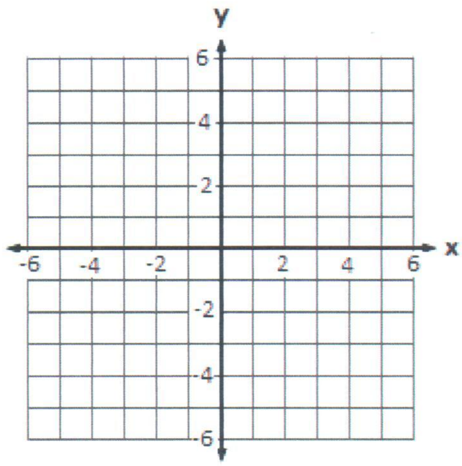
Find the x-intercepts (if they exist), the vertex and graph the function.

7. $f(x) = x^2 - 2x - 3$



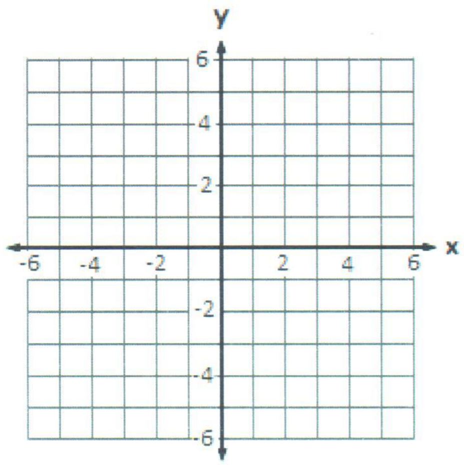
y-intercept: ____________
x-intercepts: ____________
vertex: ____________

8. $f(x) = -x^2 - 2x + 3$



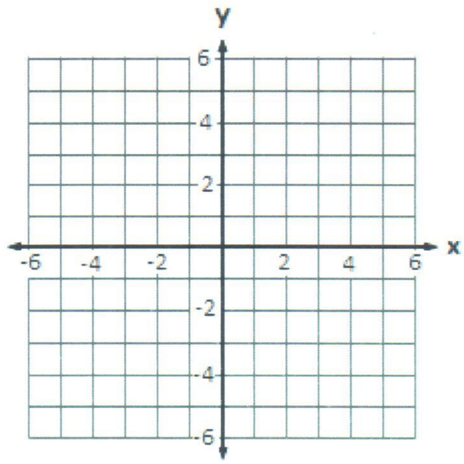
y-intercept: ____________
x-intercepts: ____________
vertex: ____________

9. $f(x) = x^2 - 4x$



y-intercept: ____________
x-intercepts: ____________
vertex: ____________

10. $f(x) = -x^2 + 4x$



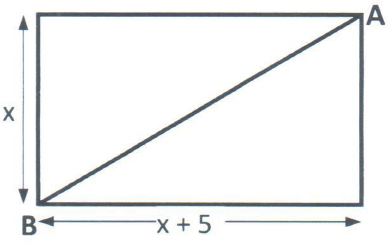
y-intercept: ____________
x-intercepts: ____________
vertex: ____________

12.4.3.6: Application of Quadratic Equation

Some problems in real life situations enable you to create a quadratic equation which you can solve to find an answer. While quadratic equations will have two solutions, it is possible that only one of them will be appropriate as an answer.

Example 1 A rectangle is 5 m longer than its width and has an area of 456 m^2. Calculate the length of the diagonal of the rectangle.

Solution: Show the situation in a diagram.



Let x be the width of the rectangle

x + 5 be the length of the rectangle

AB is diagonal of the rectangle.

Form and solve equation in x.

The area of the rectangle is 456 m^2.

Hence, $x(x+5) = 456$

$x^2 + 5x = 456$

$x^2 + 5x - 456 = 0$ General form

$(x - 19)(x + 24) = 0$ Factor the left side

$x - 19 = 0, x + 24 = 0$ Zero product property

$x = 19$ or $x = -24$

The width of the rectangle is 19 m.

The length is 19 + 5 = 24 cm.

To find the length of the diagonal AB, use the Pythagorean Theorem.

$AB^2 = 19^2 + 24^2$

$AB^2 = 361 + 576$

$AB^2 = 937$

$AB = \sqrt{937}$

$AB = 30.61045$

Therefore, the length of the diagonal is 30.6 m (1 decimal point).

Example 2

Gerry James invested K50 000 at a certain interest rate compounded semi-annually in one year. The amount of investment at the end of a year is K57 500. What is the rate of interest applied?

Solution: Let i = interest rate per conversion period, where $i = \frac{x}{2}$ (semi-annually)

x = interest rate for 1 year

Given: K57 500 = amount of investment at the end of 1 year (A)
K50 000 = principal or amount invested (P)
1 year = time

Use the formula $A = P(1 + i)^2$.
Now, by substitution, we have:

$$A = P(1 + i)^2$$

$$K57\,500 = K50\,000(1 + i)^2$$

$$\frac{K57\,500}{K50\,000} = (1 + i)^2 \qquad \text{divide both sides by K50 000}$$

$$\frac{23}{20} = (1 + i)^2 \qquad \text{simplify the left hand side}$$

$$\pm\sqrt{\frac{23}{20}} = 1 + I \qquad \text{squaring both sides}$$

$$\pm 1.\,072 = 1 + I \qquad \text{find the square root of the left side}$$

$$i = \pm 1.\,072 - 1 \qquad \text{subtract 1 from both sides}$$

$$\mathbf{i = 0.072} \text{ or } \mathbf{i = -2.072}$$

We only consider positive answer, so disregard the negative one.

Since $i = \frac{x}{2}$, then $x = 2(i)$. Now, solve for x.

$$x = 2(0.72)$$

$$x = 0.144$$

$$x = 14.4\ \%$$

Therefore, Gerry James invested his money at an interest of 14.4%.

Example 3

Find the number whose square is 5 less than 6 times the number.

Solution: Let x = the number
x^2 = square of the number
$6x - 5$ = 5 less than 6 times the number

Form the equation: $x^2 = 6x - 5$

$x^2 - 6x + 5 = 0$ general form

$(x - 5)(x - 1) = 0$ factor the left side

$x - 5 = 0$ or $x - 1 = 0$ Zero product property

$x = 5$ or $x = 1$

Checking:

For $x = 5$	For $x = 1$
$x^2 = 6x - 5$	$x^2 = 6x - 5$
$(5)^2 = 6(5) - 5$	$(1)^2 = 6(1) - 5$
$25 = 30 - 5$	$1 = 6 - 5$
$25 = 25$	$1 = 1$

Therefore, the number is 5 or 1.

Example 4

Karl has a rectangular garden whose length is 6 m more than its width. The area of the garden is 27 m^2. Find the dimensions of the garden.

Solution: Let x = the width of the garden
$x + 6$ = the length of the garden
27 m^2 = area of the rectangular garden

Using the formula, $A = LW$ for area of rectangle, form the equation:

$27 = x(x + 6)$

$27 = x^2 + 6x$ Expand bracket

$x^2 + 6x - 27 = 0$ General form

$(x + 9)(x - 3) = 0$ Factor the left side

$(x + 9) = 0$ or $(x - 3) = 0$ Zero product property

$x = -9$ or $x = 3$ m Disregard the negative answer since distance cannot be negative.

$x + 6 = 3 + 6$

$= 9$ m

Checking: When $x = 3$

$x(x + 6) = 27$

$3(3 + 6) = 27$

$3(9) = 27$

$27 = 27$

Therefore, the width of the rectangular garden is 3 m and the length is 9 m.

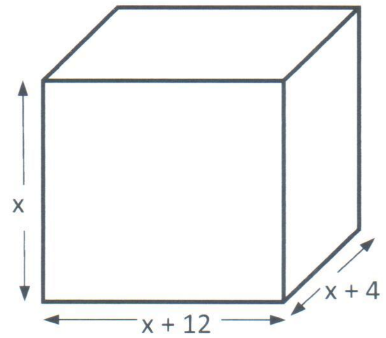
Now do the learning activity.

LEARNING ACTIVITY 12.4.3.6

20 minutes

Solve each of the following problems:

1. The area of a rectangular room is 4 m wider than it is high and it is 8 m longer than it is wide. The total area of the walls is 512 m^2. Find the width of the room.



2. The hypotenuse of a right triangle is 4 times the smallest side. The third side is $\sqrt{735}$. Find the hypotenuse and the smallest side.

3. The area of a rectangular lot is 560 m^2. The length is 3 m more than twice the width. Find the length and the width of the lot.

4. The smallest of three consecutive positive numbers is x. Three times the square of the largest is greater than the sum of the squares of the other two numbers by 67. Find x.

5. In the last planting season, a farmer planted tomatoes in a rectangular plot whose length is 3 m more than its width. Encouraged by a plentiful harvest this season, he increased the length to twice this width, thus having 44 m^2 more in area. What are the dimensions of his plot now?

SUMMATIVE TASK 12.4.2

30 minutes

A. In the following questions, choose the letter (or letters) corresponding to the correct answer (or answers).

1. If (2x – 3)(3x + 4) = 0, then x is equal to

A. $-\frac{3}{2}$ or $\frac{4}{3}$ B. $-\frac{2}{3}$ or $\frac{3}{4}$

C. $\frac{2}{3}$ or $-\frac{3}{4}$ D. $\frac{3}{2}$ or $-\frac{4}{3}$

2. If x(2x – 5) = 0, then x is equal to

A. $\frac{5}{2}$ B. $-\frac{5}{2}$

C. 0 or $-\frac{5}{2}$ D. 0 or $\frac{5}{2}$

3. If $x^2 - 5x - 2 = 0$, then x is equal to

A. $\frac{-5\pm\sqrt{33}}{2}$ B. $\frac{5\pm\sqrt{33}}{2}$

C. $\frac{-5\pm\sqrt{17}}{2}$ D. $\frac{5\pm\sqrt{17}}{2}$

4. If $3x^2 - 125 = 0$, then x is equal to

A. 0 B. $\pm\frac{1}{5}$

C. ± 5 D. $\pm\sqrt{5}$

5. Which of the following is **not** in quadratic form?

A. $x^4 + 3x^2 - 4 = 0$ B. $(x^3 + 1)^2 - (x^3 - 1) - 2 = 0$

C. $x + 2\sqrt{x} + 1 = 0$ D. $x^2 - y = x^2 + 8$

6. Which of the following quadratic equations cannot be solved by factoring?

A. $2x^2 - x - 3 = 0$ B. $16x^2 + 8x + 1 = 0$

C. $25x^2 - 9 = 0$ D. $3x^2 - 2x + 1 = 0$

7. The sum of the roots of the quadratic equation $12x^2 - x + 1 = 0$ is

A. $-\frac{1}{2}$ B. $\frac{1}{12}$

C. 12 D. -12

8. The product of the roots of the quadratic equation $12x^2 - x + 1 = 0$ is

A. $-\frac{1}{2}$ B. $\frac{1}{12}$

C. 12 D. -12

9. Which of the following is the solution set of the quadratic equation $x^2 - 4x + 4 = 121$?

A. (13, -9) B. (-13, 9)

C. (13, 9) D. (-13, -9)

10. The length of a rectangle is 5 less than twice its width. If the area of the rectangle is 63 square units, what would its dimensions be?

A. Width =7; Length = 9 B. Width = 9; Length = 7

C. Width =8; Length = 10 D. Width = 10; Length = 8

B. Solve the following quadratic equation by factoring.

1. $(3n - 2)(4n + 1) = 0$ 2. $m(m - 3) = 0$

C. Solve each equation with the quadratic formula.

1. $y^2 + 2y - 8 = 0$

2. $k^2 + 5k - 6 = 0$

D. Identify the minimum or maximum value and the vertex of each quadratic function.

1. $f(x) = -x^2 - 6x + 7$

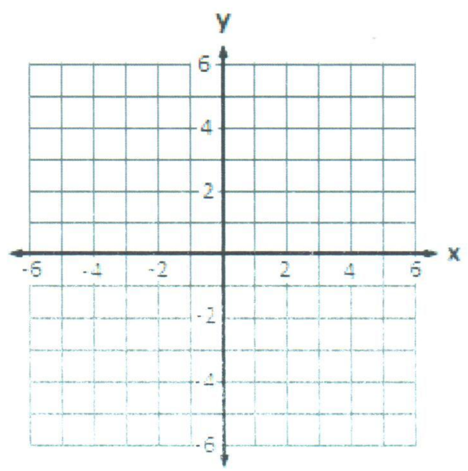
2. $f(x) = 8x - 2x^2 - 3$

3. $f(x) = x^2 - 4x + 5$

4. $f(x) = 4 + 6x + -3x^2$

E. Given the function $6 + x^2 - 4x$, find the following:

1. axis of symmetry
2. minimum or maximum value
3. coordinates of the vertex
4. the x-intercepts (if they exist) and graph the function.



12.4.4 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Exponential and logarithmic functions are important in both theory and practice. In this section we will first look at the basic concepts on exponential and logarithmic functions. Then we will look at the relationships between them.

12.4.4.1 Basic Concepts

You know that $10^2 = 100$.

Exponent

$$10^2 = 100$$

Base Power

The exponent of a number says how many times to use the number in a multiplication.

The same relationship can be thought of in another way,

Logarithm

$$10^2 = 100$$

Base Number

The logarithm is the exponent (2) to which the base (10) must be raised to give the number (100).

$$\log_{10}100 = 2$$

and is read "the logarithm of 100 to the base 10 equals 2."

EXPONENTIAL FORM LOGARITHMIC FORM

Exponent

$$10^2 = 100 \quad \Leftrightarrow \quad \log_{10}100 = 2$$

Base

When converting an equation from exponential to logarithmic form, or from logarithmic to exponential form, it may be helpful to remember that in both cases the base is written below the level of the logarithm (exponent) as indicated in the diagram above.

The symbol ($\Leftrightarrow$) means that the exponential form and the logarithmic form are equivalent.

> The logarithm of a number **y** is the exponent **x** to which the base **a** (a > 1), (a ≠ 1) must be raised to give **y.**
>
> EXPONENTIAL FORM LOGARITHMIC FORM
>
> $$a^x = y \quad \Leftrightarrow \quad x = \log_a y$$

The equivalence on the previous helps in solving logarithmic and exponential functions and needs a deep understanding. Presented below are examples of how the above relationship between the logarithmic and exponential functions may be used to transform expressions.

Example 1

Change each logarithmic expression to an exponential expression.

a. $\log_3 27 = 3$

b. $\log_{36} 6 = \frac{1}{2}$

c. $\log_2\left(\frac{1}{8}\right) = -3$

d. $\log_8 2 = \frac{1}{3}$

Solutions:

a. The logarithmic form $\log_3 27 = 3$ is equivalent to the exponential form $\mathbf{27 = 3^3}$

b. The logarithmic form $\log_{36} 6 = \frac{1}{2}$ is equivalent to the exponential form $\mathbf{6 = 36^{\frac{1}{2}}}$

c. The logarithmic form $\log_2\left(\frac{1}{8}\right) = -3$ is equivalent to the exponential form $\mathbf{\frac{1}{8} = 2^{-3}}$

e. The logarithmic form $\log_8 2 = \frac{1}{3}$ is equivalent to the exponential form $\mathbf{2 = 8^{\frac{1}{3}}}$

Example 2

Change each exponential expression to logarithmic expression.

a. $3^4 = 81$

b. $4^{\frac{1}{2}} = 2$

c. $10^4 = 10\ 000$

d. $10^{-2} = 0.01$

Solutions:

a. The exponential form $3^4 = 81$ is equivalent to the logarithmic form $\mathbf{4 = \log_3 81}$

b. The exponential form $4^{\frac{1}{2}} = 2$ is equivalent to the logarithmic form $\mathbf{\frac{1}{2} = \log_4 2}$

c. The exponential form $10^4 = 10\ 000$ is equivalent to $\mathbf{4 = \log_{10} 10\ 000}$

d. The exponential form $10^{-2} = 0.01$ is equivalent to $\mathbf{-2 = \log_{10} 0.01}$

Solutions to some logarithmic equations are best found by converting to exponential form.

An important property of logarithms and exponents useful in solving logarithmic and exponential equations involves equating exponents in exponentials expressions having the same base.

If $a^x = a^y$, then $x = y$ ($a \neq 0$ and $a \neq 1$)

For example, if we know that $5^x = 5^2$, then $x = 2$. This is used in the example below.

Example 1

Determine the numerical value of x in $x = \log_4 16$.

Solution: We convert $x = \log_4 16$ to exponential form

$$x = \log_4 16 \quad \Leftrightarrow \quad 4^x = 16$$

At this point, it might be clear that x must be 2 since $4^2 = 16$.

By writing 16 as a power of 4, $16 = 4^2$, we have

$$4^x = 4^2$$

In this form, it is obvious that $x = 2$, since the bases on both sides of the equation are 4 making the exponents to both sides to be equal.

A second property of logarithms and exponents involves equating bases on exponential expressions having the same exponents. (Remember $a > 0$, $b > 0$.)

If $a^x = b^x$, then $a = b$ ($a \neq 0$ and $a \neq 1$)

For example, if we know that $x^{-3} = 2^{-3}$, then $x = 2$. See example 2.

Example 2

Solve for x: $\log_x 27 = 3$

Solution: Convert $\log_x 27 = 3$ to exponential form

$$\log_x 27 = 3 \quad \Leftrightarrow \quad x^3 = 27$$

By writing 27 as a power of 3, $27 = 3^3$, we have

$$x^3 = 3^3$$

In this form, it is now clear that $x = 3$, since the exponents on both sides of the equation are 3 making the bases to both sides to be equal.

Here are other examples.

Example 3

Solve for x: $\log_2(x) = 4$

Solution: $\log_2(x) = 4 \Leftrightarrow 2^4 = x$

$$\mathbf{16 = x}$$

Example 5

Find $\log_8(4)$.

Solution: Let $x = \log_8(4) \Leftrightarrow 8^x = 4$

$$(2^3)^x = 2^2$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$\mathbf{x = \frac{2}{3}}$$

Now do the learning activity.

Learning Activity 12.4.4.1

20 minutes

A. Rewrite each equation in exponential form.

1. $\log_6 36 = 2$

2. $\log_{289} 17 = \frac{1}{2}$

3. $\log_{14} \frac{1}{196} = -2$

4. $\log_3 81 = 4$

5. $\log_{11}(121) = 2$

6. $\log_{10}(1000) = 3$

7. $\log_a 3 = x$

8. $a = \log_y x$

B. Rewrite each equation in logarithmic form.

1. $3^2 = 9$
2. $2^5 = 32$
3. $6^3 = 216$
4. $4^5 = 1024$
5. $2^3 = 8$
6. $64^{\frac{1}{2}} = 8$
7. $12^2 = 144$
8. $9^{-2} = \frac{1}{81}$
9. $\left(\frac{1}{12}\right)^2 = \frac{1}{144}$

C. Evaluate each expression.

1. $\log_4 64$
2. $\text{Log}_6 216$
3. $\text{Log}_4 16$
4. $\text{Log}_3 \frac{1}{243}$
5. $\log_5 125$
6. $\log_2 4$
7. $\log_{343} 7$
8. $\log_2 16$
9. $\log_{64} 4$
10. $\log_6 \frac{1}{216}$

12.4.4.2: Laws of Logarithms

There are numbers of rules known as laws of logarithms. These allow expressions involving logarithms to be rewritten in a variety of different ways. The laws apply to logarithms of any base but the same base must be used throughout calculation.

Since logarithm is simply an exponent we expect, the logarithm laws work the same as the rules for exponents.

These are the main laws of logarithms.

First Law:

$$\log_b x + \log_b y = \log_b xy \text{ or } \log_b xy = \log_b x + \log_b y$$

This law tells us how to add two logarithms together. Adding log x and log y results in the logarithm of the product of x and y, that is log xy. This is also known as the **Product Rule**.

Example: $\log_{10} 5 + \log_{10} 4 = \log_{10} (5 \times 4)$

$= \log_{10} 20$

Second Law

$$\log_b x - \log_b y = \log_b \frac{x}{y} \text{ or } \log_b \frac{x}{y} = \log_b x - \log_b y$$

This law tells us how to subtract two logarithms together. Subtracting log y from log x results in the logarithm of the quotient of x and y, that is $\log \frac{x}{y}$. It is known as the **Quotient Rule.**

Example: $\log_4 16 - \log_4 x = \log_4 \frac{16}{x}$

Third Law

$$\log_b x^n = n \log_b x$$

This law tells us that the logarithm of a number x with a rational exponent n is equal to the exponent times the logarithm. This is also known as the **Power Rule**.

Example: $\log_{10} 5^3 = 3 \log_{10} 5$

There are two other important laws.

$$\log 1 = 0 \text{ and } \log_m m = 1$$

The logarithm of 1 to any base is always 0, and the logarithm of a number to the same base is always 1. In particular, $\log_{10} 10 = 1$, and $\log_e e = 1$.

Now look at the examples.

Example 1 Expand log 7x as a sum of two logarithms.

Solution: Using the first law,

$\log 7x = \log 7 + \log x$

Note 1: This has the same meaning as $10^7 \times 10^x = 10^{7+x}$

Note 2: This question is **not** the same as $\log_7 x$, which means "log of *x* to the base 7", which is quite different.

Example 2 Express as a multiple of logarithms: $\log x^5$

Solution: Using the third law,

$\log x^5 = 5 \log x$

We have expressed it as a multiple of a logarithm, and it no longer involves an exponent.

Example 3 Simplify log 20 – log 5

Solution: Using the second law,

$$\log 20 - \log 5 = \log \frac{20}{5}$$

$$= \log 4$$

Example 4 Express $\log_2 4^3$ as a product.

Solution: Using the third law,

$\log_2 4^3 = 3 \log_2 4$

Example 5

Use the laws of logarithms to expand the following logarithmic expressions:

a. $\log_3(2xy)$ b. $\ln \frac{x}{8}$ c. $\log x^2$

Solutions:

a. Use the Power Rule for logarithms

$\log_3(2xy) = \log_3 2 + \log_3 x + \log_3 y$

b. Apply the Quotient Rule for logarithms

$$\ln\frac{x}{8} = \ln x - \ln 8$$

c. Use the Power Rule for logarithms

$$\log x^2 = 2\log x$$

Example 6

Use the laws of logarithms to write each expression as a single logarithm expression.

a. $\log_3 4 + \log_3 x$ b. $\ln(x - y) - \ln 2$ c. $3\log\frac{1}{2} + \log(x - y)$

Solutions:

a. Apply the Power Rule for logarithms

$$\log_3 4 + \log_3 x = \log_3 4x$$

b. Apply the Quotient Rule for logarithms

$$\ln(x - y) - \ln 2 = \ln\frac{x - y}{2}$$

c. $3\log\frac{1}{2} + \log(x - y) = \log\left(\frac{1}{2}\right)^3 + \log(x - y)$ By the Power Rule for logarithms

$= \log\left[\frac{1}{8}(x - y)\right]$ By the Product Rule

Example 7

Given that $\log_a 2 = 0.3010$ and $\log_a 3 = 0.4771$, find the following logarithms:

a) $\log_a 6$ b. $\log_a 9$ c. $\log_a \frac{3}{2}$

Solutions:

a. $\log_a 6 = \log_a (2)(3) = \log_a 2 + \log_a 3 = 0.3010 + 0.4771 = \mathbf{0.7781}$

b. $\log_a 9 = \log_a 3^2 = 2\log_a 3 = 2(0.47710 = \mathbf{0.9542}$

c. $\log_a \frac{3}{2} = \log_a 3 - \log_a 2 = 0.4771 - 0.3010 = \mathbf{0.1761}$

Now do the learning activity.

Learning Activity 12.4.4.2

20 minutes

1. Use the first law to simplify the following.

   a) $\log_{10} 6 + \log_{10} 3$

   b) $\log x + \log y$

   c) $\log 4x + \log x$

   d) $\log a + \log b^2 + \log c^3$

2. Use the second law to simplify the following.

   a) $\log_{10} 6 - \log_{10} 3$

   b) $\log x - \log y$

   c) $\log 4x - \log x$.

3. Use the third law to write each of the following in an alternative form.

   a) $3 \log 10^5$

   b) $2 \log x$,

   c) $\log(4x)^2$

   d) $5 \ln x^4$

   e) $\ln 1000$

4. Express as single logarithm.

   a) $2 \log_a x - \log_a y$

   b) $3 \log_a y + 2 \log_a x$

5. Given that $\log_a 3 = 0.4771$ and $\log_a 5 = 0.6990$, find the following logarithms.

   a. $\log_a 15$

   b. b. $\log_a 75$

   c. c. $\log_a \frac{5}{3}$

12.4.4.3: Exponential and Logarithmic Functions and Their Graphs

In the previous sections, we considered constant and linear functions and their graphs. In this section, we will look at two other important kinds of functions: the **exponential** and **logarithmic functions** and their graphs and see how they are related.

Exponential Function

> An **exponential function** is a function of the form **$y = f(x) = a^x$**, where **$a > 0$** and the number **a** is called the **base**.

Example of exponential functions

1. $f(x) = 2^x$ 2. $f(x) = \pi^x$ 3. $f(x) = 3^{-x}$ 4. $f(x) = \left(\frac{1}{2}\right)^x$

Graphs of Exponential Functions

Remember that in order to graph a function, we select values for x (called **domain of the exponential function)**), calculate the corresponding values of y (called **range of the exponential function**), place them in a table of values, and then plot the ordered pairs in a Cartesian coordinate system.

Example 1 Graph the function $y = f(x) = 2^x$.

Solution: We select values of x and calculate the corresponding values of y to obtain the table of values in the table below.

x	$y = f(x) = 2^x$	(x, y)
-4	$f(-4) = (2)^{-4} = \frac{1}{2^4} = \frac{1}{16}$	$(-4, \frac{1}{16})$
-3	$f(-3) = (2)^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$(-3, \frac{1}{8})$
-2	$f(-2) = (2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(-2, \frac{1}{4})$
-1	$f(-1) = (2)^{-1} = \frac{1}{2}$	$(-1, \frac{1}{2})$
0	$f(0) = (2)^0 = 1$	(0,1)
1	$f(1) = (2)^1 = 2$	(1, 2)
2	$f(2) = (2)^2 = 4$	(2, 4)
3	$f(3) = (2)^3 = 8$	(3, 8)
4	$f(3) = (2)^4 = 16$	(4, 16)

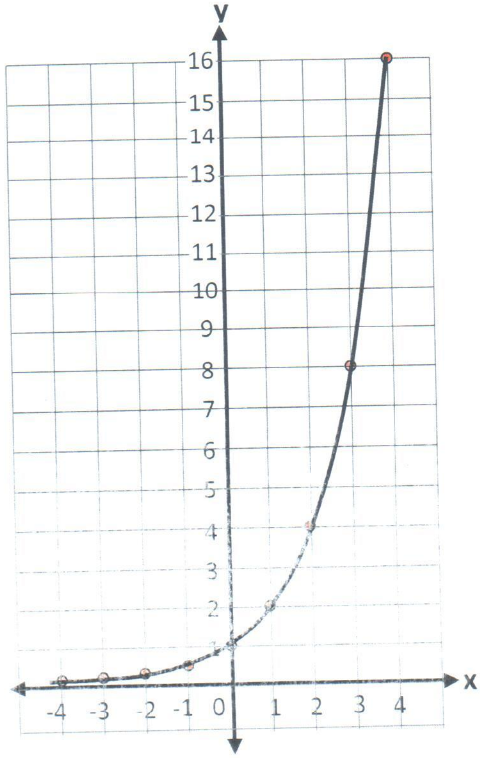


Figure 12.4.4.1

Example 2

Graph $y = f(x) = \left(\frac{1}{2}\right)^x$

Solution: First we construct the table of values.

x	$y = f(x) = \left(\frac{1}{2}\right)^x$	(x, y)
-4	$y = \left(\frac{1}{2}\right)^{-4} = 2^4 = 16$	(-4, 16)
-3	$y = \left(\frac{1}{2}\right)^{-3} = 2^3 = 8$	(-3, 8)
-2	$y = \left(\frac{1}{2}\right)^{-2} = 2^2 = 4$	(-2, 4)
-1	$y = \left(\frac{1}{2}\right)^{-1} = 2^1 = 2$	(-1, 2)
0	$y = \left(\frac{1}{2}\right)^{0} = 2^0 = 1$	(0, 1)
1	$y = \left(\frac{1}{2}\right)^{1} = \frac{1}{2}$	$(1, \frac{1}{2})$
2	$y = \left(\frac{1}{2}\right)^{2} = \frac{1}{2^2} = \frac{1}{4}$	$(2, \frac{1}{4})$
3	$y = \left(\frac{1}{2}\right)^{3} = \frac{1}{2^3} = \frac{1}{8}$	$(3, \frac{1}{8})$
4	$y = \left(\frac{1}{2}\right)^{4} = \frac{1}{2^4} = \frac{1}{16}$	$(4, \frac{1}{16})$

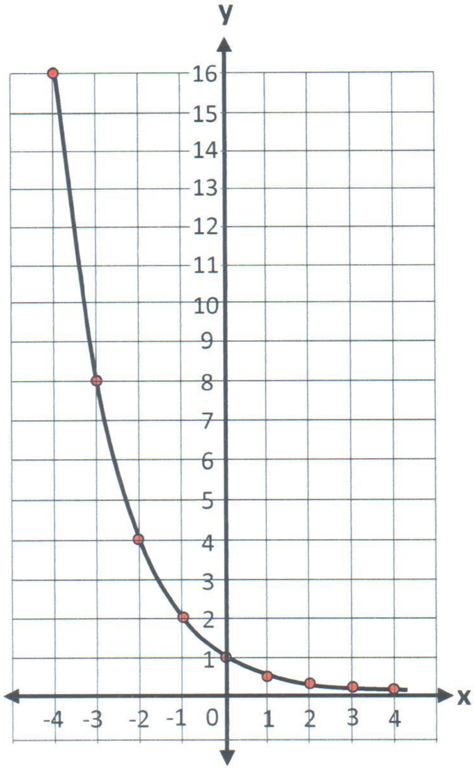


Figure 12.4.4.2

To compare exponential functions, in the next example we graph several functions in the same coordinate system.

Example 3

Graph the functions $y = 2^x$, $y = \left(\frac{1}{2}\right)^x$, $y = 3^x$, $y = \left(\frac{1}{3}\right)^x$, $y = 10^x$, and $y = \left(\frac{1}{10}\right)^x$.

Solution:

First we construct a table of values. The scale for our graphs in Figure 12.4.3 does not allow us to plot all the points listed in the table, but the values are included there for the purpose of comparison as shown on the next page.

Table of values for the functions $y = 2^x$, $y = \left(\frac{1}{2}\right)^x$, $y = 3^x$, $y = \left(\frac{1}{3}\right)^x$, $y = 10^x$, and $y = \left(\frac{1}{10}\right)^x$.

x	$y = 2^x$	$y = \left(\frac{1}{2}\right)^x$	$y = 3^x$	$y = \left(\frac{1}{3}\right)^x$	$y = 10^x$	$y = \left(\frac{1}{10}\right)^x$
-4	$\frac{1}{16}$	16	$\frac{1}{81}$	81	$\frac{1}{10000}$	10000
-3	$\frac{1}{8}$	8	$\frac{1}{27}$	27	$\frac{1}{1000}$	1000
-2	$\frac{1}{4}$	4	$\frac{1}{9}$	9	$\frac{1}{100}$	100
-1	$\frac{1}{2}$	2	$\frac{1}{3}$	3	$\frac{1}{10}$	10
0	1	1	1	1	1	1
1	2	$\frac{1}{2}$	3	$\frac{1}{3}$	10	$\frac{1}{10}$
2	4	$\frac{1}{4}$	9	$\frac{1}{9}$	100	$\frac{1}{100}$
3	8	$\frac{1}{8}$	27	$\frac{1}{27}$	1000	$\frac{1}{1000}$
4	16	$\frac{1}{16}$	81	$\frac{1}{81}$	10000	$\frac{1}{10000}$

The Graphs of the Functions

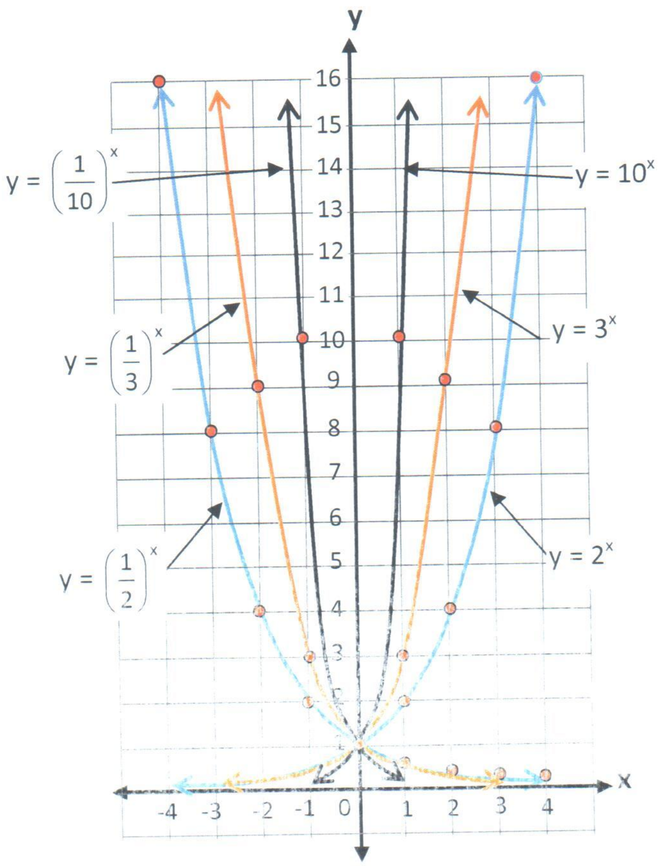


Figure 12.4.4.3

Several observations can be made from Example 3.

1. In both cases, the graph passes through the point (0, 1) since $\mathbf{a^0 = 1}$ for any value of **a** (except 0).
2. If a > 1, the graph rises to the right, and toward the left it gets closer and closer (but never crosses) the x-axis.
3. If a < 1, the graph decreases from left to the right, and toward the right it gets closer and closer (but never crosses) the x-axis.

Logarithmic Functions

> **A logarithmic function** is a function defined by an equation of the form $\mathbf{y = \log_a x}$.

Since logarithmic equations and exponential equations are closely related, to graph $\mathbf{y = \log_a x}$ we need only to graph its equivalent exponential form $\mathbf{a^y = x}$.

When doing so, we keep in mind that the roles of x and y have been interchanged from our previous work when we graph $y = a^x$.

Example 4

Graph $y = \log_2 x$.

Solution: Since $y = \log_2 x$ is equivalent to the exponential form $x = 2^y$, graph $y = \log_2 x$ by plotting $x = 2^y$.

First, we construct the table of values by finding x for a given value of y.

Table of values

$x = 2^y$	y
8	3
4	2
2	1
1	0
$\frac{1}{2}$	-1
$\frac{1}{4}$	-2
$\frac{1}{8}$	-3

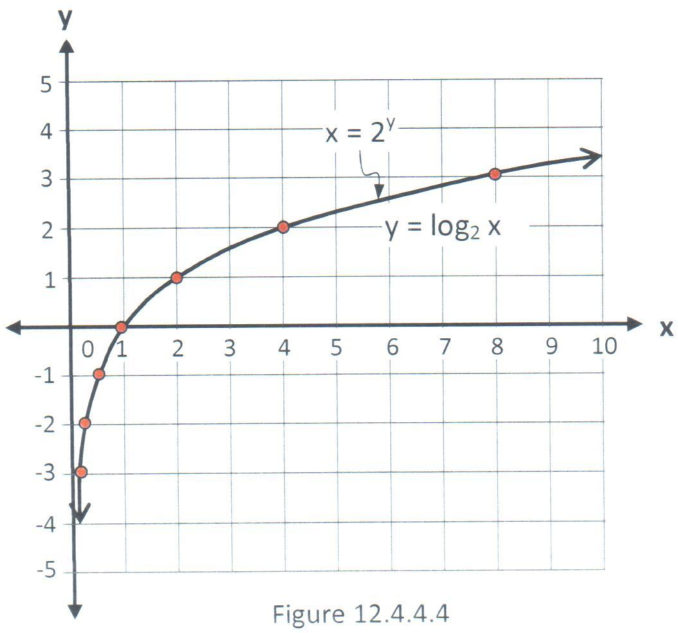
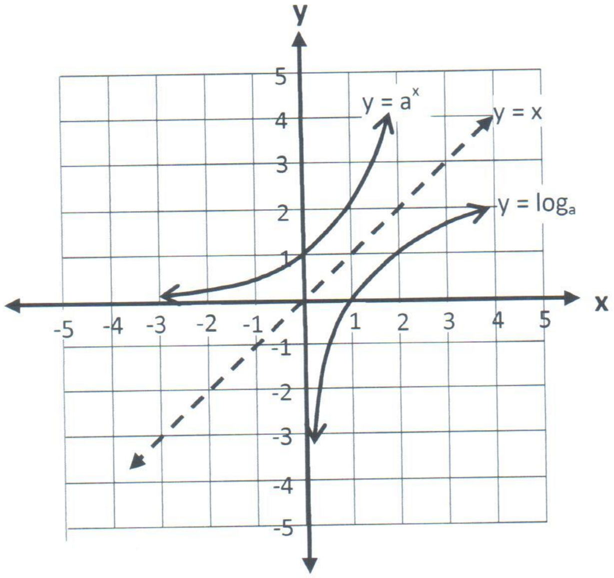


Figure 12.4.4.4

The logarithmic and exponential functions $y = \log_a x$ and $y = a^x$ are related in an important way. The graph of each is a reflection of the graph of the other across the line $y = x$. (See Figure 12.4.5). Functions of this property are called inverses of each other, and if the graph of either is known, the graph of the other can be obtained.

Figure 12.4.4.5



Generally the most interesting exponential and logarithmic functions are those with the base **a** where a > 1.

The graph of every exponential functions $y = a^x$, $a > 1$ has the same basic shape shown in Figure 12.4.6 and passes through point (0, 1). The negative x-axis is an **asymptote** as $x \to \infty$, meaning the curve does not cross or cut the x-axis at all.

The graph of every logarithmic functions $y = \log_a x$, $a > 1$ has the same basic shape as shown in Figure 12.4.7 and passes through point (1, 0). The negative y-axis is an **asymptote**. There is no horizontal asymptote for the curve as $x \to \infty$, $\log_a x \to \infty$ also.

Asymptote is a line that the curve approaches but never touches or crosses it.

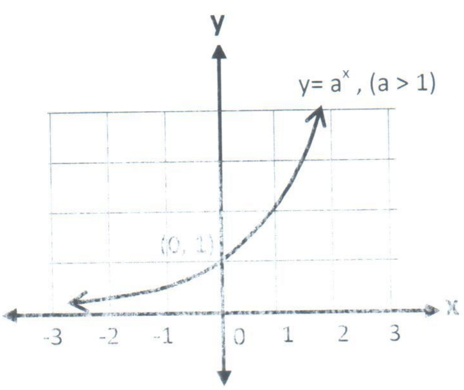


Figure 12.4.4.6

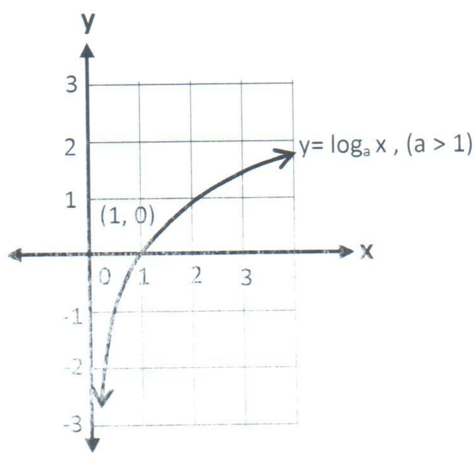


Figure 12.4.4.7

Common and Natural Logarithmic Functions

> **Common logarithmic function** is logarithmic function with base **10.** The function $f(x) = \log_{10} x$ is usually written as $f(x) = \log x$.

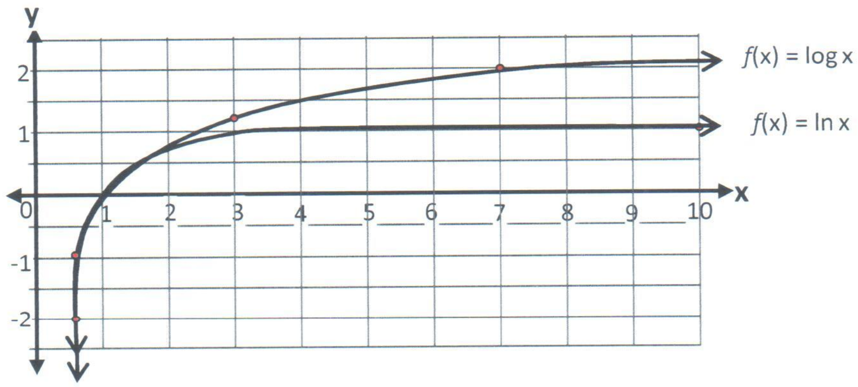
> **The natural logarithmic function** is logarithmic function with base **e.** The function $f(x) = \log_e x$ is usually written as $f(x) = \ln x$.

Common logarithms are also called **Briggsian logarithms.** Natural logarithms **are also called Napierian logarithms.** Basic properties of common and natural logarithms directly follow from the basic rule of logarithms.

Basic Properties of Common and Natural Logarithms

Common Logarithms		Natural Logarithms	
$\log 1 = 0$	since $10^0 = 1$	$\ln 1 = 0$	since $e^0 = 1$
$\log 10 = 1$	since $10^1 = 10$	$\ln e = 1$	since $e^1 = e$

The graphs of $f(x) = \log x$ and $f(x) = \log_e x$ are shown below.



With the use of calculators, the logarithmic values can be easily obtained even without using the logarithmic tables.

Many calculators have function keys labeled $\boxed{\text{LOG}}$ and $\boxed{\text{LN}}$.

The function key $\boxed{\text{LOG}}$ can be used to evaluate common logarithms and the function key $\boxed{\text{LN}}$ can be used to evaluate natural logarithms.

For example, log 2 can be easily evaluated using calculator.

Logarithm	Keystroke	Answer Display
$\log 2$	LOG 2 =	0.30103
$\ln\left(\frac{1}{2}\right)$	LN (1 ÷2) =	-0.69315

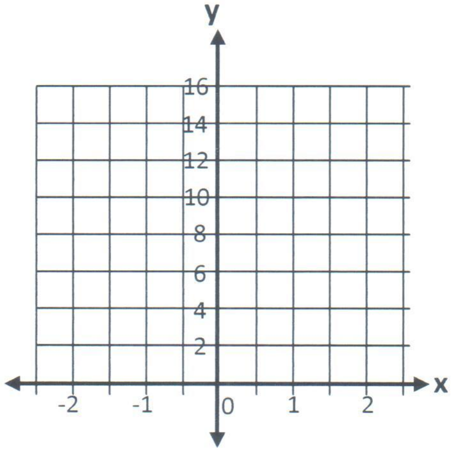
Learning Activity 12.4.4.3

20 minutes

Answer the following:

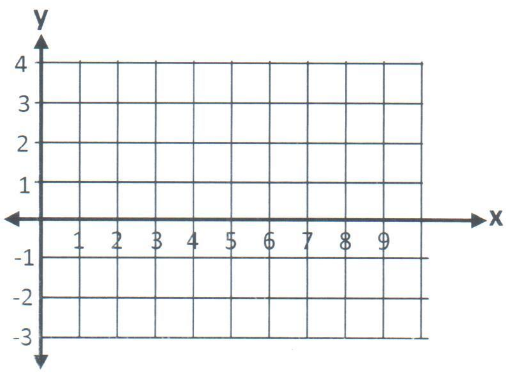
1. Given an exponential function $y = a^x$, a is called the ________________.
2. The graph of the logarithmic function $y = \log_a x$ can be determined by plotting the points that satisfy the exponential function ________________.
3. The graph of y $\log_3 x$ can be obtained by reflecting the graph of $y = 3^x$ across the line with equation ________________.
4. The graph of $y = 5^x$ and y = ________________are reflections of each other across the line y = x.
5. Complete the following table and sketch the graph of the functions $y = 4^x$

x	y
-2	
-1	
0	
1	
2	



6. Graph Sketch the graph of $y = \log_3 x$ by plotting $x = 3^y$.

x	y
	-2
	-1
	0
	1
	2



7. Evaluate the following common logarithms using calculator.

   a. log 100   b. log 25

8. Evaluate the following common logarithms using calculator.

   a. ln 10   b. $\ln \frac{1}{10}$

12.4.4.4: Solving Exponential and Logarithmic Equations

In this section, we will look at the application of logarithms and the rules of logarithms in solving exponential and logarithmic equations.

First let us define exponential equations and logarithmic equations.

An **exponential equation** is an equation with exponential expressions that contain a variable in one of its exponents.

Examples:

a. $3^x = 27$ b. $10^{x-2} = \left(\frac{1}{1000}\right)^{3x}$ c. $e^x - e^{-x} = 1$ d. $(5.26)^{x+1} = 75.4$

A **logarithmic equation** is an equation with logarithmic expressions that contain a variable.

Examples:

a. $\log_5 x = 625$ b. $\log x = \log\frac{2}{x} + 8$ c. $\ln(x-2) - \ln x = 32$

Solving exponential equations

When both sides of an exponential equation can be expressed as a power of the same base, it is easier to equate the corresponding exponents (set them equal to each other) and solve.

Example 1: Solve $64^x = \frac{1}{4}$

Solution: $64^x = \frac{1}{4}$

$(2^6)^x = \frac{1}{2^2}$ Express both sides as powers of the same base 2.

$2^{6x} = 2^{-2}$

Therefore, $6x = -2$ Equate the exponents equal to each other.

$x = -\frac{2}{6} = -\frac{1}{3}$ Simplest form

The solution set is $\left\{-\frac{1}{3}\right\}$.

Example 2

Solve the following exponential functions.

a. $5^{3x-2} = \frac{1}{125}$

b. $e^{x^2} - e^{4x-3} = 0$

Solutions:

a. $5^{3x-2} = \frac{1}{125}$

$5^{3x-2} = 5^{-3}$	Express both sides as powers of the same base 5.
$3x - 2 = -3$	Equate the exponents equal to each other.
$3x = -3 + 2$	Addition property
$3x = -1$	Solve for x.
$x = -\frac{1}{3}$	

Therefore, the solution set is $\left\{-\frac{1}{3}\right\}$.

Check by substituting $-\frac{1}{3}$ in the original equation.

Check:
$$5^{3\left(-\frac{1}{3}\right)-2} = 5^{-3}$$
$$5^{-1-2} = 5^{-3}$$
$$5^{-3} = 5^{-3}$$

b. $e^{x^2} - e^{4x-3} = 0$

$x^2 = 4x - 3$	Equate the exponents equal to each other.
$x^2 - 4x + 3 = 0$	General form
$(x-3)(x-1) = 0$	Factor the left side
$x - 3 = 0;\ x - 1 = 0$	Zero product property
$x = 3 \quad ;\ x = 1$	Solve for x

Therefore, the solution set is $\{3, 1\}$.

Check by substituting the solutions 3 and 1 in the original equations.

Check:
$$e^{(3)^2} - e^{4(3)-3} e = 0$$
$$e^9 - e^{12-3} = 0$$
$$e^9 - e^9 = 0$$
$$0 = 0$$

Remember:

To solve exponential equations,	
Step 1:	Express both sides as a power of the same base, and equate the resulting exponents.
Step 2:	If Step 1 fails, take the logarithm of both sides and use the power rule to eliminate the variable exponents.
Step 3:	Solve the resulting equations and check in the original.

Solving logarithmic equations

Solving logarithmic equations requires a thorough knowledge of properties of logarithms to simplify both sides of the equation.

Example 1

Solve for x in the logarithmic equation log (x + 1) – log (x + 1) = 1.

Solution: $\log(x + 10) - \log(x + 1) = 1$

$$\frac{\log(x+10)}{\log(x+1)} = 1 \qquad \text{Quotient rule on left side}$$

$$\frac{(x+10)}{(x+1)} = 10^1 \qquad \text{Convert to exponential form}$$

$$x + 10 = 10(x + 1) \qquad \text{Clear fraction}$$

$$x + 10 = 10x + 10 \qquad \text{Expand the right side}$$

$$9x = 0 \qquad \text{Subtraction Property}$$

$$x = 0 \qquad \text{Solve for x.}$$

The solution is 0.

Check by substituting x = 0 in the original equation.

$$\log(0 + 10) - \log(0 + 1 = 1$$

$$\log 10 - \log 1 = 1$$

$$1 - \log 0 = 1$$

$$1 = 1$$

Example 2

Solve: log (2x + 1) + log 5 = log (x + 6)

Solution:

$$\log(2x+1) + \log 5 = \log(x+6)$$

$$\log(2x+1)5 = \log(x+6) \qquad \text{Rule 1(Product Rule)}$$

Therefore,

$$(2x+1)5 = x+6$$

$$10x + 5 = x + 6$$

$$9x = 1$$

$$\mathbf{x = \frac{1}{9}}$$

Check:

$$\log(2x+1) + \log 5 = \log(x+6)$$

$$\log\left[2\left(\frac{1}{9}\right)+1\right] + \log 5 = \log\left(\frac{1}{9}+6\right)$$

$$\log\left(\frac{11}{9}\right) + \log 5 = \log\left(\frac{55}{9}\right)$$

$$\log\left(\frac{11}{9}\right)5 = \log\left(\frac{55}{9}\right)$$

$$\log\left(\frac{55}{9}\right) = \log\left(\frac{55}{9}\right)$$

Example 3 Solve log (x – 1) + log (x + 2) = log 4

Solution:

$$\log(x-1) + \log(x+2) = \log 4$$

$$\log(x-1)(x+2) = \log 4$$

Therefore,

$$(x-1)(x+2) = 4$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x + 3 = 0;\ x - 2 = 0$$

$$x = -3;\ x = 2$$

Check for x = 2:

$$\log(x-1) + \log(x+2) = \log 4$$

$$\log(2-1) + \log(2+2) = \log 4$$

$$\log 1 + \log 4 = \log 4$$

$$0 \quad + \log 4 = \log 4$$

$$\log 4 = \log 4 \ \text{ true}$$

Therefore, x = 2 is a solution.

Check for x = 2:

$$\log(x - 1) + \log(x + 2) = \log 4$$

$$\log(-3 - 1) + \log(-3 + 2) = \log 4$$

$$\log(-4) + \log(-1) = \log 4$$

Logarithms of negative numbers are not real numbers.

Therefore, x = -3 is not a solution.

We summarized what we have learned.

To solve logarithmic equations,

- Use the rules of logarithms to write each side of the equation as the logarithm of a single expression.
- Equate the expressions.
- Solve the resulting equation from the unknown.
- Check apparent solutions in the given logarithmic equation.

Now do the learning activities.

Learning Activity 12.4.4.4

20 minutes

1. Solve the following exponential expressions by expressing each side as a power of the same base.

   a. $27^x = \frac{1}{9}$

   b. $8^x = \frac{1}{16}$

   c. $125^x = \frac{1}{25}$

2. Solve by taking the logarithm of both sides of the exponential equations.

   a. $2^x = 3$

b. $5^x = 4$

c. $5^{2x} = 8$

3. Solve the logarithmic equation $\log(3x - 1) + \log 4 = \log(9x + 2)$ and check.

4. Solve the exponential equation $25^{2x + 2} = 5^{x - 1}$ by expressing each side as a power of the same base.

12.4.4.5: Application of Logarithmic and Exponential Equations

Strategies in solving exponential and logarithmic equations are useful in solving numerous real life applications such as the time it takes an investment to grow, time it takes for a population to grow, rate of radioactive decay, severity of an earthquake and many more.

Investments with Compounded Interest

Exponential and logarithmic equations are used to find the total amount of money invested after a given time and the time it takes for the money to double or to grow a specified amount.

Example 1

A K1 000 deposit is made at a bank that pays 12% compounded annually. How much will you have in your account at the end of 10 years?

Solution: Use the compound interest formula $\mathbf{A = P(1 + r)^t}$ with **P** = K1 000, **r** = 0.12 and **t** = 10 years.

The resulting equation is:
$$\begin{aligned} A &= 1\,000(1 + 0.12)^{10} \\ &= 1\,000(1.12)^{10} \\ &= 1\,000(3.1058) \\ &= \text{K}3\,105.8 \end{aligned}$$

Example 2

Ron deposited a portion of his money amounting to K10 000 at an annual interest rate of 4% compounded yearly. How long will it take for Ron's money to double?

Solution: Use the compound interest formula $\mathbf{A = P(1 + r)^t}$ with **P** = K10 000 and **r** = 0.04.

Let **t** be the time for Ron's money to double, That is A = 20 000.

The resulting equation is:

$$10\,000(1 + 0.04)^t = 20\,000$$

$$(1.04)^t = \frac{20000}{10000}$$

$$(1.04)^t = 2$$

Now take the logarithm of each side of the equation.

$$\log (1.04)^t = \log 2$$

$$t \log(1.04) = \log 2$$

$$t = \frac{\log 2}{\log 1.04}$$

$t = 17.67$ (approximate value using calculator)

Therefore, Ron's money will be doubled after 17.67 years or 17 years and 8 months.

Population Growth

Exponential and logarithmic equations are also useful in finding population growth rate.

Example 2

Suppose that the population of country **C** can be modelled by an exponential function

$$\mathbf{A} = 76.5e^{0.0204x}$$

where **A** approximates the country **C** population in millions and **x** is the number of years after 2015. In what year will the population of country **C** reach 100 million?

Solution: Use the population model with A = 100.

$$76.5e^{0.0204x} = 100$$

$$e^{0.0204x} = \frac{100}{76.5}$$

$$e^{0.0204x} = 1.3072$$

Now take the natural logarithms of both sides of the equation.

$$\ln e^{0.0204x} = \ln(1.3072)$$

$$0.0204^{x} = \ln(1.3072)$$

$$x = \frac{\ln(1.3072)}{l0.0204}$$

$$x = 13.13 \text{ years (using calculator)}$$

Hence, Country C population is expected to reach 100 million using this model in 2023.

Example 3

In an experiment, the bacteria increased from population of 200 to 600 in 3 hours. How long will it take for the population to reach 1000 bacteria? Use the model $\mathbf{A = Pe^{kt}}$ with P = 200, A = 600 and t = 3

Solution: $200e^{k(3)} = 600$

$$e^{3k} = \frac{600}{200}$$

$$e^{3k} = 3$$

$$\ln e^{3k} = \ln 3$$

$$3k = \ln 3$$

$$k = \frac{\ln 3}{3}$$

To find the time for the population to reach 1000 bacteria, substitute A by 1000 and solve for t.

$$\mathbf{200\,e}^{\left(\frac{\ln 3}{3}\right)t} = 1000$$

$$e^{\left(\frac{\ln 3}{3}\right)t} = \frac{1000}{200}$$

$$e^{\left(\frac{\ln 3}{3}\right)t} = 5$$

$$\ln e^{\left(\frac{\ln 3}{3}\right)t} = \ln 5$$

$$t\left(\frac{\ln 3}{3}\right) = \ln 5$$

$$t = \frac{3\ln 5}{3}$$

$$t = 4.4 \text{ hours}$$

Thus, the population will reach 1000 bacteria in about 4.4 hours

Another application of exponential functions is to **model amount of decay**, often referred to **negative growth**. This is essential in real life situations involving radioactive decay and age of fossils.

Radioactive materials are often used in medical treatments and as power sources. The time of radioactive materials decay exponentially over time. The rate of decay varies for different radioactive isotopes. The measure of the rate of decay is called the **half-life** of the material which is the time it takes for half the **amount** of the material to **decay**.

The half-life decay model is $\mathbf{A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}}$ **or** $\mathbf{A = P(2)^{-\frac{1}{2}}}$.

Where, **A** is the amount at a time **t**.
P is the present amount
h is the half-life.

Example 4

A radioactive isotope used in medical diagnosis of diseases such as malignant tumours is gallium 67(^{67}Ga). It has a half-life of 78.26 hours. If the present amount of ^{67}Ga is 2 milligrams, how many milligrams will be left after 1 day?

Solution:

The half-life decay model is $\mathbf{A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}}$ with **P** = 2, **h** = 78.26 and t = 1 d or 24 hours

The resulting equation is: $A = 2\left(\frac{1}{2}\right)^{\frac{24}{78.26}}$

$$A = 2(.5)^{.3066}$$

$$= 1.617 \text{ or} 1.62 \text{ mg}$$

Hence, about 1.62 mg will be left after a day.

Learning Activity 12.4.4.5

20 minutes

1. Toby deposited a portion of his money amounting to K50 000 at an annual interest rate of 5% compounded yearly. How long will it take for Toby's money to double?

2. What is the half-life of radioactive iodine (^{131}I) if 0.84 g of an initial 2 g remains after 10 days? Use the half-life decay model $\mathbf{A = P(2)^{-\frac{t}{h}}}$.

3. In Chemistry, the number pH is a measure of the acidity or alkalinity of a solution. If (H+) is the hydronium ion concentration measured in moles per litre, then

$$\mathbf{pH = -\log (H+)}.$$

Find the pH of a solution with a hydronium ion concentration of $4.0(10^{-3})$.

SUMMATIVE TASK 12.4.4

30 minutes

A. Read the following carefully. Choose the letter of the correct answer.

1. Which of the following is equivalent to $y = 7^x$?

   A. $\log_7 x = y$ B. $\log_y x = 7$ C. $\log_7 y = x$ D. $\log_x y = 7$

2. Which of the following is **not** true?

   A. $\log xy = \log x + \log y$ C. $\log \frac{x}{y} = \log x - \log y$

   B. $\log (x + y) = \log x + \log y$ D. $\log_b x^c = c \log_b x$

3. What is the value of $\log_2 128$?

   A. 1 B. 2 C. 6 D. 7

4. What is the value of x in $3^x = 8$?

   A. $\frac{8}{3}$ B. $\frac{3}{8}$ C. $\frac{\log 3}{\log 8}$ D. $\frac{\log 8}{\log 3}$

5. What is the solution of the equation $\log x - \log 100 = 1$?

   A. 0 B. 10 C. 100 D. 1000

6. Which is the solution of $5^{x+1} = \frac{1}{125}$?

   A. -4 B. -2 C. 2 D. 3

7. What is the value of x in $e^x = 2$?

   A. 2 B. e^2 C. ln 2 D. ln e

8. What is the solution set of $\log_3(x+3) + \log_3(x + 2) = \log_3(2x^2 + 10)$?

   A. $\{\ \}$ B. $\{1\}$ C. $\{4\}$ D. $\{1, 4\}$

9. The amount of money in a savings account earns interest at 2% compounded annually. After how many years will this amount double?

   A. 2 years B. 10 years C. 25 years D. 35 years

10. The radioactive substance Cesium-137 has a half-life of 30 years. If the present amount of this radioactive substance is 100 g, how many grams will be left after 20 years?

    A. 70 g B. 63 g C. 60 g D. 53 g

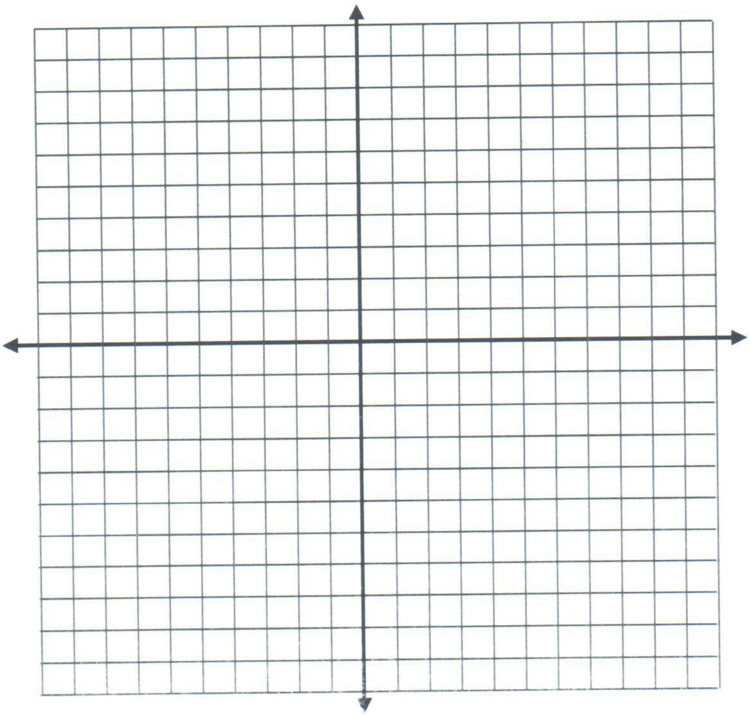
B. Complete the table of values below for the given exponential and logarithmic functions and sketch the graphs in the same coordinate axis.

a. $f(x) = \left(\frac{1}{2}\right)^x$

x	$y = f(x) = \left(\frac{1}{2}\right)^x$
-3	
-2	
-1	
0	
1	
2	
3	

b. $f(x) = \log_{\frac{1}{2}} x$

x	$y = f(x) = \log_{\frac{1}{2}} x$
	-3
	-2
	-1
	0
	1
	2
	3



UNIT SUMMARY

This summary outlines the important key ideas and concepts to be remembered.

12.4.1 Linear Equations and Inequalities

- **Linear equation** is an equation in which the variable is raised to the first power only. This is also known as a **first degree equation** and is written in the form **ax + b = c.**
- An **identity or identical equation** is an equation which is true for all values of the variable involved.
- A **conditional equation** is an equation which is true only for a certain value (or values) of the variable.
- An equation which has no solution is called **contradiction**.
- The set of all solutions of an equation is called the **solution set** of the equation.
- **To solve a linear equation**
  1. Simplify both sides by removing grouping symbols and collecting like terms.
  2. Eliminate fractions or decimals by multiplying both sides of the equation by an appropriate factor (the LCD for fractions or a power of 10 for the decimals).
  3. Use the addition-subtraction rule to isolate all variable terms on one side and all constant terms on the other side. Collect like terms where possible.
  4. Use multiplication-division rule to obtain a variable with coefficient of 1.
  5. Check the solution by substituting in the original equation.
  6. If an identity results, the original equation has every real numbers as a solution. If a contradiction results, there is no solution.
- **Linear inequality** is a statement which uses the symbols **<, >, ≥,** and **≤.**
- A **solution** to an inequality is a number which, when substituted for a variable, makes the inequality true.
- **To solve a linear inequality**
  1. Simplify both sides by removing grouping symbols and collecting like terms. Fractions or decimals may be eliminated by multiplying both sides by an appropriate factor (the LCD for fractions or a power of 10 for the decimals).
  2. Use the addition-subtraction rule to isolate all variable terms on one side and all constant terms on the other side. Collect like terms where possible.
  3. Use multiplication-division rule to obtain a variable with coefficient of 1. Remember to reverse the symbol of inequality when multiplying or dividing by a negative number.
- **To solve word problems**
  1. Read the problem very carefully to determine what is unknown. What is being asked for? **Do not** try to solve the problem at this time.
  2. Represent one unknown number by a letter.
  3. Reread the entire word problem, breaking it up into small pieces that can be represented by algebraic expressions.
  4. After each of the pieces has been written as an algebraic expression, fit them together into an equation.

5. Solve the equation for the unknown letter by the methods used in solving equations.
6. Check the solution in the word problem not with the equation.

- **To graph linear equation with one variable**, first solve the equation, and then find the point (or points) on a number line which corresponds to the solution (or solutions) of the equation.
- **To graph an inequality**, first solve the inequality and then plot the solution set on the number line. The graph of an inequality shows all the numbers that satisfy the inequality. When graphing inequalities on a number line, use solid circles ( ) for ≤ and ≥ and open circles ( )for < and >.
- A **linear equation in two variables** has as its graph a **straight line**. It is easy to recognize a linear equation since the variables are raised to the first power only and it can always be written in the general form **ax + by + c = 0.**
- **To graph an equation in the two variables x and y**
  1. Make a table of values. These values represent the ordered pair solutions of the equation.
  2. Plot the points which correspond to these solutions in a Cartesian coordinate system.
  3. Connect the points with a straight line.
- Any first degree inequality (in no more than two variables) has a graph that is a **half-plane.**
- The boundary line is a **solid line** when equality is included.
- The boundary line is a **dashed line** when equality is not included.
- **How to determine the correct half-plane**
  1. **If the boundary line does not go through the origin,** substitute the coordinates of the origin (0, 0) into the inequality.
     - ❖ If the resulting inequality is true, the solution is the half-plane containing the origin.
     - ❖ If the resulting inequality is false, the solution is the half-plane not containing the origin.
  2. **If the boundary line goes through the origin,** select a point not on the boundary. Substitute the coordinates of this point into the inequality.
     - ❖ If the resulting inequality is true, the solution is the half-plane containing the point selected.
     - ❖ If the resulting inequality is false, the solution is the half-plane not containing the point selected.

12.4.2 System of Equations

- A **system of equations** is a set of two or more equations that are considered together for a common solution.
- The system is called **linear system**, if each equation of a system is a first-degree equation.
- The system is said to be **inconsistent** if it has no solution.
- The system is **dependent** if it is infinitely many solutions.
- A system of equations in the same number of unknowns where each equation is consistent with the other or others and independent of them is called a **simultaneous system**.

- **To solve a system of equations by Elimination by Addition or Subtraction**
  1. If necessary multiply either or both of the given equations by numbers which will make the coefficients of one of the variables the same or alike in both equations.
  2. If these same coefficients have like signs, subtract one equation from the other; if they have unlike signs, add the equations to eliminate the variable or letter whose coefficients were made the same.
  3. The result will be an equation in one variable. Solve the equation.
  4. Substitute the value thus obtained in one of the given equations to obtain the value of the other unknown.
  5. Check in the original equations.
- **To solve a system of equations by Substitution method**
  1. Solve one of the equations for one letter in terms of the other.
  2. Substitute the expression obtained in Step 1 into the other equation.
  3. Solve that equation.
  4. Substitute the value thus obtained in one of the given equations or in the equation obtained in Step 1 to obtain the value of the other letter.
  5. Check in the original equations.
- **To solve a system of two linear inequalities with two variables graphically**
  1. Graph the first inequality, shading the half-plane that represents its solution.
  2. Graph the second inequality on the same set of axes. Use different types of shading for the half-plane that represents its solution.
  3. The solution of the system is represented by the area with both types of shading.
- **To solve a linear system of two equations by graphical method:**
  1. Graph each equation of the system on the same set of axes.
  2. There are three possible cases of the graph of the system:

     Case 1: **The lines intersect at one point.** The solution is the ordered pair representing the point of intersection.

     Case 2: **The lines never cross or meet**. They **are parallel**. There is no solution.

     Case 3: **Both equations have the same line for their graph.** Any ordered pair that represents a point on the line is a solution.
- **To solve a system of two linear inequalities with two variables graphically**
  1. Graph the first inequality, shading the half-plane that represents its solution.
  2. Graph the second inequality on the same set of axes. Use different type of shading for the half-plane that represents its solution.
  3. The solution of the system is represented by the area with both types of shading.

12.4.3 Solving Quadratic Equation

- A **quadratic equation** is a polynomial with a second degree term as its highest-degree term.
- The general form of quadratic equation $\mathbf{ax^2 + bx + c = 0}$
- An **incomplete or pure quadratic equation** is one in which **b** or **c** (or both) is zero. The only letter that cannot be zero is **a**. If **a** is zero, the equation would not be a quadratic.
- **To solve quadratic equation by factoring**
  1. Arrange the equation in general form $ax^2 + bx + c = 0$.
  2. Factor the polynomial on the left side.

3. Set each factor equal to zero (Zero- product property)
4. Solve the unknown letter.
5. Check

- **To solve quadratic equation by completing the Square**
  1. Place the unknown terms in the left member and the known term in the right member.
  2. Complete the square by adding to both sides of the equation the square of one-half of the coefficient of x. The left member of the resulting equation is now a perfect trinomial square.
  3. Factor the left side.
  4. Extract the square root of both sides of the equation, prefixing the ± sign to the right member. Simplify the result.
  5. If **a ≠ 1**, divide both sides of $ax^2 + bx + c = 0$ by **a** and apply steps 1, 2, 3 and 4.
  6. Check the roots obtained.
- **To solve quadratic equation by Formula**
  1. Arrange the equation in general form.
  2. Substitute the values of a, b, and c into the quadratic formula.
  3. Simplify your answers.
  4. Check your answers by substituting them in the original equation.
- A function whose equation can be written in the form $\mathbf{y = ax^2 + bx + c}$ is called **quadratic functions**
- **To graph a quadratic function $f(x) = ax^2 + bx + c$**
  1. Determine whether the graph opens up ($a > 0$) or down ($a < 0$).
  2. Find the vertex and axis of symmetry using the fact that the x-coordinate of the vertex is $-\frac{b}{2a}$ and the axis of symmetry is $x = -\frac{b}{2a}$.
  3. The y-coordinate of the vertex is found by evaluating $f\left(-\frac{b}{2a}\right)$.
  4. If the vertex is located above the x-axis and the graph opens down, or if the vertex is located below the x-axis and opens up, find the x-intercepts by solving the equation $ax^2 + bx + c = 0$.
  5. Plot the vertex and the x-intercepts (or two other points on either side of the line of symmetry if there are no x-intercepts) and sketch the parabola.
- The graph of a quadratic function is called a **parabola.**

12.4.4 Exponential and Logarithmic Functions

- The logarithm of a number **y** is the exponent **x** to which the base **a** ($a > 1$), ($a \neq 1$) must be raised to give **y.**
- Laws of Logarithms
  - Product Rule: $\log_b xy = \log_b x + \log_b y$
  - Quotient Rule: $\log_b x - \log_b y = \log_b \frac{x}{y}$
  - Power Rule: $\log_b x^n = n \log_b x$

- An **exponential function** is a function of the form $\mathbf{y = f(x) = a^x}$, where $\mathbf{a > 0}$ and the number **a** is called the **base**.
- **A logarithmic function** is a function defined by an equation of the form $\mathbf{y = \log_a x}$.
- **Common logarithmic function** is logarithmic function with base **10.** The function $\mathbf{f(x) = \log_{10} x}$ is usually written as $\mathbf{f(x) = \log x}$.
- **The natural logarithmic function** is logarithmic function with base **e.** The function $\mathbf{f(x) = \log_e x}$ is usually written as $\mathbf{f(x) = \ln x}$.
- An **exponential equation** is an equation with exponential expressions that contain a variable in one of its exponents.
- **To solve exponential equations**
  1. Express both sides as a power of the same base, and equate the resulting exponents.
  2. If Step 1 fails, take the logarithm of both sides and use the power rule to eliminate the variable exponents.
  3. Solve the resulting equations and check in the original.
- A **logarithmic equation** is an equation with logarithmic expressions that contain a variable.
- **To solve logarithmic equations,**
  1. Use the rules of logarithms to write each side of the equation as the logarithm of a single expression.
  2. Equate the expressions.
  3. Solve the resulting equation from the unknown.
  4. Check apparent solutions in the given logarithmic equation.
- **Application of exponential and logarithmic Functions**
  - ❖ Population Growth Model : $\mathbf{A = Pe^{rt}}$
  - ❖ Half-life Decay Model: $\mathbf{A = P\left(\frac{1}{2}\right)^{\frac{t}{h}} = P(2)^{-\frac{t}{h}}}$
  - ❖ Investment Earning Compound Interest: 1. $\mathbf{A = P(1+r)^t}$
    2. $\mathbf{A = Pe^{rt}}$

REVISE SECTIONS 12.4.1 TO 12.4.4 THEN DO MODULE ASSESSMENT 4

ANSWERS TO LEARNING ACTIVITIES 12.4.1.1 to 12.4.4.5 AND SUMMATIVE TASKS 12.4.1 TO 12.4.4

Learning Activity 12.4.1.1

A.	1.	X	2.	17	3.	$2x - 5$	4.	No
	5.	No	6.	Yes	7.	Yes	8.	11

B. 1. $x = -4$ 2. $x = 6$ 3. $x = 15$ 4. $X = -9$ 5. -4

Learning Activity 12.4.1.2

A.	1.	False	2.	True	3.	True
	4.	True	5.	True	6.	True

B. 1. $x < 4$ 2. $x < 3$ 3. $Z > 2$ 4. $x \geq 4$ 5. $x \leq -9$

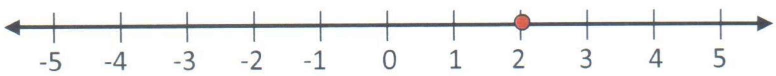
C. 1. $2x > 8$; $x > 4$ 2. $x - 3 > 20$; $x > 23$ 3. $2x - 8 \leq 12$; $x \leq 10$

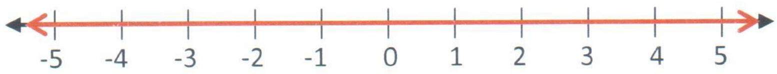
Learning Activity 12.4.1.3

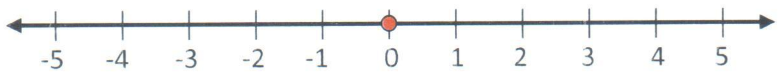
1. a. $t - 4$ b. $w + 10$ c. $2S$ d. $c - 1\,000$

2. a. $x = 11$
   b. John's age = 39; Peter's age = 48
   c. K600
   d. slower car = 60 km/h; faster car = 70 km/h
   e. 7.5 days
   f. 8, 9 and 10
   g. 5 m and 7 m

Learning Activity 12.4.1.4

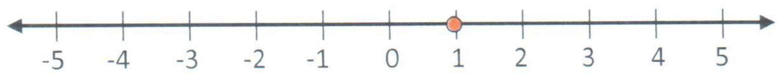
A. 1.


2.


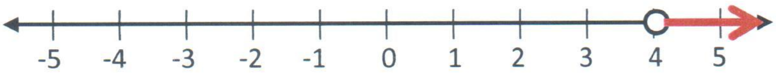
3.


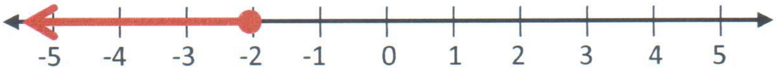
4.

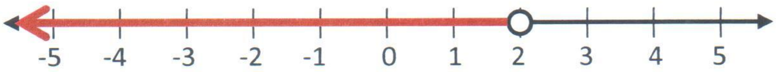

5.

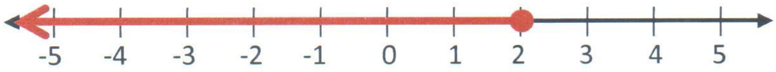

6.


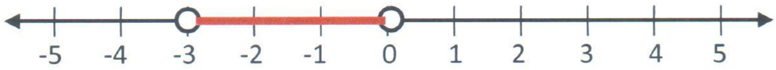
B.

1. 

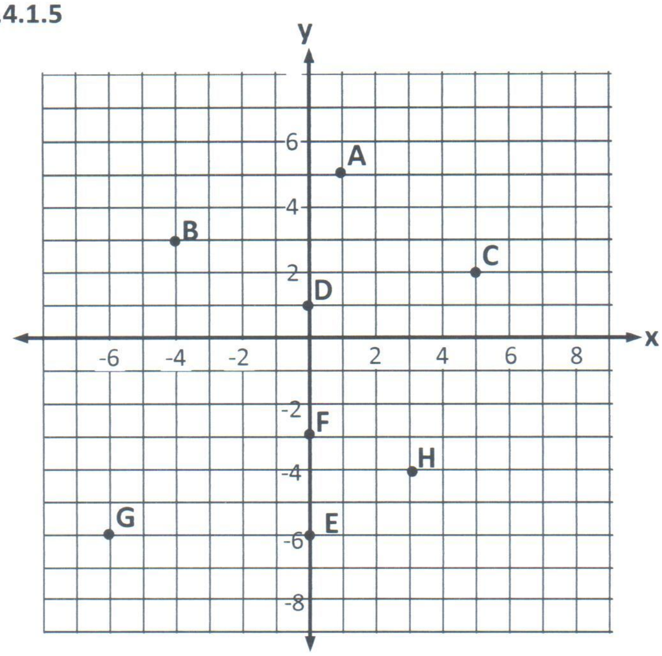
2. 

3. 

4. 

5. 

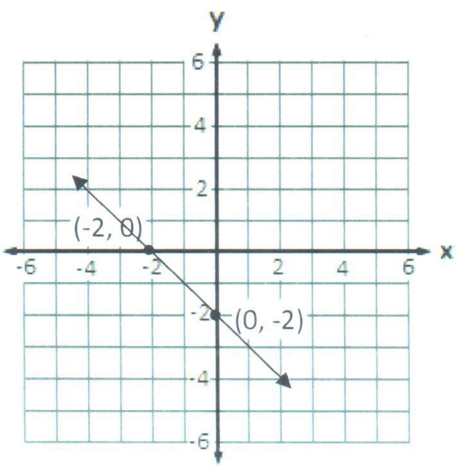
Learning Activity 12.4.1.5

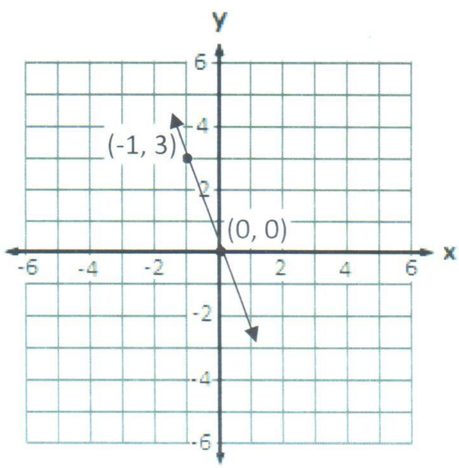
1. 

2. M(-3, 5) N(5, 4) O(0, -5) P(-4, 0) Q(7, 0) R(6, -4) S(-6, -8)

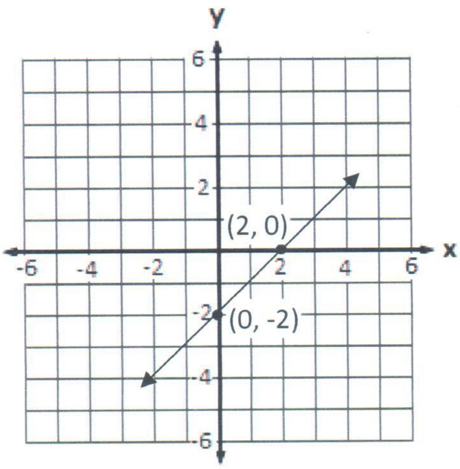
3. a. x-intercept (-2, 0)
y-intercept (0, -2)

b. x-intercept (0, 0)
y-intercept (0, 0)

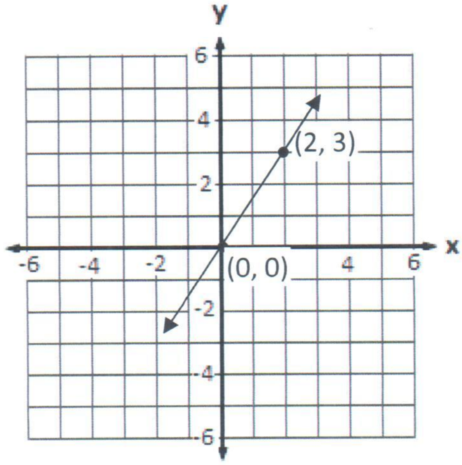




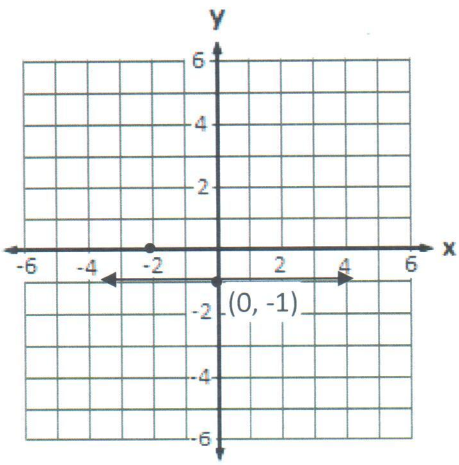
c. x-intercept (2, 0)
y-intercept (0, -2)



d. x-intercept (0, 0)
y-intercept (0, 0)



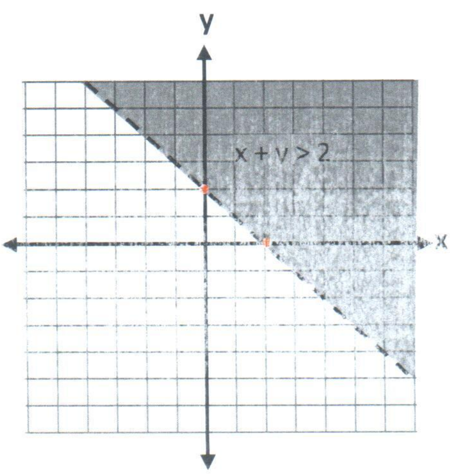
e. no x-intercept
y-intercept (0, -1)



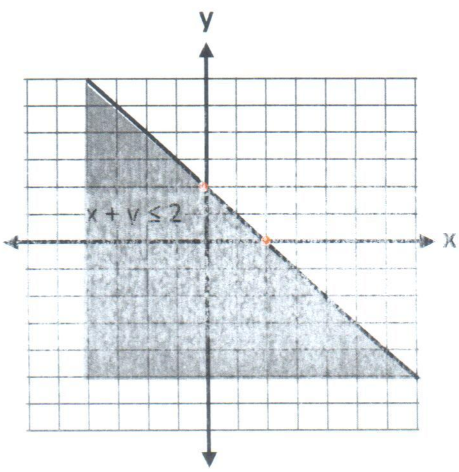
f. x-intercept are all lines in the x-axis
y-intercept (0, 0)
The graph is the x- axis.

Learning Activity 12.4.1.6

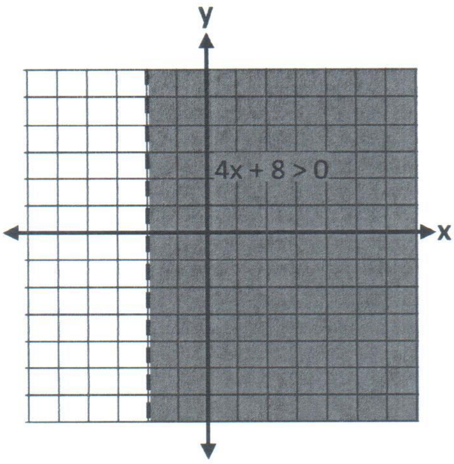
1. a. $x + y > 2$



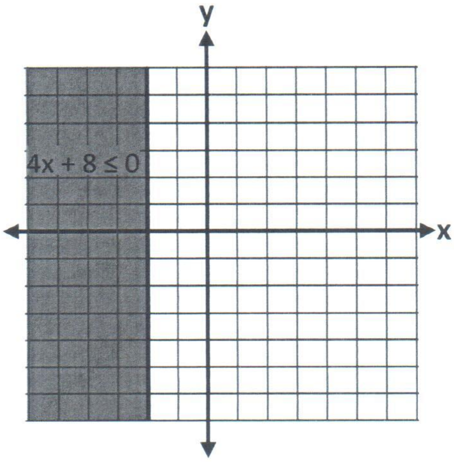
b. $x + y \leq 2$



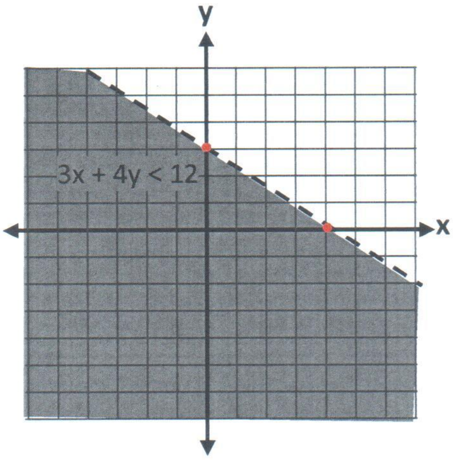
c. $4x + 8 > 0$



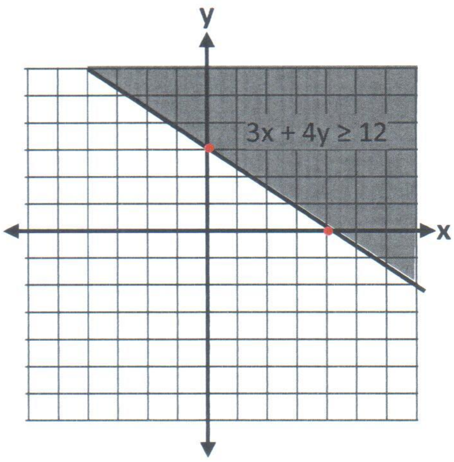
d. $4x + 8 \leq 0$



e. $3x + 4y < 12$



f. $3x + 4y \geq 12$



Summative Task 12.4.1

A.

1.	A	6.	D
2.	B	7.	C
3.	C	8.	B
4.	B	9.	B
5.	A	10.	A

B.
1. conditional equation; $x = 5$
2. contradiction; no solution
3. identity; true for any values of x
4. conditional equation; $x = 5$
5. identity; true for any values of x

C.
1. K6000 at 8%; K4000 at 4%
2. 17, 18 and 19
3. Width = 48 cm; Length = 90

Learning Activity 12.4.2.1

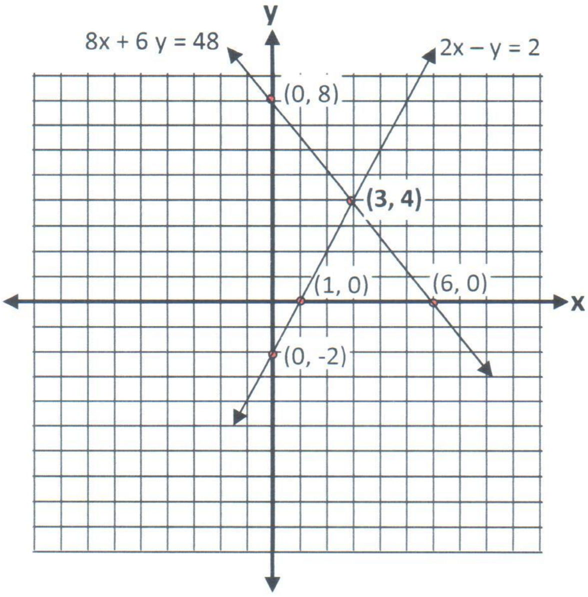
1. (2, -5) 2. (-3, 4) 3. (7, -3) 4. (6, 3) 5. (2, 3)

Learning Activity 12.4.2.2

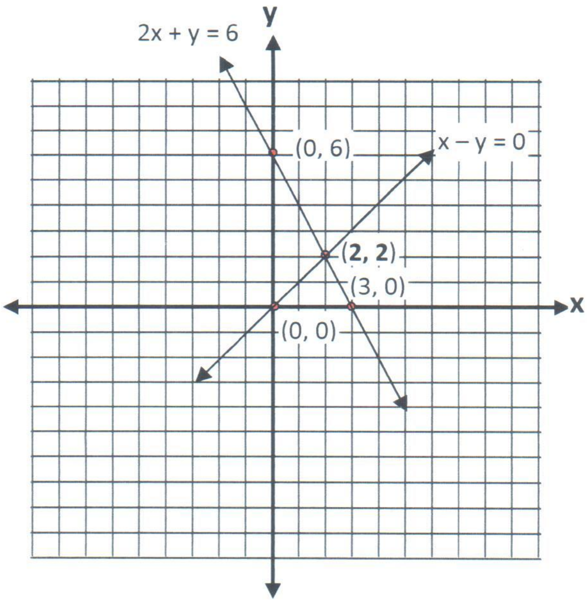
1. (5, -10) 2. (2, -2) 3. $(\frac{5}{4}, \frac{3}{2})$ 4. (4, -6) 5. (-9, -11)

Learning Activity 12.4.2.3

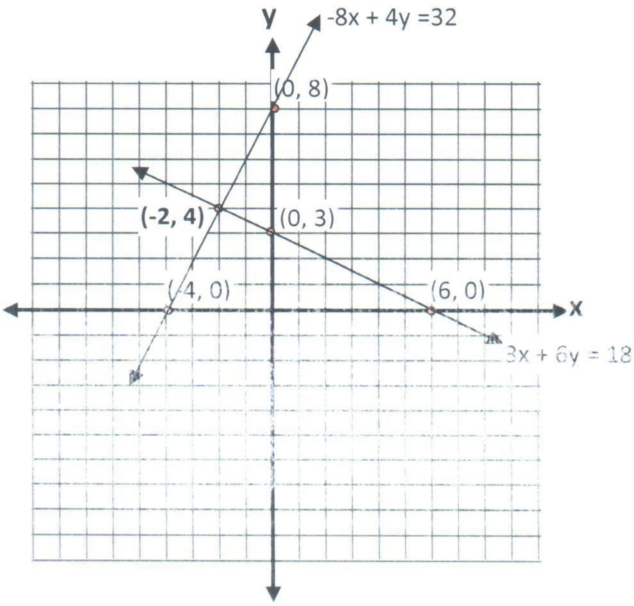
1. Solution is (3, 4).



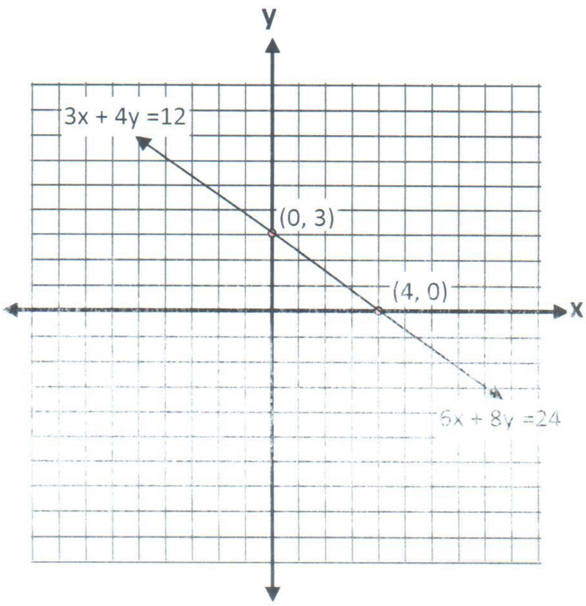
2. Solution is (2, 2).



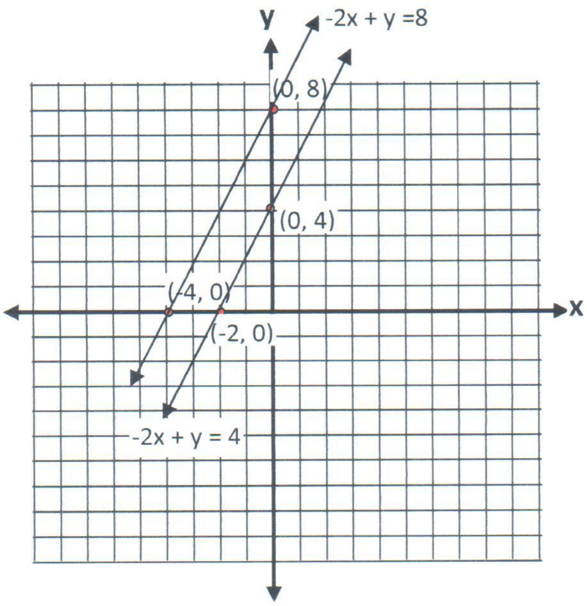
3. Solution is (-2, 4).



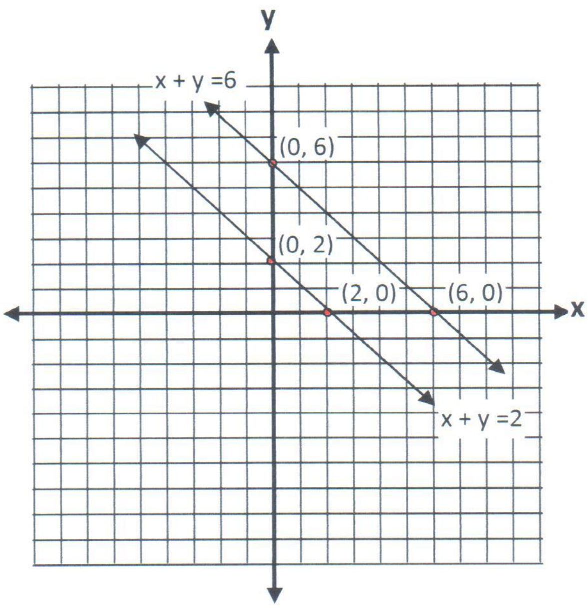
4. The graph of the lines is a single line. That is, infinitely many solutions.



5. The lines are parallel. No Solution.

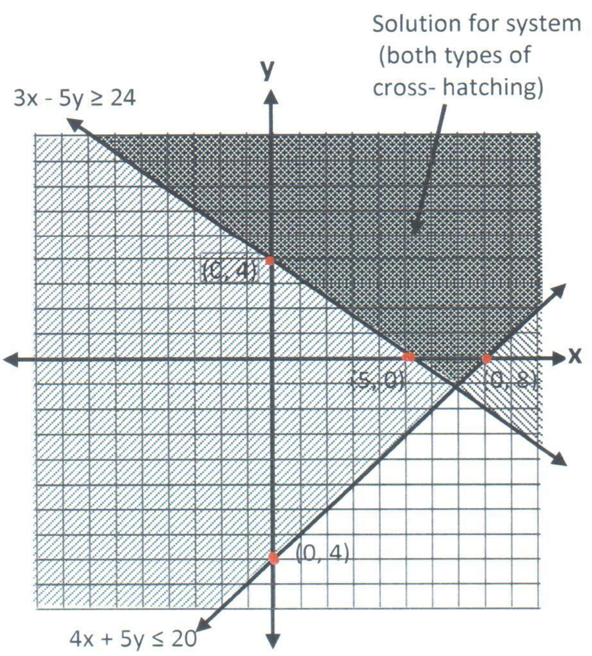


6. The lines are parallel. No Solution.

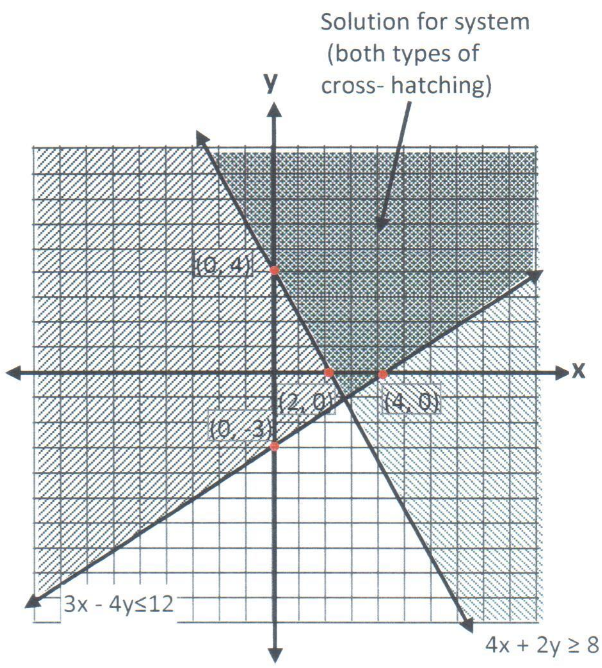


Learning Activity 12.4.2.4

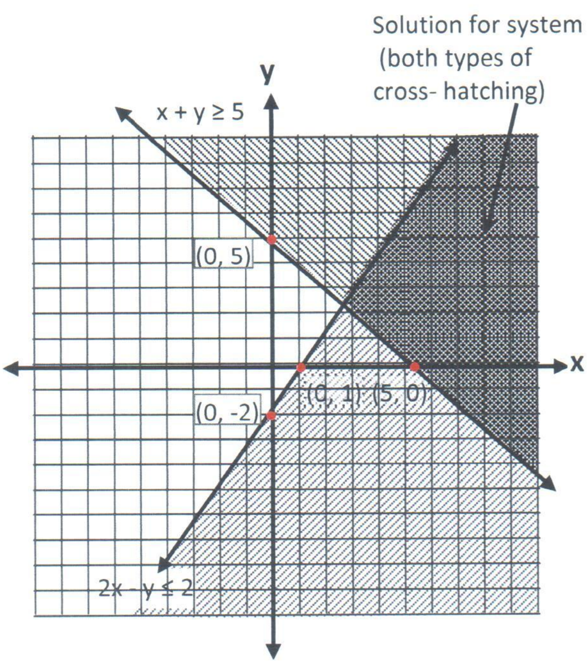
1.



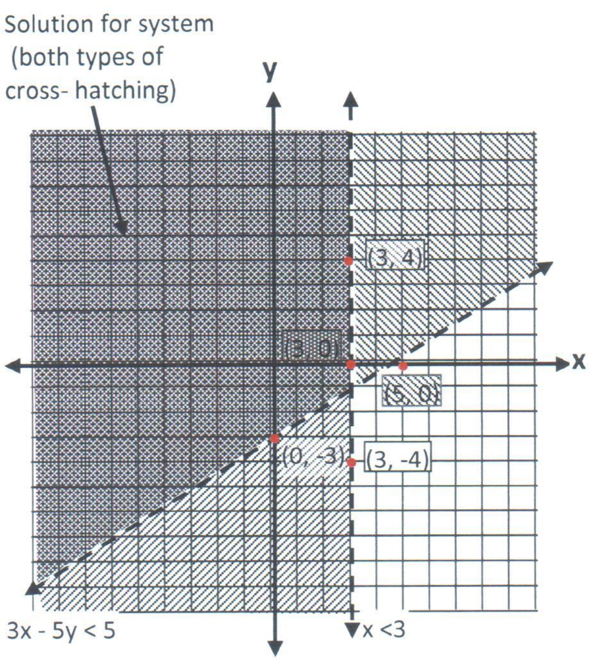
2.



3.



4.

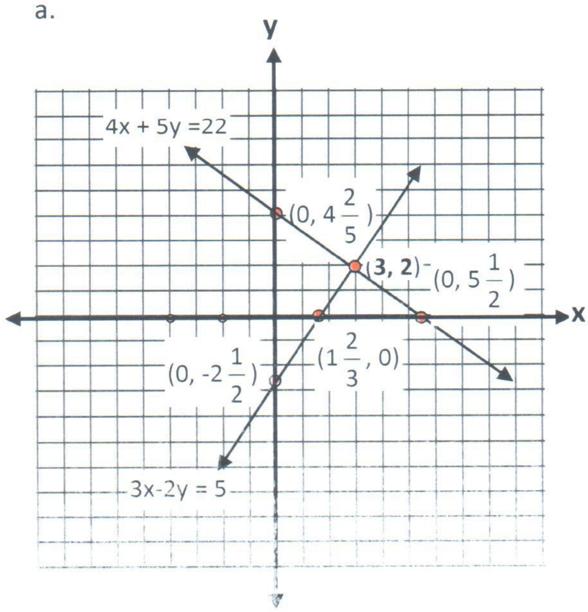


Summative task 12.4.2

1. a. $(2, \frac{1}{2})$ b. (4, 1) c.

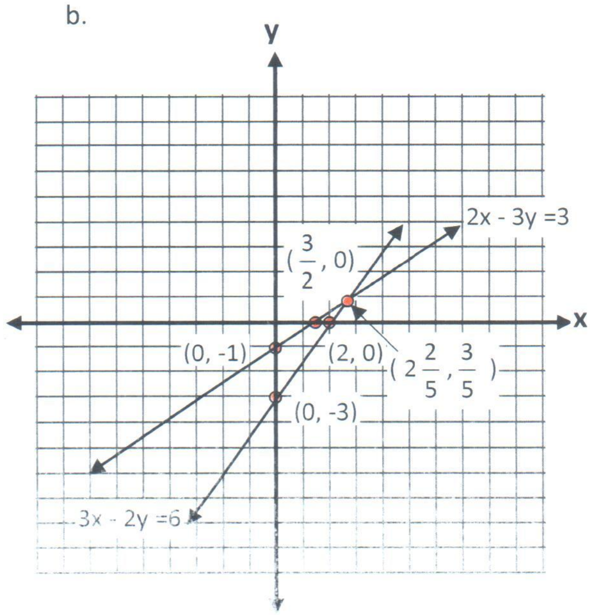
2. a. (7, 5) b. (4, 9)

3. a.



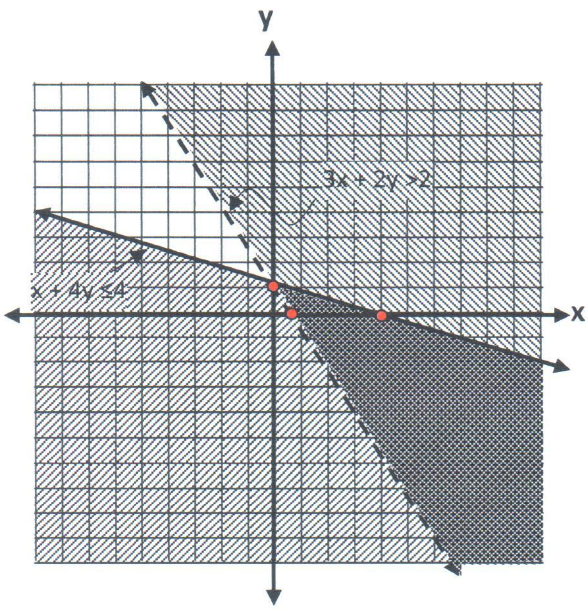
Solution: (3, 2)

b.

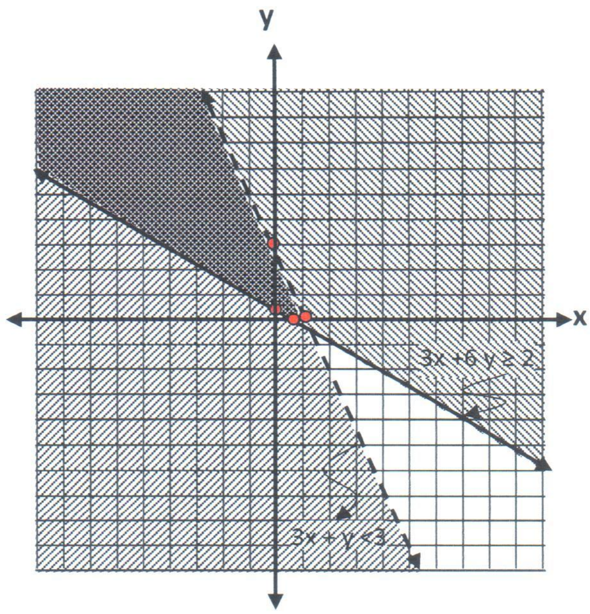


Solution: $(2\frac{2}{5}, \frac{3}{5})$

c.



d.



Learning activity 12.4.3.1

1. $3x^2 + 5x - 2 = 0$; $a = 3, b = 5, c = -2$

2. $2x^2 - 3x - 5 = 0$; $a = 2, b = -3, c = -5$

3. $3x^2 - 4 = 0$; $a = 3, b = 0, c = -4$

4. $3x^2 + 4x - 2 = 0$; $a = 3, b = 4, c = -2$

5. $3x^2 - 4x + 12 = 0$; $a = 3, b = -4, c = 12$

6. $x^2 + 2x - 4 = 0$; $a = 1, b = 2, c = -4$

Learning activity 12.4.3.2

1. $x = 1$ and $x = -2$
2. $x = 1$ and $x = -5$
3. $x = 0$ and $x = -5$
4. $x = \pm \frac{\sqrt{6}}{2}$
5. $x = 0$ and $x = 6$
6. $x = \pm 12$
7. $x = -3$ and $x = -2$
8. $x = 7$ and $x = -8$
9. $y = 3$ and $y = -3$
10. $x = 0$ and $x = -\frac{3}{2}$

Learning activity 12.4.3.3

1. $x = 3 \pm 2\sqrt{5}$
2. $x = 5 \pm 2\sqrt{3}$
3. $x = 2 \pm \sqrt{13}$
4. x = 8, x = -2
5. x = 4, x = -1
6. x = 13, x = -3

Learning activity 12.4.3.4

1. x = 6, x = 2
2. $x = 2 \pm \sqrt{2}$
3. x = 3 or $x = -\frac{3}{2}$
4. x =1 or $x = -\frac{2}{3}$
5. $x = \frac{1}{3}$ or $x = -\frac{2}{3}$

Learning activity 12.4.3.5

1. x-intercepts: (1, 0) and (3, 0)
   Vertex: (2, -1)
   Opens upward

2. x-intercepts: (-1, 0) and (-3, 0)
   Vertex: (-2, -1)
   Opens upward

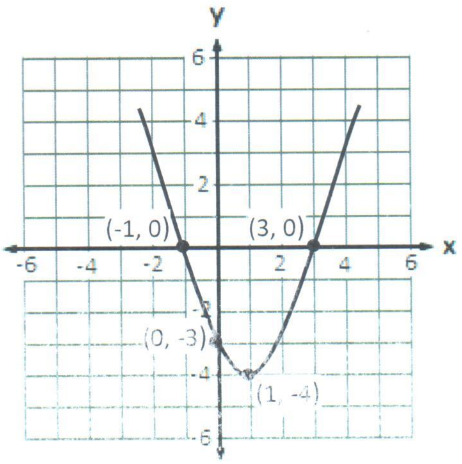
3. no x- intercepts
   Vertex: (-1, 1)
   Opens upward

4. x-intercepts: (0, 0) and (-8, 0)
   Vertex: (-4, -16)
   Opens upward

5. x-intercepts: (2, 0) and (3, 0)
   Vertex: $(\frac{5}{2}, \frac{1}{4})$
   Opens downward

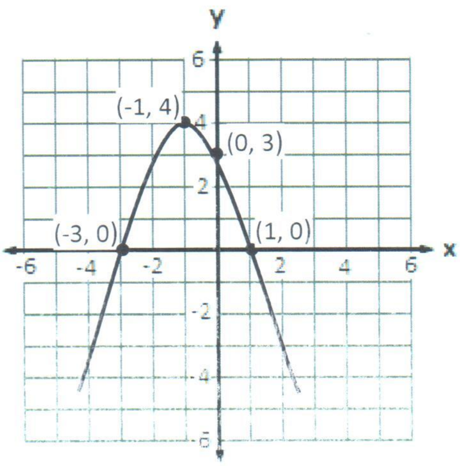
6. no x- intercepts
   Vertex: (1, -1)
   Opens downward

7.



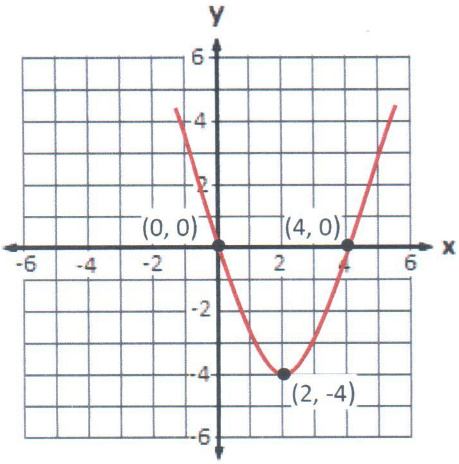
y-intercept: (0, -3)
x-intercepts: (-1, 0) and (3, 0)
vertex: (1, -4)

8.



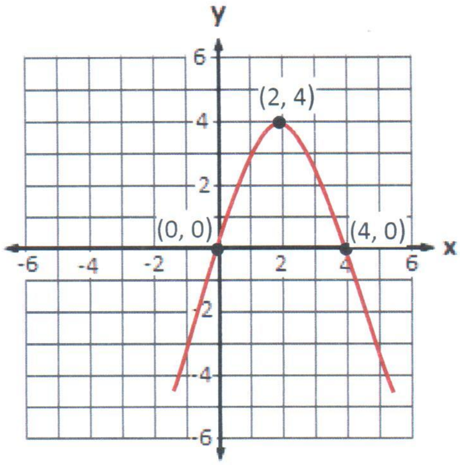
y-intercept: (0, 3)
x-intercepts: (-3, 0) and (1, 0)
vertex: (-1, 4)

9.



y-intercept: (0, 0)
x-intercepts: (0, 0) and (4, 0)
vertex: (2, -4)

10.



y-intercept: (0, 0)
x-intercepts: (0, 0) and (4, 0)
vertex: (2, 4)

Learning activity 12.4.3.6

1. w = 12 m
2. Hypotenuse = 28
   Smallest ide = 7
3. Width = 16 m
   Length = 35 m
4. x = 4 Or x = - 14
5. Width = 6 m; Length = 12 m

Summative Task 12.4.3

A.
1. D
2. D
3. B
4. C
5. D
6. D
7. B
8. A
9. A
10. A

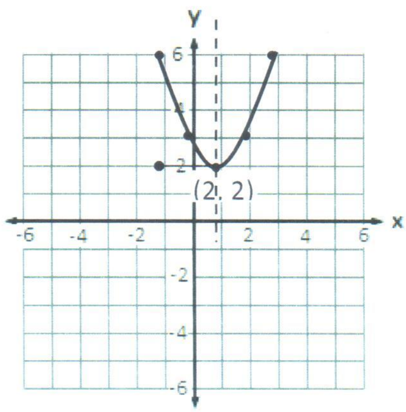
B.
1. $\frac{2}{3}$ or - $\frac{1}{4}$
2. 3 or 0

C.
1. 2 or -4
2. 1 or -6

D.
1. Minimum value = -2
   Vertex = (3, -2)
2. Minimum value = -2
3. Minimum value = 1
   Vertex = (2, 1)
   Vertex = (1, 7)
4. Maximum value = 7

E.
1. axis of symmetry = 2
2. minimum value = 2
3. vertex = (2, 2)
4. No x= intercepts
5. Find points.

x	$f(x)$
1	3
0	6



Plot symmetrical points and draw graph.

Learning activity 12.4.4.1

A.

1. $6^2 = 36$
2. $289^{\frac{1}{2}} = 17$
3. $14^{-2} = \frac{1}{196}$
4. $3^4 = 81$
5. $121 = 11^2$
6. $1000 = 10^3$
7. $a^x = 3$
8. $y^a = x$

B.

1. $\log_3(9) = 2$
2. $\log_2(32) = 5$
3. $\log_6(216) = 3$
4. $\log_4(1024) = 5$
5. $\log_2(8) = 3$
6. $\log_{64} 8 = \frac{1}{2}$
7. $\log_{12}144 = 2$
8. $\log_9 \frac{1}{81} = -2$
9. $\log_{\frac{1}{12}} \frac{1}{144} = 2$

C.

1. 3
2. 3
3. 2
4. -5
5. 3
6. 2
7. $\frac{1}{3}$
8. 4
9. $\frac{1}{3}$
10. -3

Learning activity 12.4.4.2

1. a. $\log_{10}18$
   b. log xy
   c. log 4x 2
   d. $\log ab^2c^3$

2. a. $\log_{10} 2$
   b. $\log \frac{x}{y}$
   c. log 4

3. a. $\log_{10} 5^3$ or $\log_{10} 125$
   b. $\log x^2$
   c. 2 log (4x)
   d. 20 ln x or $\ln x^{20}$
   e. $1000 = 10^3$ so ln 1000 – 3 ln 10

4. a. $\log_a \frac{x^2}{y}$ b. $\log_a y^3x^2$

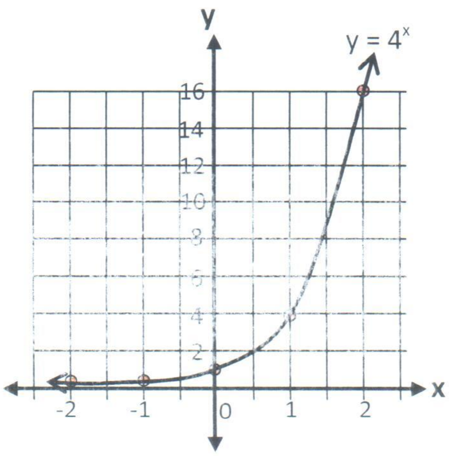
5. a. 1.1761 b. 1.8751 c. 0.2219

Learning activity 12.4.4.3

1. base 2. $a^y = x$ 3. y = x 4. $y = \log_5 x$

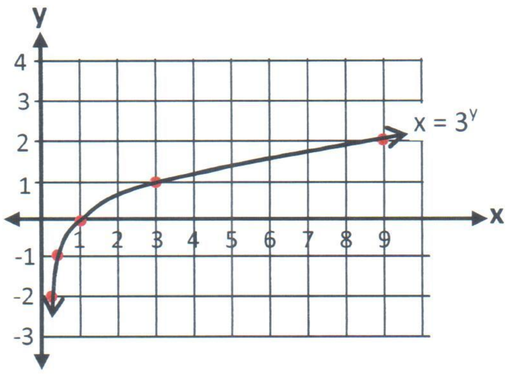
5.

x	y
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16



6.

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2



7. a. 2 b. 1.39794

8. a. 2.30259 b. -2.30259

Learning activity 12.4.4.4

1. a. $x = -\frac{2}{3}$ b. $x = -\frac{4}{3}$ c. $x = -\frac{2}{3}$

2. a. x = 1.583 b. x = 0.8614 c. x = 0.645

3. x = 2

4. $x = -\frac{7}{3}$

Learning activity 12.4.4.5

1. 14.21 years
2. 8 days
3. 2.3979 or 2.4

Summative task 12.4.4

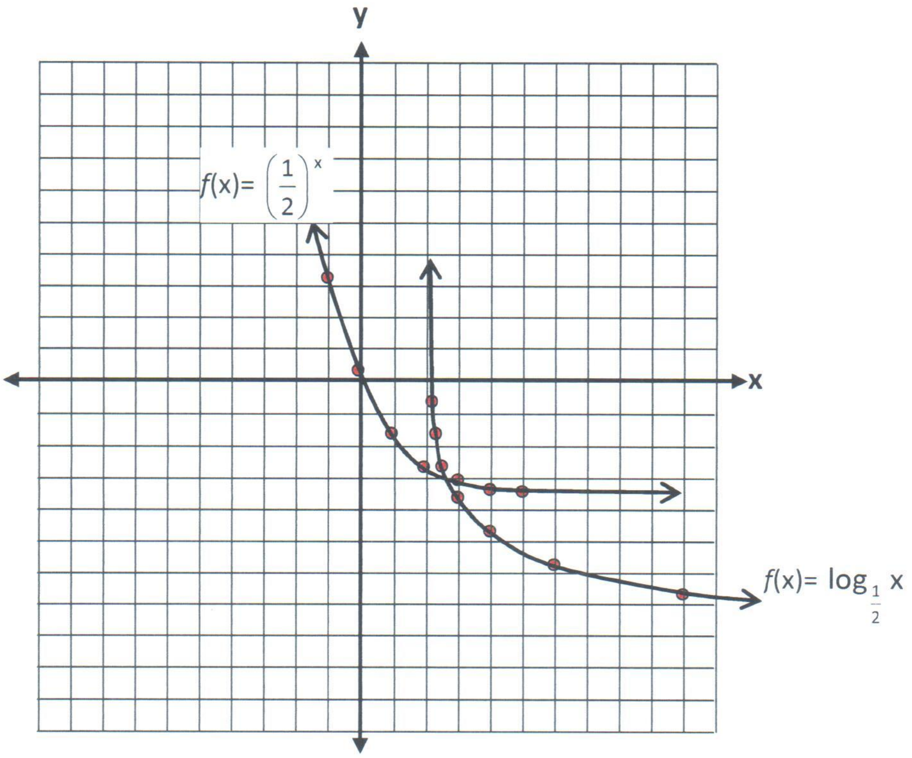
A.
1. C
2. B
3. D
4. D
5. D
6. A
7. C
8. D
9. D
10. B

B.

x	$y = f(x) = \left(\frac{1}{2}\right)^x$
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$

x	$y = f(x) = \log_{\frac{1}{2}} x$
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{8}$	3

Graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = \log_{\frac{1}{2}} x$ are shown below.



END OF UNIT MODULE 12.4

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Grade Levels	Subjects
Grades 7 and 8	1. English 2. Mathematics 3. Science 4. Social Science 5. Making a Living 6. Personal Development 7. English
Grades 9 and 10	1. English 2. Formal Mathematics 3. Practical Mathematics 4. Science 5. Social Science 6. Commerce 7. Design and Technology- Computing 8. Personal Development
Grades 11 and 12	1. English • Applied English • Language and Literature 2. Mathematics • Mathematics A • Mathematics B 3. Science 4. Social Science • History • Geography • Economics 5. Business Studies 6. Personal Development 7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified.

Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1
Maths 2
History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology
Chemistry, Physics and Social Science Streams: Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional