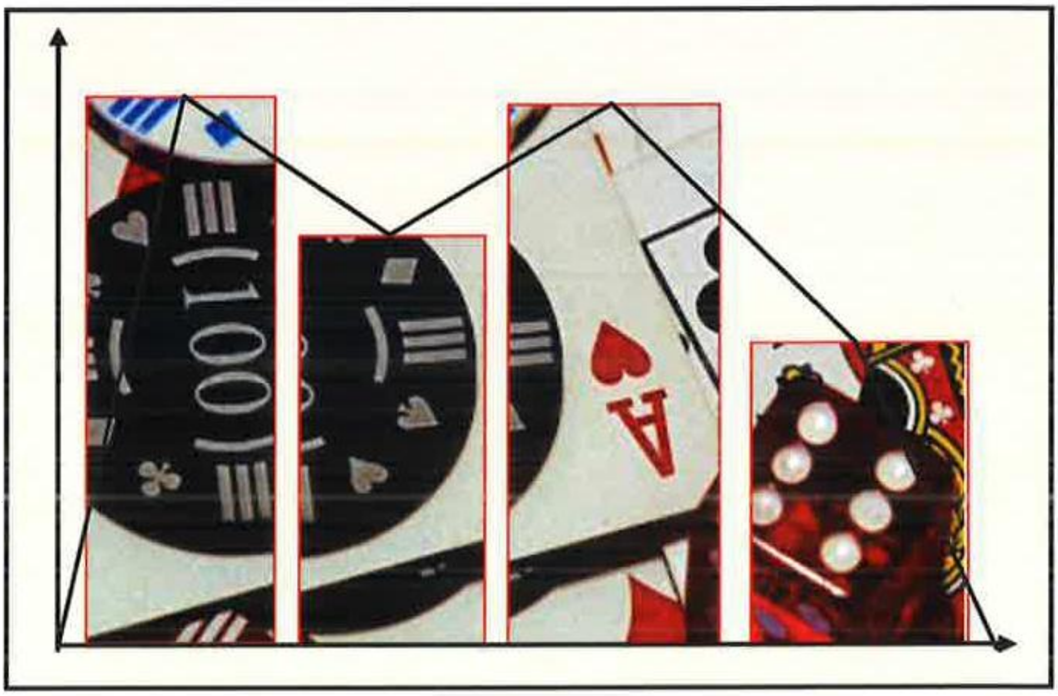
DEPARTMENT OF EDUCATION

GRADE 9

MATHEMATICS

UNIT 4



STATISTICS GRAPHS AND PROBABILITIES

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PAPUA NEW GUINEA

UNIT 4

STATISTICS GRAPHS
AND
PROBABILITIES

Acknowledgements

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Mathematics Department- CDAD
Mathematics Subject Review Committee-FODE
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MR. DEMAS TONGOGO
Principal-FODE

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Written by: Mr. Renato Ortega
Mrs. Luzviminda B. Fernandez

Finalised and compiled by: Mrs. Luzviminda B. Fernandez
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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the national OBE curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as a alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.

……………………………………………
DR. UKE KOMBRA, PhD
Acting Secretary for Education

Unit Introduction

Dear Student,

This is the Fourth Unit of the Grade 9 Mathematics Course. It is based on the CDAD Lower Secondary Syllabus and Curriculum Framework for Grade 9

This Unit consists of four topics:

Topic 1: **Reading and Interpreting Graphs**
Topic 2: **Straight Line Graphs**
Topic 3: **Probabilities**
Topic 4: **Statistical Estimation**

In Topic 1- **Reading and Interpreting Graphs**- You will learn to read and use mathematics to interpret different types of graphs

In Topic 2- **Straight Line Graphs**- You will learn to explore the Number Plane, draw line graphs, learn the importance of gradient and write the equation of a straight line.

In Topic 3- **Probability**- You will learn the language of chance, and calculate the chance of something happening. Furthermore, you will explore the different ways to compute probabilities.

In Topic 4- **Statistical Estimation**- You will learn to use Statistics to estimate different aspect of samples and population parameters. You will also learn the different sampling methods.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercise are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy using this unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

Step 1: Start with Topic 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of TOPIC 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:

[√] Lesson 1: Reading and Interpreting graphs

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Contents Page, like this [√] .This helps you to check on your progress.

Step 6: Revise the Topic using Topic 1 Summary, then, do Topic test 1 in Assignment 1.

Then go on to the next Topic. Repeat the process until you complete all of the four Topics in Unit 1.

<u>Assignment:</u> (Four Topic Tests and a Unit Test)
When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment Book. The Course Book tells you when to do each Topic Test.

When you have completed the four Topic Tests, revise well and do the Topic test. The Assignment Book tells you when to do the Unit Test.

> The Topic Tests and the Unit test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

UNIT 4

TOPIC 1

READING AND INTERPRETING GRAPHS

Lesson 1:	**Reading Bar Graphs**
Lesson 2:	**Reading Histogram**
Lesson 3:	**Interpolation & Extrapolation**
Lesson 4:	**Population Pyramid**
Lesson 5:	**Travel Graphs**

TOPIC 1: READING AND INTERPRETING GRAPHS

Introduction

In the field of Statistics, one should undergo the steps or aspect of collecting, organising, displaying and interpreting of data.

For example, a teacher asked a class to write their favourite subject. The students wrote them on pieces of papers and were collected.

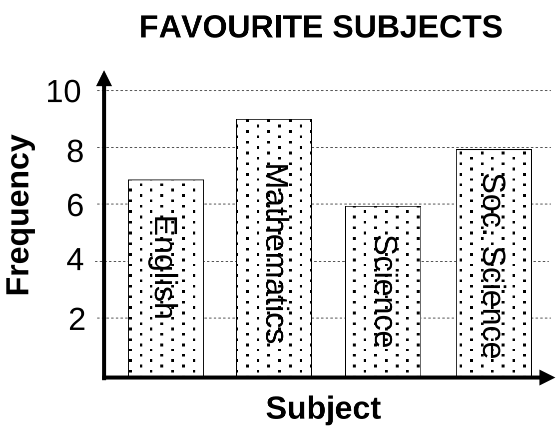
The data collected were presented below:

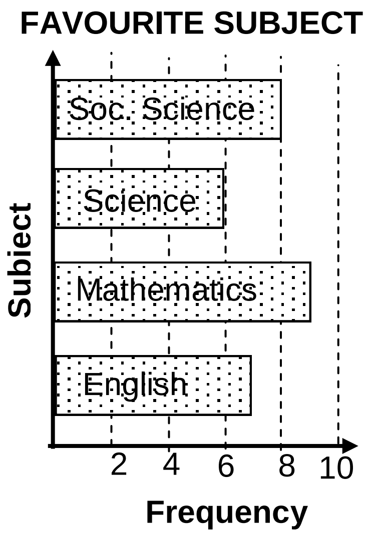
Mathematics	Science	Science	Social Science	Mathematics
Social Science	English	Mathematics	English	Science
Social Science	English	Mathematics	Science	Social Science
English	Social Science	Science	Social Science	Mathematics
Mathematics	English	Mathematics	English	Social Science
Mathematics	Science	Mathematics	English	Social Science

If the data would be organised in a frequency table:

Subject	Frequency
English	7
Mathematics	9
Science	6
Social Science	8
Total	30

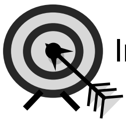
The data above can be displayed in the following forms:





Lesson 1: Reading Bar Graphs

You learnt something about bar graphs in your Upper Primary Mathematics.



In this lesson, you will:

- revise bar graphs
- read bar graphs.

In the previous unit, we learnt about the different types of data. Let us recall them.
If we collect data such as number of 'kulau'. Which of the following will not belong to the group?

12, 13.5, 10, 5, 6.5, 7, …

The numbers 13.5 and 6.5 do not belong to the group. 13.5 and 6.5 will not have any meaning when you are counting the number of 'kulau'.

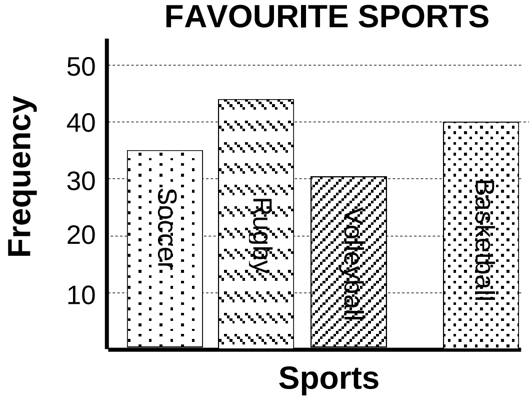
The others are what we call discrete data.

Discrete data is information that is collected by counting.

Discrete data is illustrated with a **bar graph**.
You have to remember the following in drawing a bar graph:

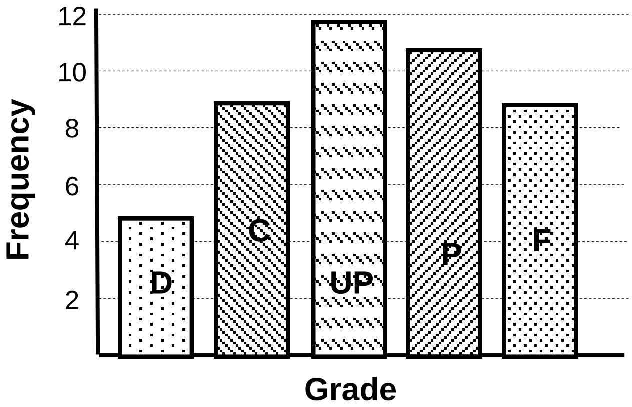
- The graph must have a title
- The axes must have labels
- The bar can be horizontal or vertical. Often, horizontal ones are called **bar graphs** while vertical ones are **column graphs**.
- The bars should be of the same width
- The bars should be the same distance apart
- The bars can be of different colours or of different shading

If we examine each of the following column graphs, determine what is missing in the graph?



In the example given, notice that the bars are of different distances apart.

There should be the same distance apart for each bar.



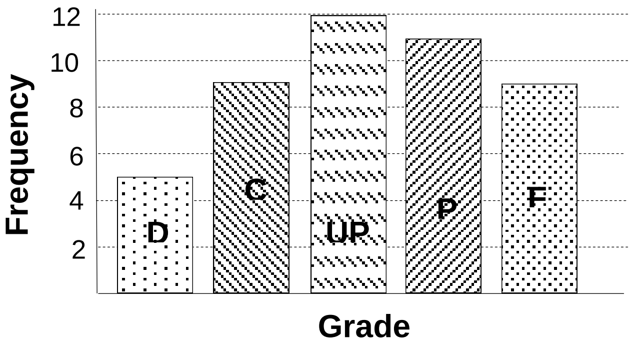
What is lacking in the second example?

What would be the appropriate title for this bar graph?

In the discussions on a field of mathematics, such as Statistics, we usually combine skills in order to improve on them and practice the skills at hand. Using problem solving skills, mathematical skills, as well as statistical skills, you would be able to solve many problems.

For an illustrative example, consider the column graph below and answer the following questions below.

GRADE 10 MATHEMATICS RESULTS



(i) How many students sat the Mathematics examination?

(ii) How many students were awarded Credit grade?

(iii) What percentage of the students was awarded a Credit grade?

(iv) What grade was the mode?

(v) What percentage were awarded Pass grade?

(vi) How many students were awarded Upper Pass or better?

(vii)Find the Mean Rating Index (MRI) of the group.

To obtain the Mean Rating Index of a group, the total number of students who got Upper Pass or better were added, divided by the number of students who sat the exam and multiplied by 100%.

Thus, to answer the first question, you need to add the number of frequency for each grade:

D = 5, C = 9 , UP = 12, P = 11, and F = 9

(i) The total number of students who sat for the Mathematics examination

Total = 5 + 9 + 12 + 11 + 9 = 46 students

(ii) C = 9. Therefore there are 9 who were awarded Credit.

(iii) Percentage = $\frac{9}{46} \times 100 = 19.56\%$

(iv) Mode is the item with the highest frequency. Thus, Upper Pass is the mode

(v) $\frac{11}{46} \times 100 = 23.91\%$

(vi) Upper Pass or better would mean, UP, C and D. Thus 5 + 9 + 12 = 26

26 students got Upper Pass or better.

(vii) Mean Rating Index = $\frac{26}{46} \times 100 = 56.52\%$

REMEMBER:
You have to remember the following in drawing a bar graph:

- the graph must have a title
- the axes must have labels
- the bar can be horizontal or vertical. Horizontal one are called **bar graphs** while vertical ones are **column graphs**
- the bars should be of the same width
- the bars should be the same distance apart
- the bars can be of different colours or of different shading.

Different skills can be combined such as mathematical skills, problem solving skills as well as statistical skills in order to solve problems.

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

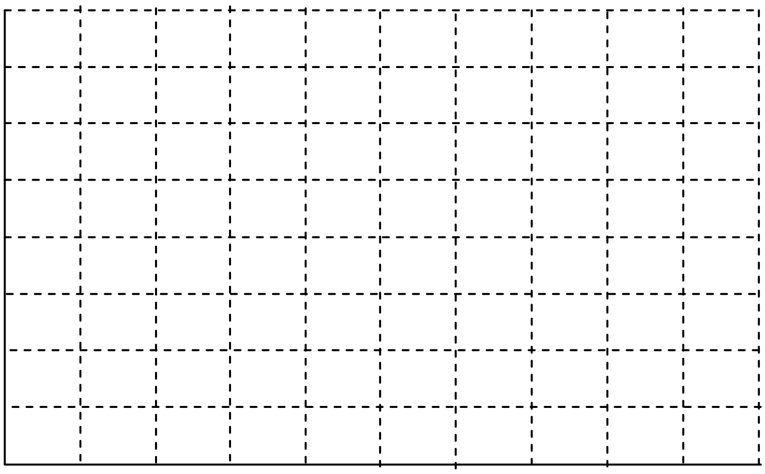
1. On a day the Airlines PNG computer at Tokua received bookings to different destinations as shown in the table below.

Destinations	Passengers
Buka	35
Madang	10
Lihir	30
Kavieng	25
Hoskins	15

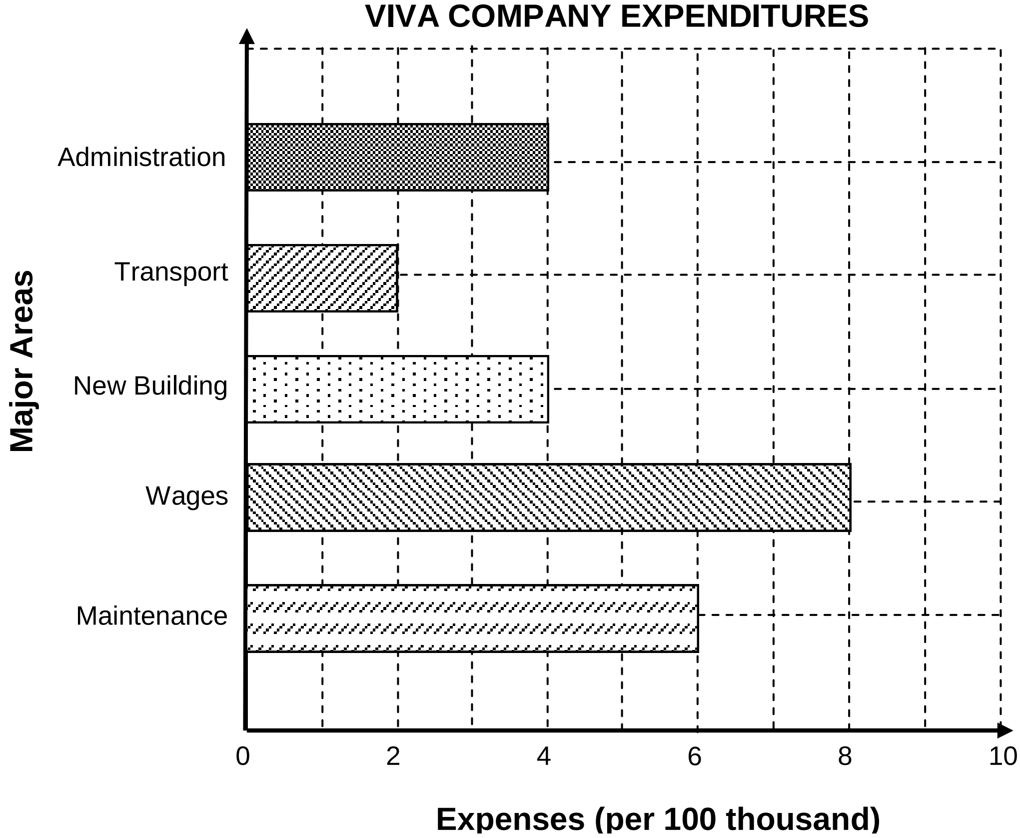
(a) Draw a column graph to illustrate the data.

Use the grid below.

Remember the properties of a bar graph



2. The bar graph shows the major areas of expenditure in a company, in hundred thousand of Kina.



(i) What is the total amount of expenditure of Viva Company?

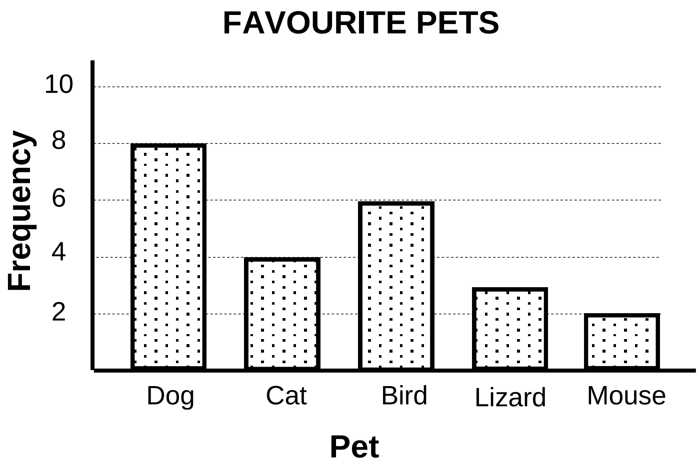
(ii) What percentage of the total expenditure is allotted for the New Building?

(iii) What percentage of the expenditure is the difference between Administration and Maintenance?

(iv) The next year's expenditure has increased by 5%, what is the amount that would be allotted for the Wages next year?

(v) If a pie chart would be made to display this data, how many degrees will be allocated for Transport?

3. The graph below represents the favourite pets of a particular Year 9 Class.



(i) How many students preferred a dog as a pet?

(ii) How many students in the class had a favourite pet?

(iii Which was the least favoured pet?

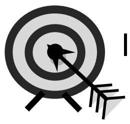
(iv) How many times more popular than mouse are dogs?

(v) If there are 28 students in the class, how many do not have a favourite pet?

CORRECT YOUR WORK, ANSWER ARE AT THE END OF TOPIC 1

Lesson 2: Reading Histograms

You learnt how to read bar graphs in Lesson 1.



In this lesson you will:

- revise histograms
- interpret histograms.

In lesson 1, you recalled about discrete data. This time let us recall continuous data.

In the previous example of the number of 'kulau' in which our examples were 12, 13.5, 10, 5, 6.5, 7, …, it was noted that 13.5 and 6.5 were not discrete data.

If we considered the diameter of the 'kulau', then 13.5 and 6.5 would be included. These are called **continuous data.**

Continuous data is information that is collected by measuring.

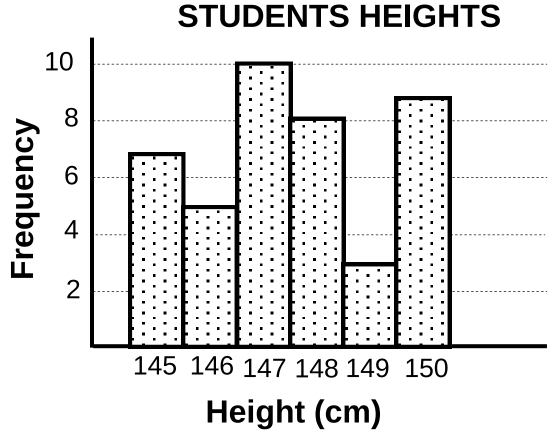
Continuous data can be illustrated using a **histogram.**

A histogram is a type of bar graph where there are no gaps between the bars, used to represent continuous data.

You have to remember the following in drawing a histogram:

- the histogram must have a title
- the axes must have labels
- the bars are usually vertical
- the bars should be of the same width
- the bars should be the same distance apart
- there are no gaps between the bars.
- the bars should have the same colour or have the same shading

An example of a histogram is shown:

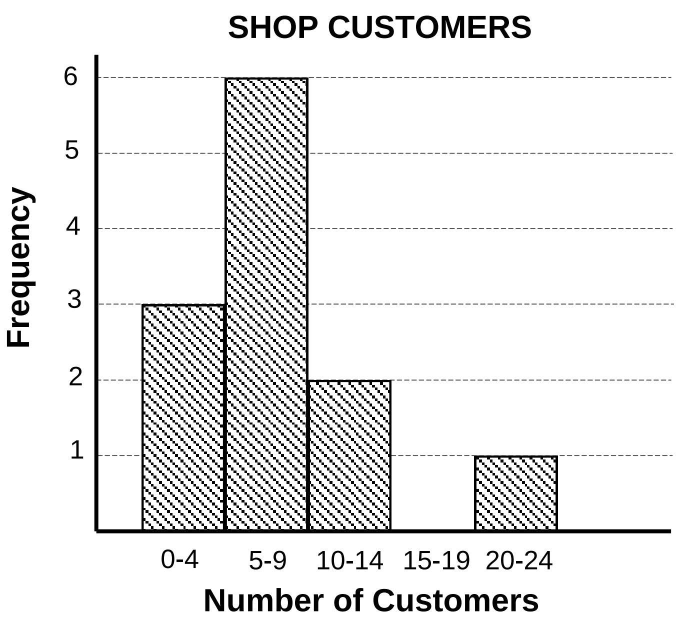


For example, consider the table representing the number of customers, per 5-minute interval, to enter a shop.

Number of Customers	Frequency
0 - 4	3
5 - 9	6
10 - 14	2
15 - 19	0
20 - 24	1

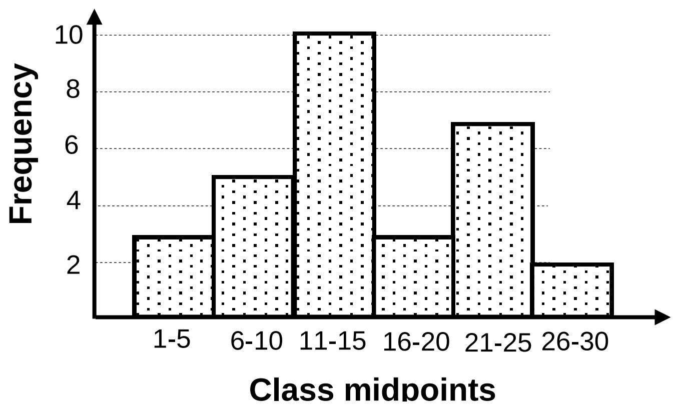
Drawing a histogram, you should consider the following steps,

1. Draw up a set of axes with frequency marked on the vertical axis.
2. Decide on your column width and start the values for the horizontal axis one-half column width from the vertical axis.
3. Draw the columns to represent each frequency leaving no gaps between them. You should note that a frequency of 0 means no column gets drawn, but a gap must be left in the appropriate place or position.



Let us consider the histogram showing the number of eggs produced daily by the School Poultry project on the next page.

NUMBER OF EGGS PRODUCED



i) What is the modal class number of eggs?

(ii) In which class would you find the median value?

(iii) Calculate the mean number of eggs produced daily.

(iv) Write the numbers of days in which 16 or more eggs are produced as a percentage?

Solutions:

(i) The modal class is one that has the highest frequency. The highest frequency is 10, thus the modal class is 11-15.

(ii) There are 30 days of recording the median will be the 15th and 16th recording.
It would be in 11—15 and 16- 20.

(iii) Considering the midpoint of each class, 1- 5 midpoint is 3, 6-10 midpoint is 8, ...etc. Thus the mean can be calculated as:

Class Midpoint

$$\text{Mean} = \frac{(3\times 3) + (8\times 5) + (13\times 10) + (18\times 3) + (23\times 7) + (28\times 2)}{(3 + 5 + 10 + 3 + 7 + 2)}$$

Frequency

$$\text{Mean} = \frac{450}{30} = 15$$

(iv) The total number of days where there are more than 16 eggs produced is 3 + 7 + 2 = 12.

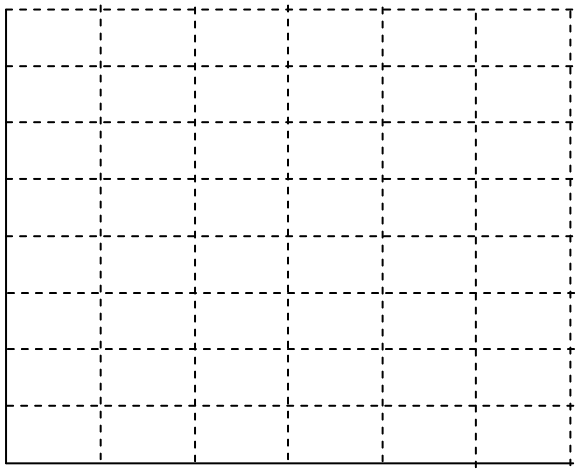
$$\text{Percentage of days} = \frac{12}{30} \times 100 = 40\%$$

NOW DO PRACTICE EXERCISE 2

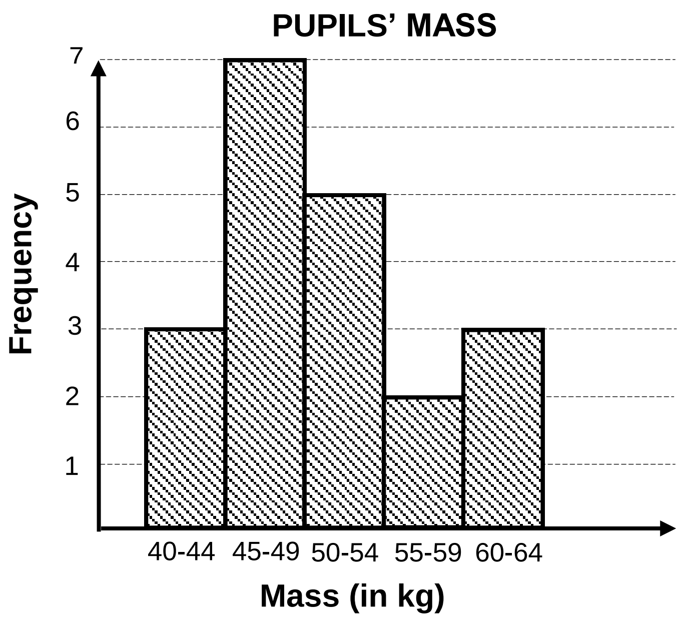
Practice Exercise 2

1. Use the grid below to draw the histogram of the following. Note the different properties you have to consider in drawing histogram.

Number of Brothers and Sisters	Frequency
0	3
1	10
2	5
3	2
4	1



2. Refer to the histogram representing the mass of pupils in a class to answer the questions that follow.



(i) What is the modal class mass?

(ii) What is the median class mass?

(iii) What percentage of the pupils has masses of 55 kg or more?

(iv) Calculate the mean mass of the pupils.

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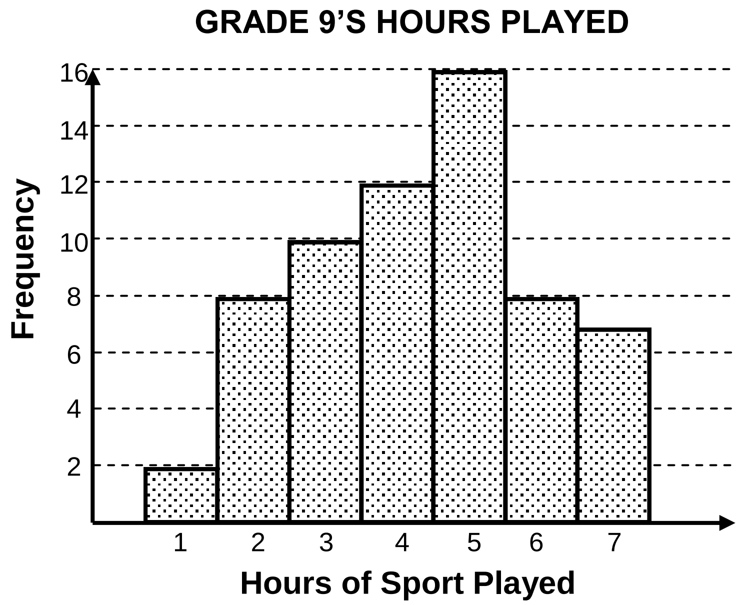
3. Below is the histogram showing the number of hours played per week by Year 9 students.



(i) Which is the most common number of hours of sports played per week?

(ii) Which is the least common number of hours of sports played per week?

(iii) How many students play at least 6 hours of sports per week?

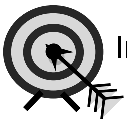
(iv) How many students were included in the survey?

(v) What is the average number of hours of sports played per week?

CORRECT YOUR WORK, ANSWERS ARE AT THE END OF TOPIC 1

Lesson 3: Interpolation and Extrapolation

You learnt to interpret and read histogram in the last lesson.



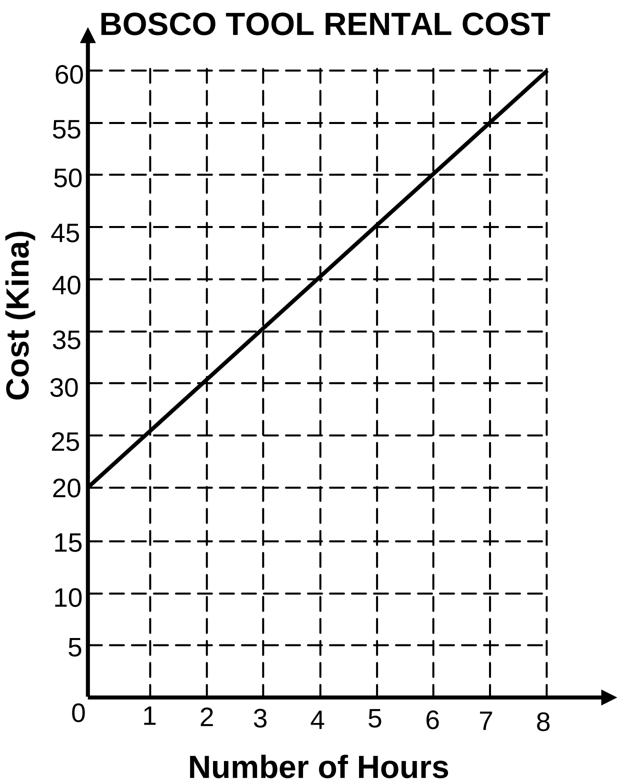
In this lesson you will:

- define interpolation and extrapolation
- identify steps to interpret data by interpolation and extrapolation
- read data using interpolation and extrapolation

Interpolation and extrapolation are mathematical terms given to the process of reading graphs

Interpolation - estimating information within a graph.
Extrapolation - extending the graph to estimate information.

To understand the concept above, let us use the graph of the cost for rental to a certain tool in Bosco Rentals.

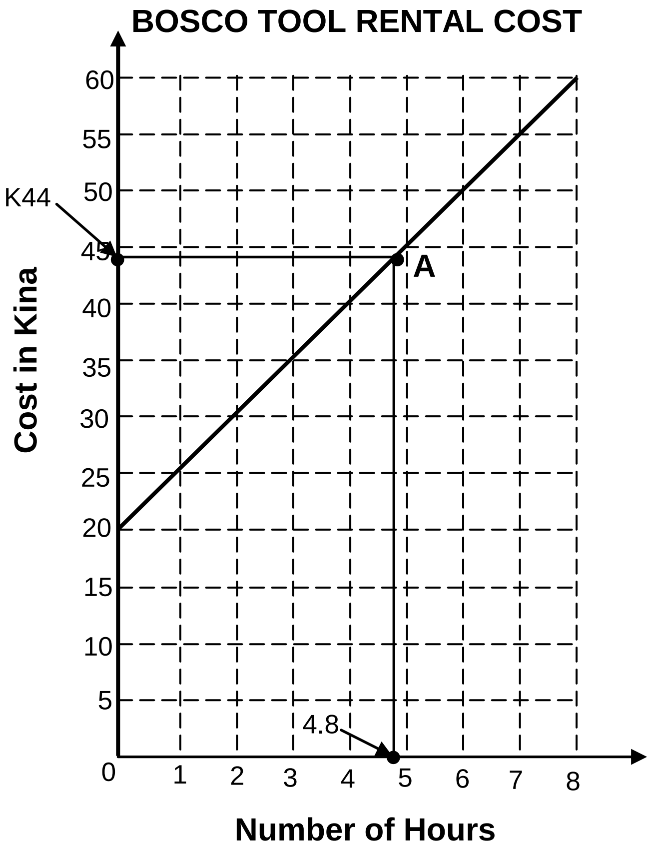


(i) How many hours can you rent the tool for K44.00?
(ii) How much will it cost to rent the tool for 3.5 hours?
(iii) How much will it cost to rent the tool for 10 hours?
(iv) How many hours can you rent the tool for K80.00?

We can answer the first two questions by looking at the line between the plotted points. This is called **interpolation**. The result obtained in this way is usually very accurate.

The other two questions can be solved by looking at the line extended beyond the plotted points. This is called **extrapolation**. The results obtained in this way may not always be accurate.

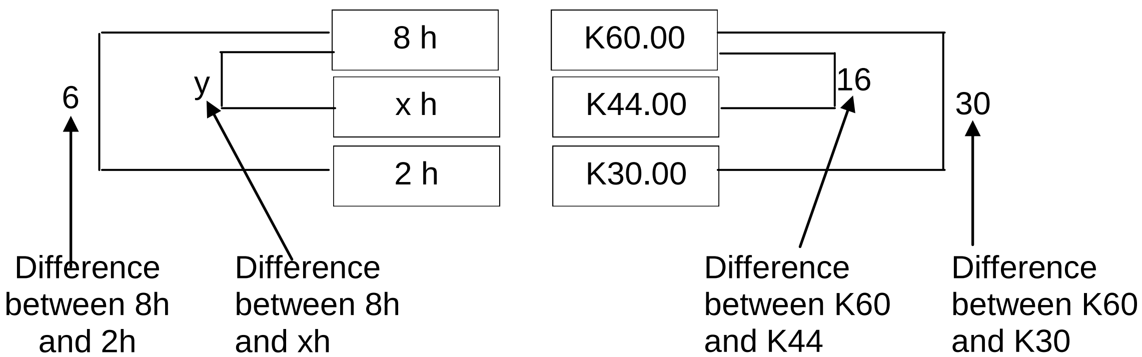
Let us use the graph to estimate the answer to questions (i) by interpolation.



STEPS:

1. Locate the point corresponding to K44 on the cost, as shown. Draw a line parallel to the horizontal axis till it intersects the graph at point A.
2. Draw another line perpendicular to the line drawn from the point A till it intersect the axis for number of hours.
3. Estimate the number of hours. Thus it is approximately 4.8 hour

Another solution is using algebra and proportion. There is another need to find two known points on the graph. We consider the points, (8h, K60) and (2h, K30)s.



To solve for y, take $\frac{y}{6} = \frac{16}{30}$

$$y\,(30) = 6(16)$$

$$y = \frac{96}{30} = 3.2$$

Therefore, y = 3.2

To get x, subtract 3.2 from 8. Hence, you have 8 – 3.2 = **4.8 hours.**

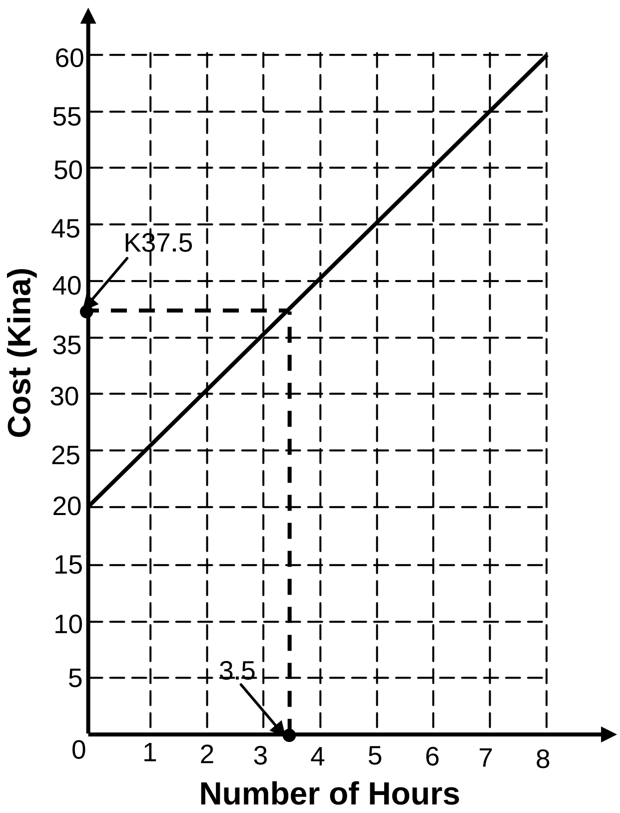
Using the graph or using algebra, let us try solving questions (ii) How much will it cost to rent the tool for 3.5 hours?

Use the graph below, to answer question (ii).

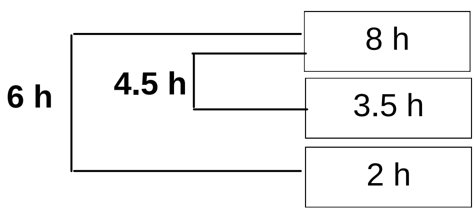
Steps:

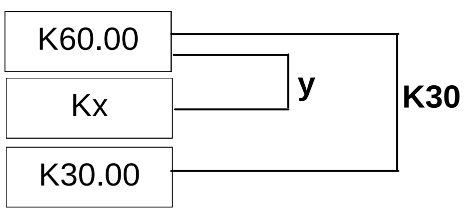
1. Locate the point 3.5 h by drawing an arrow.
2. Estimate the corresponding cost for 3.5 hours. Indicate the cost to the graph. Thus it is approximately K37.5.

BOSCO TOOL RENTAL COST



Use the diagram below to solve question (ii) using algebra. Write the numbers and variables necessary.





Use the space below to show the solution:

$$\frac{4.5}{6} = \frac{y}{30}$$

$$6y = 4.5(30)$$

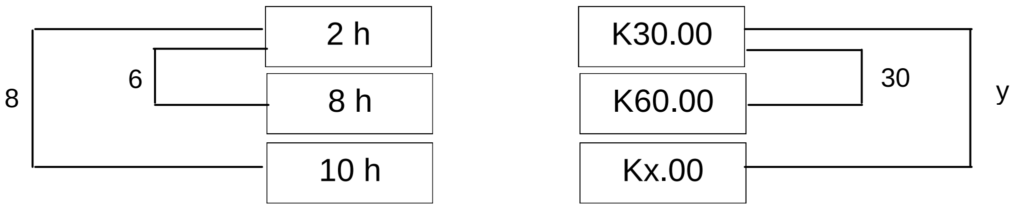
$$y = \frac{135}{6} = 22.5$$

To get x, subtract K22.5 from K60. Hence you have K60 – K22.5 = K37.5.

The answer should be K37.50

(iii) How much will it cost to rent the tool for 10 hours?

Solving questions (iii) by extrapolation,



To solve for y, $\frac{6}{8} = \frac{30}{y}$

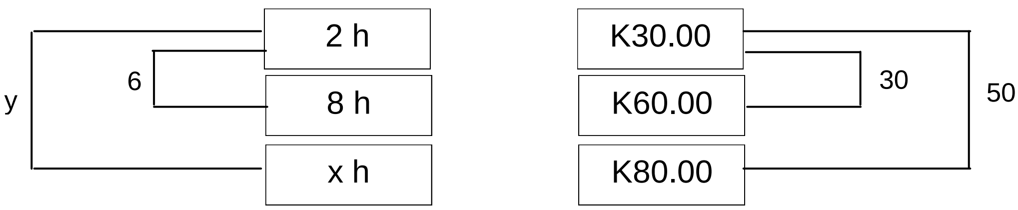
$$6y = 8(30)$$

$$y = \frac{240}{6} = 40$$

To get x, add 40 to 30 thus **x = K70.00**

(iv) How many hours can you rent the tool for K80.00?

Solving question (iv) by extrapolation,



To solve for y, $\frac{6}{y} = \frac{30}{50}$

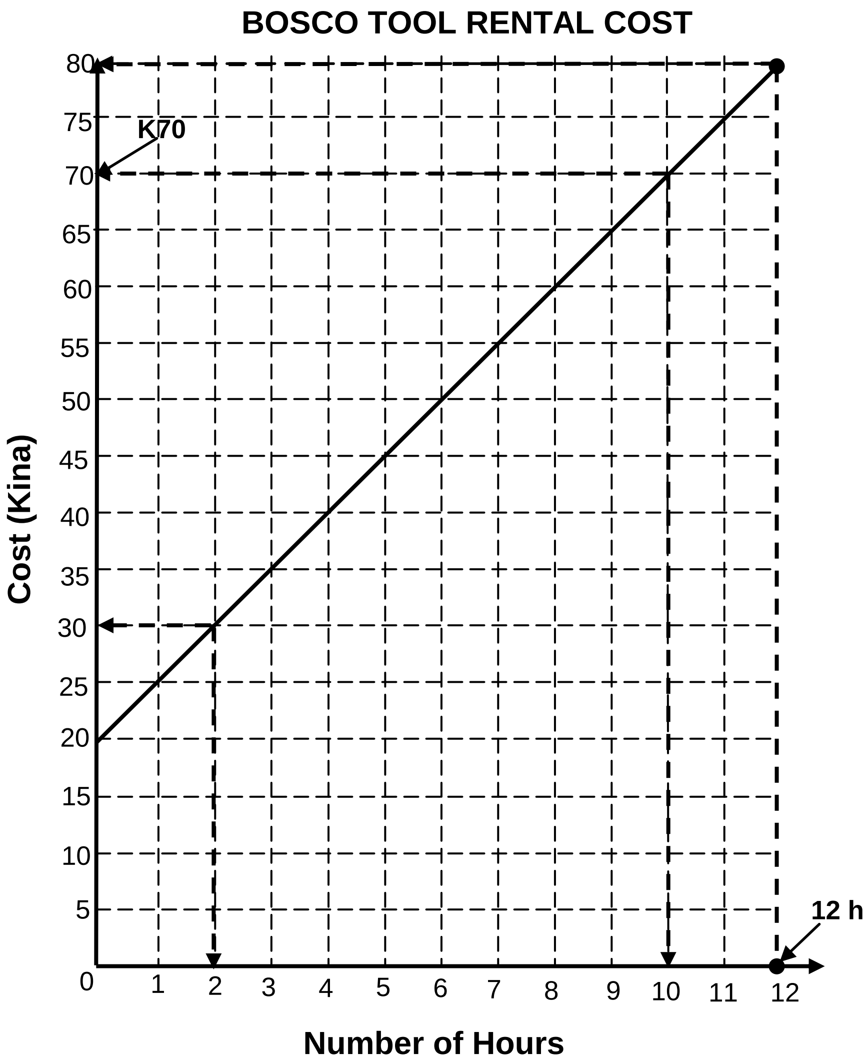
$$y = \frac{300}{30} = 10$$

To find x add 10 h to 2 h, then x = 2 + 10 = **12 hours**

To use extrapolation in the graph, there is a need to extend the graph in order to find solution to problems (iii) and (iv).

(i) Locate the point 10 hr on the horizontal axes. Draw the necessary lines till it intersects the drawn line. Indicate on the graph, the required cost.

(ii) Locate on the vertical axes K80.00. Draw the necessary lines till they intersect the drawn line. Indicate on the graph the required number of hours.



Remember:

Interpolation is the reading of data between the line of points used to construct the graph.

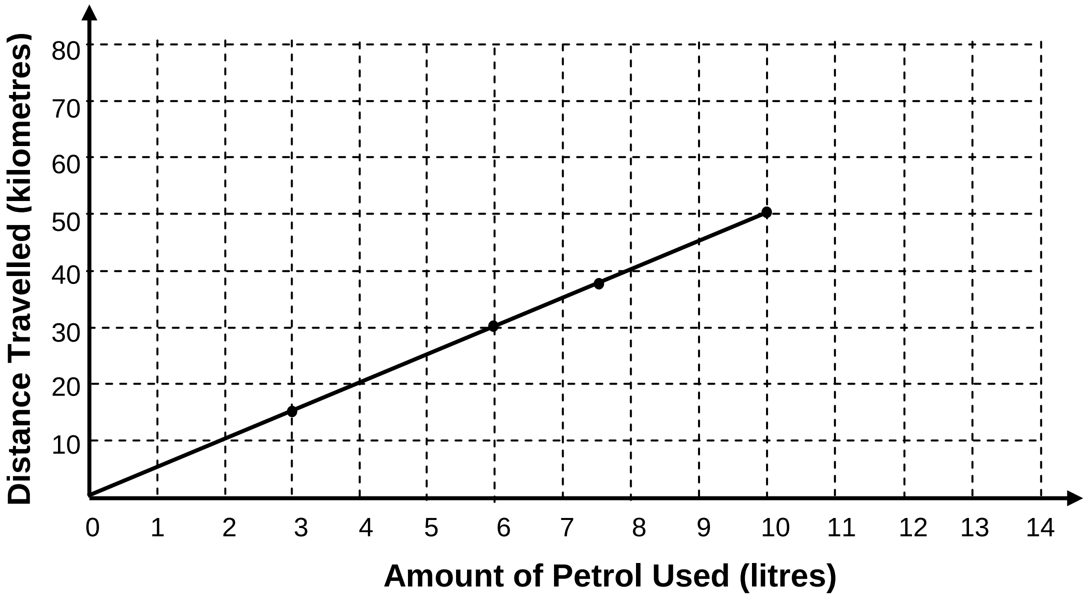
Extrapolation is the reading of data outside the line of points used to construct the graph.

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. A car owner checked how far his car would travel on an amount of petrol. He did this four times. The graph below has the four points plotted and a line of best fit drawn.

PETROL USE RATE



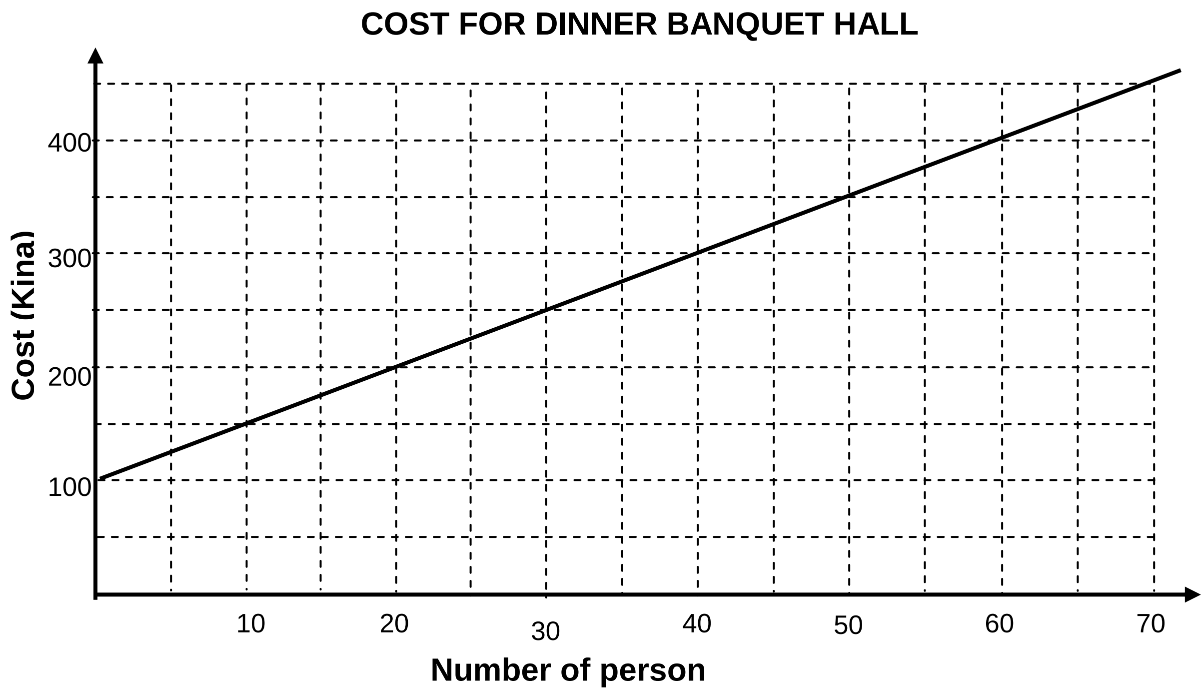
(i) How far would a car be expected to travel on 8.5 litres of petrol?

(ii) How much petrol would you expect to use on a journey of 40 kilometres?

(iii) How much petrol would you expect to use on a journey of 10 kilometres?

(iv) How far would the car be expected to travel on 12 L of petrol?

2. The graph shows the cost of renting a hall for a dinner function.



(i) Use interpolation to determine the cost for 25 guests

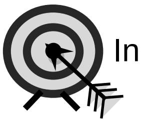
(ii) Verify the answer in (i) using algebra.

(iii) How many guests could pay for K1100? Use extrapolation or algebra to find the answer.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

Lesson 4: Population Pyramids

You learnt what interpolation and extrapolation were in the last lesson. You also learnt to interpret data on graphs using interpolation and extrapolation.



In this lesson you will:

- define a population pyramid
- read population pyramids

We can use many diagrams to display and compare continuous data. Some of the main ones were discussed earlier in Unit 3 such as the following:

1. Stem and leaf diagrams
2. Frequency tables and frequency polygons
3. Histograms
4. Cumulative Frequency polygon

Remember that continuous data is usually collected in groups, so most of these diagrams use groups to display data.

In this section, we are going to display and compare two sets data using the **population pyramid**.

A population pyramid is a pyramid shaped diagram which shows the age distribution of a population. The youngest are represented by a rectangle at the base and the oldest by one at the apex or top.

A **population pyramid**, also called an **age picture diagram**, is a graphical illustration that shows the distribution of various age groups in a population (usually that of a country or region of the world) which forms the shape of a pyramid when the population is growing.

In Ecology a population pyramid is used to determine the overall age distribution of a population, this is an indication of the reproductive capabilities and likelihood of the continuation of a species.

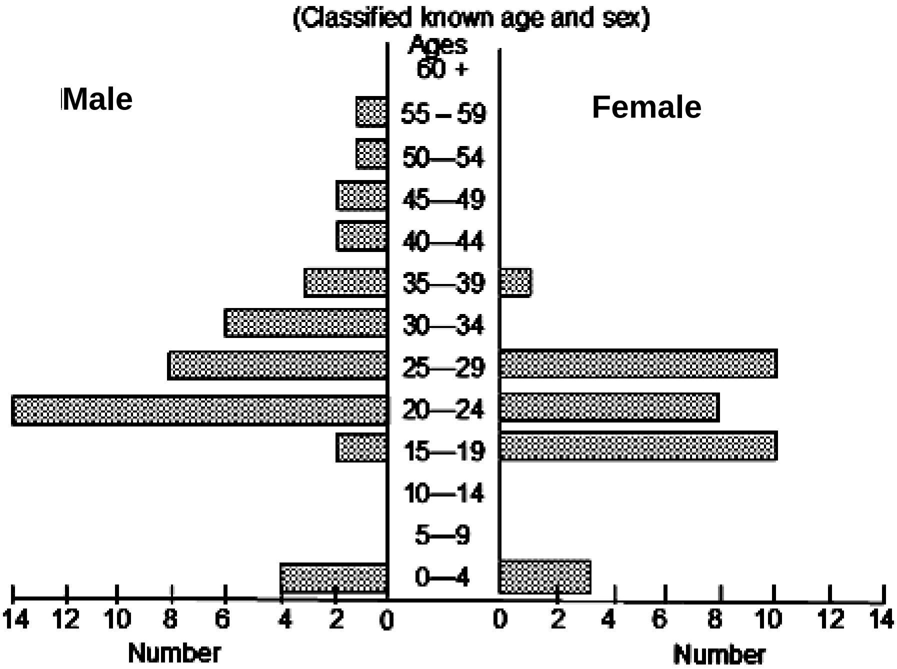
Usually, a population pyramid consists of two back-to-back bar graphs, with the **population** plotted on **the x-axis** and **age** on **the y-axis**, one showing the number of males and one showing females in a particular population in five-year age group. Males are conventionally shown on the left and females on the right, and they may be measured by a raw number or as a percentage of the total population.

A population pyramid is useful for comparing two sets of continuous data. It looks like a bar chart with the bars back to back.

Example 1

Shown below is a population pyramid showing the number of people in PNG, infected with HIV in the year 1987 to 1992.

PEOPLE INFECTED WITH HIV IN PNG 1987 - 1992



Using the population pyramid, we can answer the following questions:

(i) What was the total number of people infected with HIV from 1987 to 1992?

(ii) Find the total number of people, between the ages of 15 to 29, who were HIV infected from 1987 to 1992?

(iii) Find the percentage of people, between the ages 15 to 29, who were infected with HIV from 1987 to 1992.

(iv) Discuss why most of the people infected with HIV are between the ages of 15 to 29.

Solutions:

(i) No. of Male = 4 + 0 + 0 + 2 + 14 + 8 + 6 + 3 + 2 + 2 + 1 + 1 + 0 = 43 Males

No. of Female = 3 + 0 + 0 + 10 + 8 + 10 + 0 + 1 + 0 + 0 + 0 + 0 = 32 Females

Total = 75 people

(ii) 2 + 10 + 14 + 8 + 8 + 10 = 52 people

(iii) $\frac{52}{75} \times 100\% = 69.3\%$

(iv) This is the age in which people are most sexually active.

Example 2

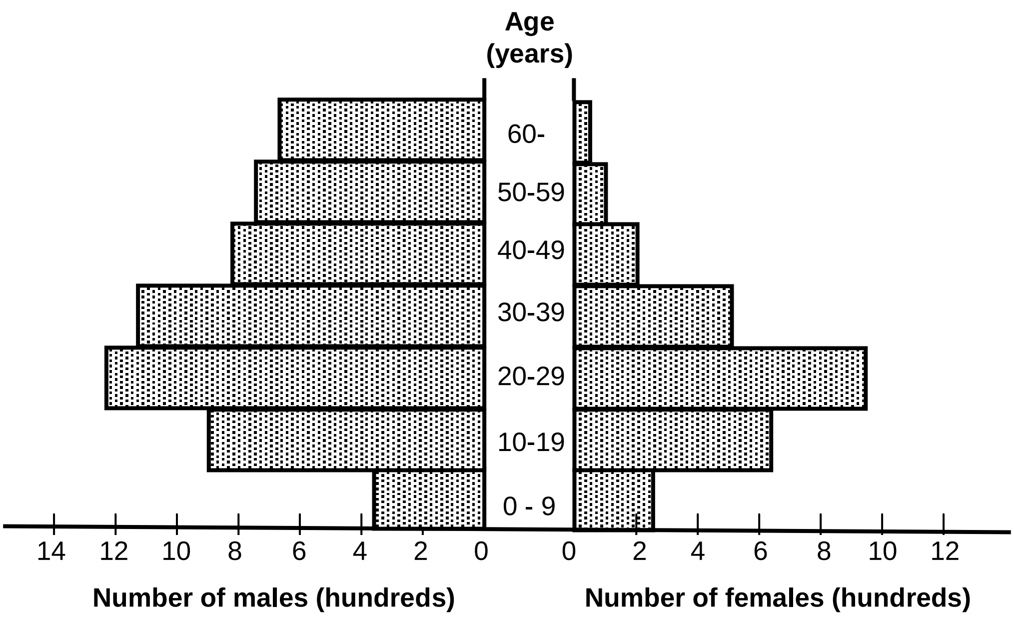
Use a population pyramid to illustrate this data showing the distribution of supporters of a soccer match. (All data has been rounded to the nearest ten.

Age (years)	Male	Female
0-9	380	270
10-19	890	620
20-29	1250	970
30-39	1140	510
40-49	830	210
50-59	740	80
60 -	680	50

Solution:

Step1: Draw the age groups in the centre. Mark axes on either side.

Step 2: Draw the data for males.
Draw the data for females.



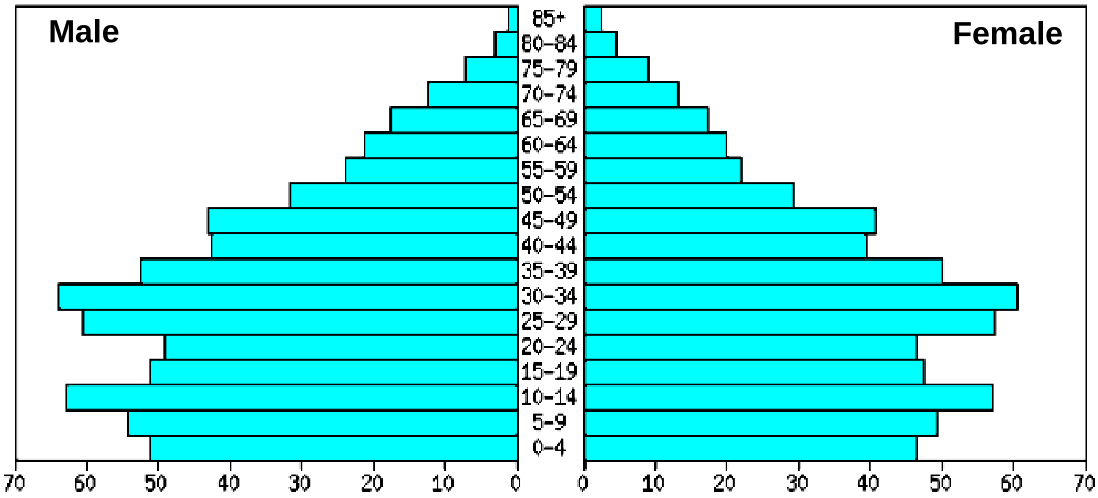
Note that the final age group in the table doesn't have an endpoint. In the pyramid we have chosen 69 as the endpoint so the rectangle is the same width as the other age group.

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. The population pyramid below shows the distribution of the population of China in 2000. Use the population pyramid to answer the following questions:

POPULATION OF CHINA IN 2000



Population (in Millions)

Source: U.S. Census Bureau, International Data Base.

(i) How many Chinese males are there in the age group 30—34 years old?

(ii) How many Chinese females are there in the age group of 30—34?

(ii) What age group has about 30 000 000 for each gender?

(v) What is the ratio of the number of males in the age groups 35 -39 to 55 - 59?

2. The age distribution for two suburbs of a city **A** and **B are shown as follows:**

Age Group	0-14	15-29	30-44	45-59	60-74	75+
A	1000	1660	950	1120	1690	680
B	1110	1570	1380	1710	1250	550

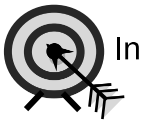
Represent the data using population pyramid.

15 18

CORRECT YOUR WORK. ANSWER ARE AT THE END OF TOPIC 1

Lesson 5: Travel Graph

You learnt how to read and interpret population pyramid in the previous lesson.



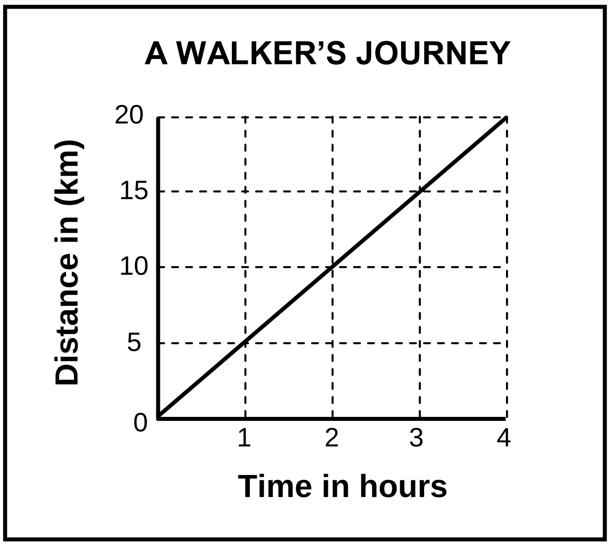
In this lesson you will:

- define travel graph
- read travel graph.

Travel Graph

Travel graphs are line graphs that show the relationship between distance and time travel. The distance travelled is represented on the vertical axis and the time taken to travel the distance is represented on the horizontal axis.

Below is an example of a time graph which indicates a travel at a constant speed.



1. What distance has the walker travelled after 2 hours?

2. What is the average speed of the walker? (Note: To get the average speed is to divide a speed with the time travelled)

3. Did the walker stop at any time?

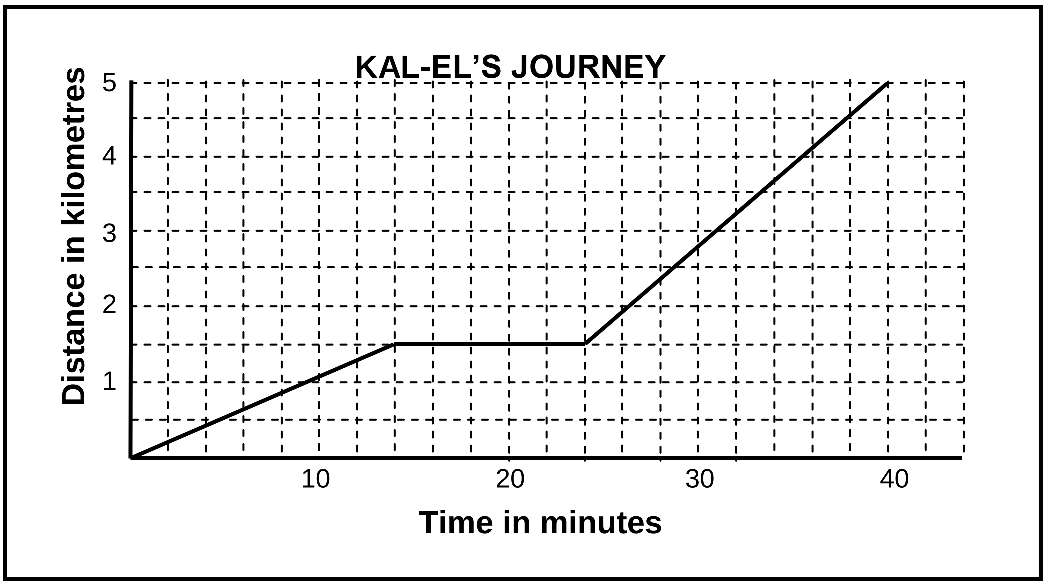
Solution:

(1) The distance travelled after 2 hours as shown in the graph is 10 km

(2) Dividing 10 km by 2 hours will give us the average speed of 5 km / hr.

(3) From the graph given, the walker did not stop. Stops can be indicated by horizontal line on a given time or period of time.

Let us have another example of a travel graph. The graph shows Kal-el journey when he visited his friend. He stopped for refreshment before he reached his friend.



Using the graph, answer the following questions:

(i) How far away does his friend live?

(ii) After what time did he stop for the refreshment? How far was this from home?

(iii) How long did he stop for refreshment?

(iv How long did it take Kal-el to reach his friend?

Solutions:

(I) Since the graph shows Kal-el journey to see his friend. The place of his friend will be at the end of the graph. Thus his friend lives 5 km from home

(iii) He stopped after 14 minutes travelling from home, which is 1.5 kilometres from home.

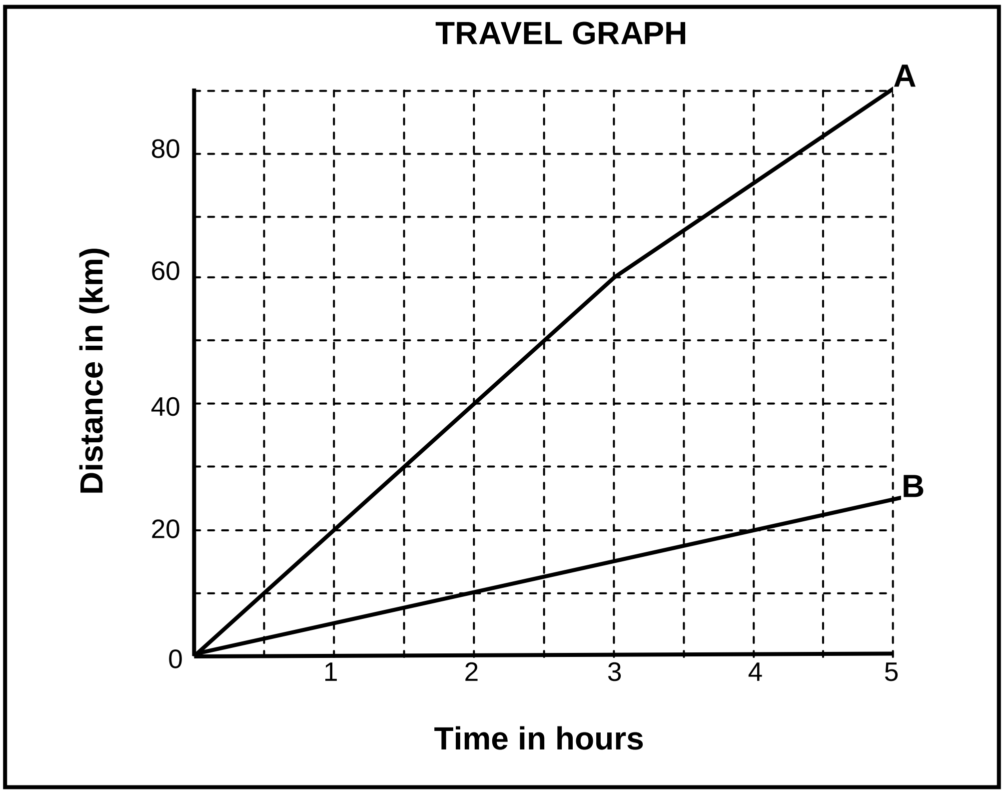
(iii) The scale on the horizontal axis is 1 unit = 2 minutes. The horizontal line on the graph represents the stop, and it is 5 units long. Thus the stop is 10 minutes long.

(iv) It took him 40 minutes to reach his friend's place.

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

Refer to the graph below to answer Questions 1 and 2.



One line represents the journey of a cyclist, the other, the progress of a walker.

1. Which graph do you think represents the

   (i) walker?

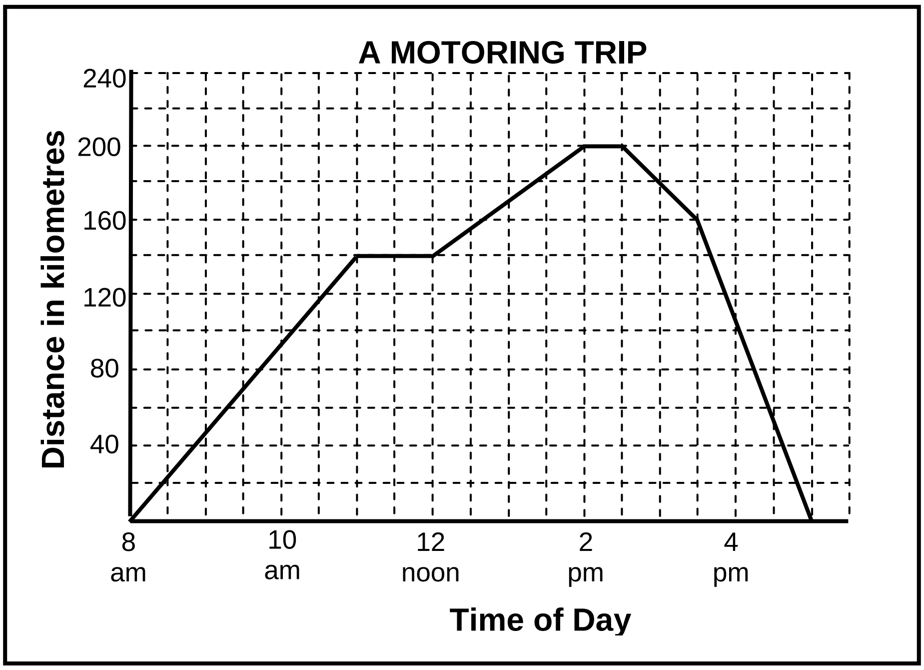
   (ii) the cyclist's journey?

2. (i) What was the total distance the cyclist travelled in 5 hours?

   (ii) What is the average speed of the cyclist?

   (iii) What is the average speed of the walker?

3. The graph of a motoring trip is shown below, study it carefully and answer the questions concerning the trip.



(i) What time did the motorist begin his trip?

(ii) How far did he go in the first three hours?

(iii) What was his speed in the first three hours?

(iv) How many stops did the motorist make during the journey?

(v) How far from home was the motorist at 3 p.m.?

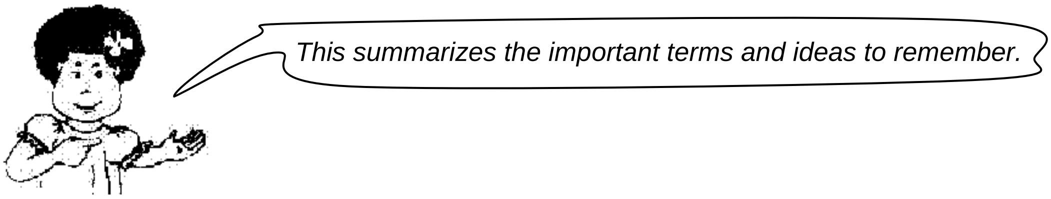
(v) What was the total distance travelled on the trip?

(vii) At what time did the motorist begin the trip home?

Viii At what time did he arrive home?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

TOPIC 1: SUMMARY



- **Discrete** data refers to information that is collected by counting.

- You have to remember the following in drawing a bar graph:
  (a) The graph must have a title.
  (b) The axes must have labels.
  (c) The bars can be horizontal or vertical. Horizontal ones are called **bar graphs** while vertical ones are called **column graphs**.
  (d) The bar should be of the same width.
  (e) The bars should be the same distance apart.
  (f) The bars can be of different colours or of different shading.
  (g) Different skills can be combined such as mathematical skills, problem solving skills as well as statistics skills in order to solve problems.

- **Continuous data** is information that is collected by measuring.

- A **histogram** is a type of bar graph where there are no gaps between the bars. It is used to represent continuous data.

- **Interpolation** is estimating information within a graph.

- **Extrapolation** is extending the graph to estimate information.

- **Population Pyramid** usually consists of two back-to-back bar graphs, with the population plotted on the x-axis and age on the y-axis. One shows the number of males and the other shows the number of females in a particular population in five-year age groups.

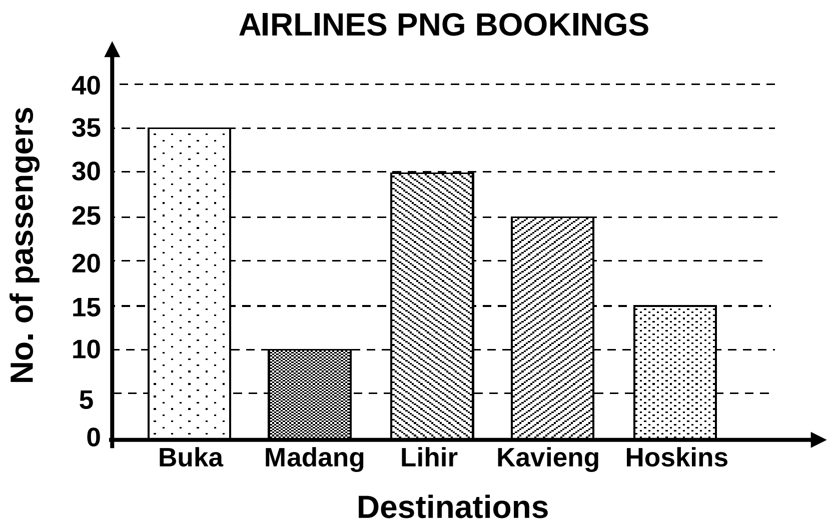
- **Travel graphs** are line graphs that show the relationship between distance and time travelled. The distance travelled is represented on the vertical axis and the time taken to travel the distance is represented on the horizontal axis.

REVISE LESSONS 1-5 THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 4.

ANSWERS TO PRACTICE EXERCISES 1-5

Practice Exercise 1

1.



2. (i) K 2 400 000

(ii) 16 2/3 % or 16.67%

(iii) 8 1/3% or 8.33 %

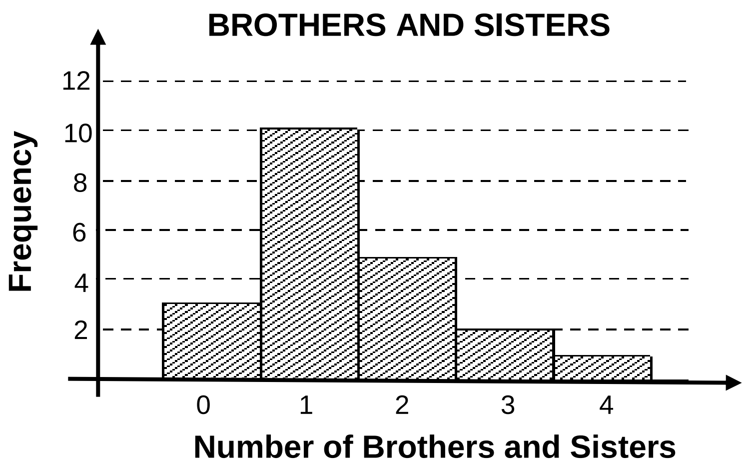
(iv) K 840 000

(v) 30°

3. (i) 8 (ii) 23 (iii) Mouse (iv) 4 times (v) 5

Practice Exercise 2

1.



2. (i) 45-49 (ii) 50-54 (iii) 25% (iv) 50.75

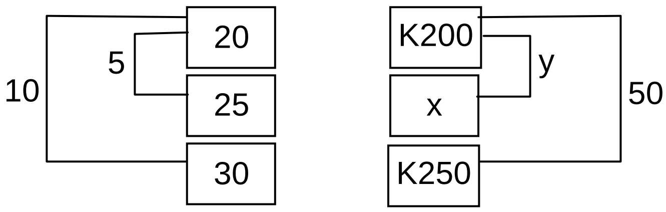
3. (i) 5 hours
   (ii) 1 hour
   (iii) 15 students
   (iv) 63 students
   (v) $4.33\bar{3}$ or $4\frac{1}{3}$ hours

Practice Exercise 3

1. (i) 42.5 km
   (ii) 8 litres
   (iii) 2 litres
   (iv) 60 km

2. (i) K225

   (ii)



$$\frac{5}{10} = \frac{y}{50}$$

y = 25

x = K200 + y

x = K200 + K25 = K225

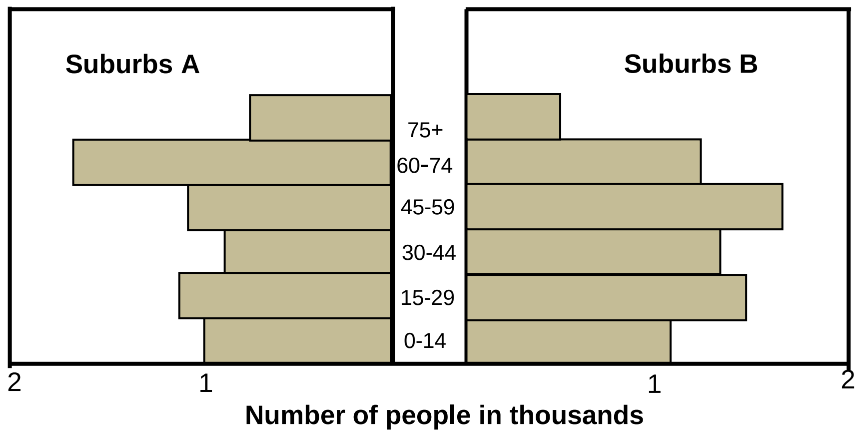
   (iii) 200 guests

Practice Exercise 4

1. (i) 65 million
   (ii) 60 million
   (iii) 50-54 age group
   (iv) 2:1

2.

AGE DISTRIBUTION OF TWO SUBURBS



Practice Exercise 5

1. (i) B (ii) A

2. (i) 90 km (ii) 18 km / h (iii) 5 km / h

3. (i) 8 am (ii) 140 km (iii) 46 2/3 km / h (iv) 2

(v) 180 km (vi) 400 km (vii) 2.15 p.m. (viii) 5 p.m.

END OF TOPIC 1

TOPIC 2

STRAIGHT LINE GRAPHS

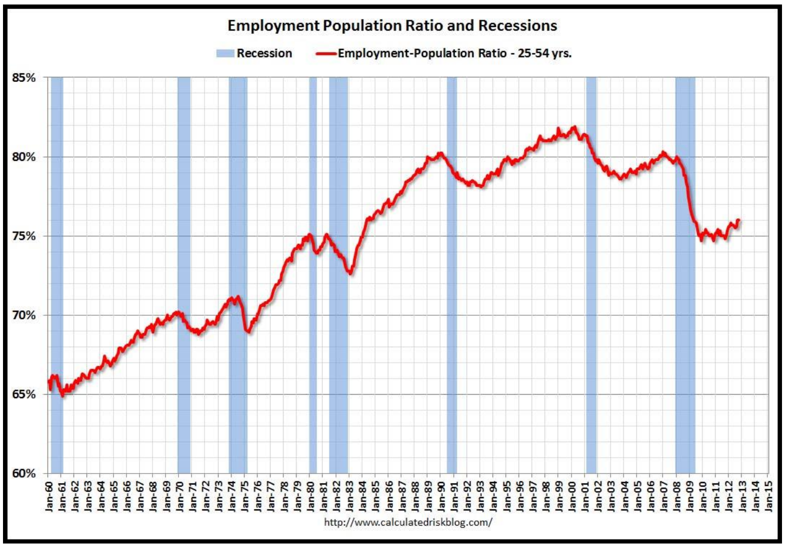
TOPIC 2: STRAIGHT LINE GRAPHS

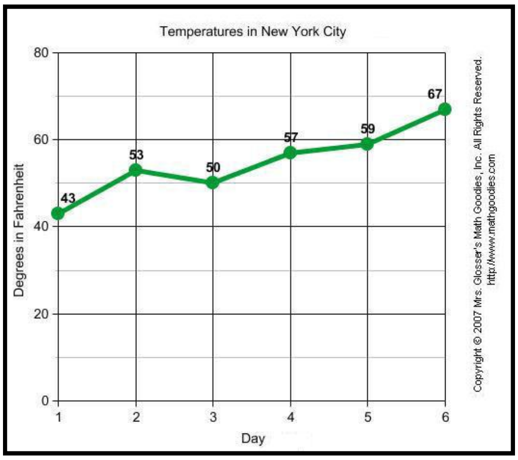
Introduction

Welcome to the second Topic of your Grade 9 Mathematics Unit 4.

Graphs are found in many areas of everyday life. Often, in different media such as television, newspaper, and even on the internet, graphs are shown to enhance the facts that are being presented.

The graphs below are examples of such graphs.





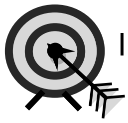
In lesson 5, you were able to read travel graphs. However, drawing such graph is necessary in other areas too.

It is important for you to learn how to draw graphs in order to use them in problems solving. Finding the rules of a given straight line will also be given emphasis.

These are the things that you will learn in this course.

Lesson 6: Cartesian Planes

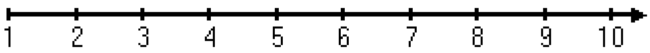
You learnt something about coordinate planes in your Grade 8 Mathematics.



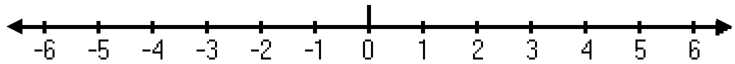
In this lesson you will:

- define Cartesian Plane and identify its parts
- locate points on a Cartesian Plane
- plot points on a Cartesian Plane

You learned about the basic (counting) number line back in your primary school:

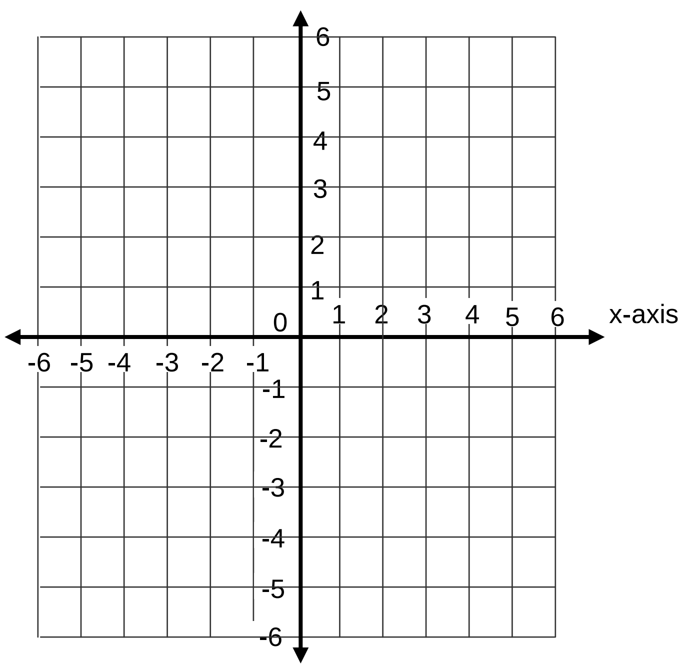


Later on, you were introduced to zero and <u>negatives</u>, which completed the number line:



In Grade 7 and 8, you learnt the Rectangular Coordinate System commonly known as the **Cartesian Coordinate Plane**, named after Rene Descartes who popularized its use in Analytic Geometry.

Descartes' breakthrough was in taking a second number line, standing it up on its end, and crossing the first number line at zero.

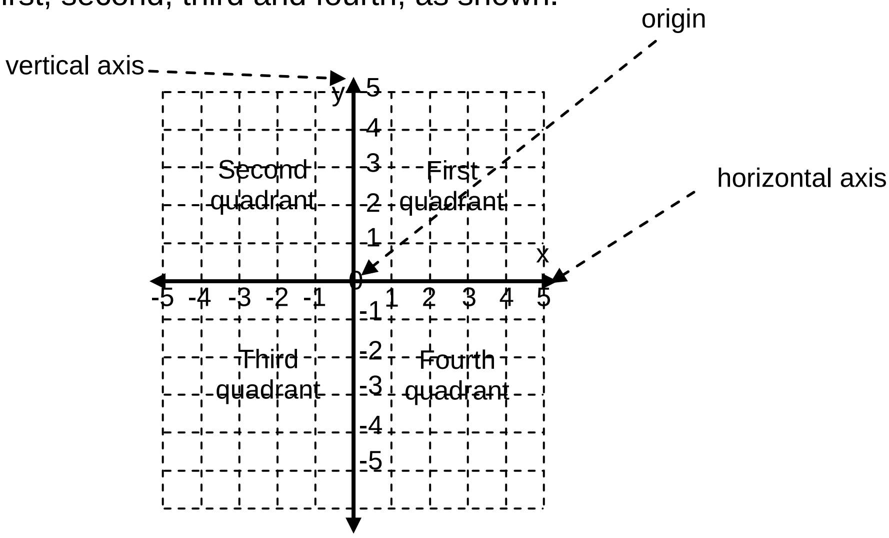


When two perpendicular number lines intersect, a Cartesian plane is formed. The point where the axes intersect is called the **origin.**

The axis going from left to right on a page is called the **horizontal axis** and the axis from bottom to top of a page is referred to as the **vertical axis**.

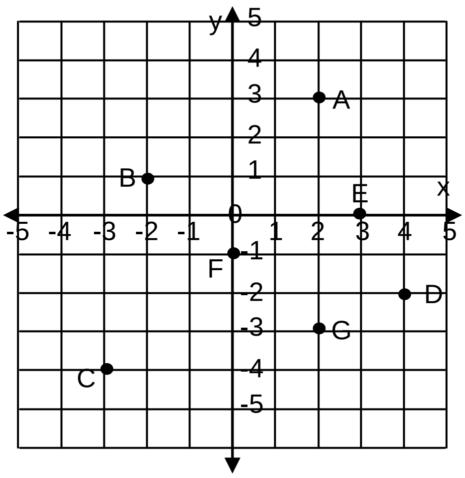
It is important that the axes are scaled accurately. On each of the axes the length representing a unit must be the same all along the axis. However, the length representing a unit does not have to be the same on the vertical as it is on the horizontal axis.

The axes divide the Cartesian plane into four **quadrants.** The quadrants are commonly labelled first, second, third and fourth, as shown.



Points on the Cartesian plane are called **'ordered pairs'**.

When identifying a point on a Cartesian plane, the **coordinates** relating to the position on the axes are quoted in brackets with the coordinate that relates to the horizontal axis given first.



The **coordinates** of a point is in the ordered pair form of (x , y).

Thus A has the x-coordinate of 2 and a y-coordinate of 3. Thus the coordinates of point A are (2 , 3)

B has the x-coordinate of –2 and a y-coordinate of 1. Thus the coordinates of point B (-2 ,1)

The coordinates of the other points C, D, E, and F are (-3 , -4), (4, -2) , (3 , 0) and (0, -1), respectively.

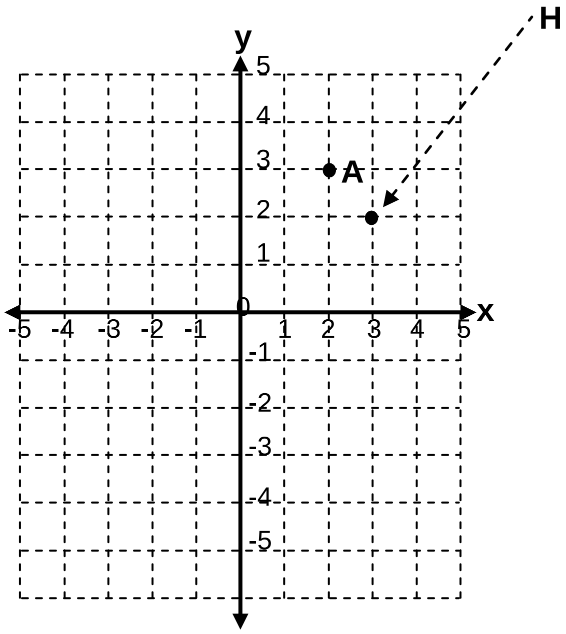
What are the coordinates of point G?

Let us now learn to plot points on the Cartesian plane.

Let us locate the point H whose coordinates are (3, 2). Would it be the same point as A on the number plane above?

Definitely not.

Point H is the point as shown.



Try to plot the following points.

Point I with the coordinates of (4, - 5)

Point J with the coordinates of (-4, 5)

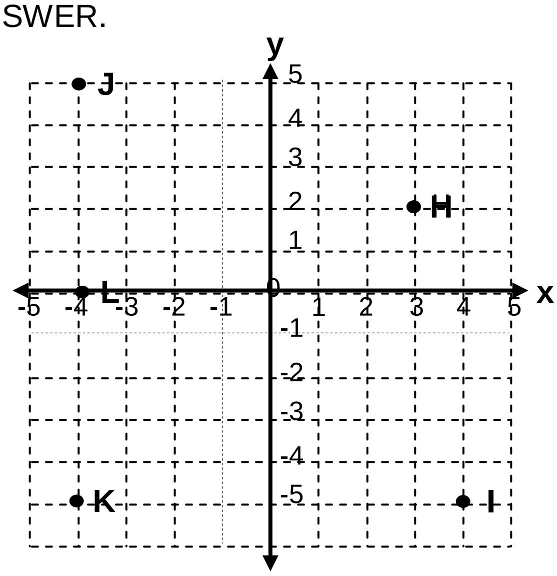
Point K with the coordinates of (-4, -5)

And L with the coordinates of (-4, 0)

Examine the points you plotted. Place the letter beside the point.

Try to examine it again.

If you are sure of your ANSWERS, then try to look at the next page to check your ANSWER.



How many points have you located correctly?

Encircle how many?

0 1 2 3 4

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Complete the following statements.

   (a) The x– axis and the y-axis intersect at a point called ____________

   (b) The horizontal axis is commonly called the _______________

   (c) The vertical axis is commonly called the _________________

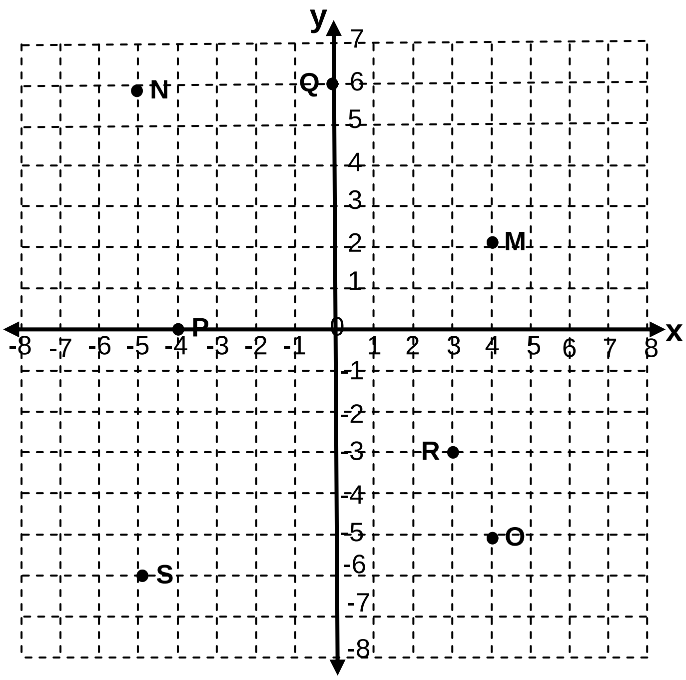
   (d) The ordered arrangement of numbers in the form (x, y) is called

   _____________ _______________.

   (e) (-3, 2) can be found in _____________ quadrant.

   (f) The coordinates of the point of origin are _________.

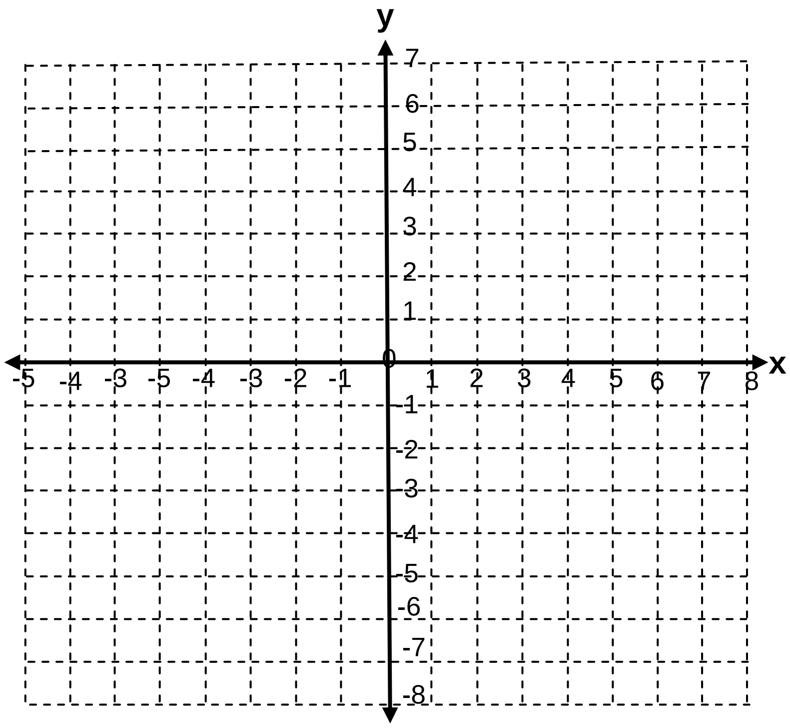
2. Write the coordinates of the named points on the space provided.



Point	Coordinates
M	
N	
O	
P	
Q	
R	
S	

3. Plot the named points on the Cartesian Plane on the next page. Indicate the dot and write the letter corresponding to the coordinates.

Point	Coordinates
T	(-3 , 5)
U	(0 , -8)
V	(4, 6)
W	(-4, 2)
X	(-2 , -4)
Y	(7 , -4)
Z	(5 , -3)

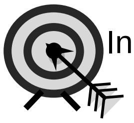


4. From the plotted points above, name the parallelogram formed.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 7: The Gradient

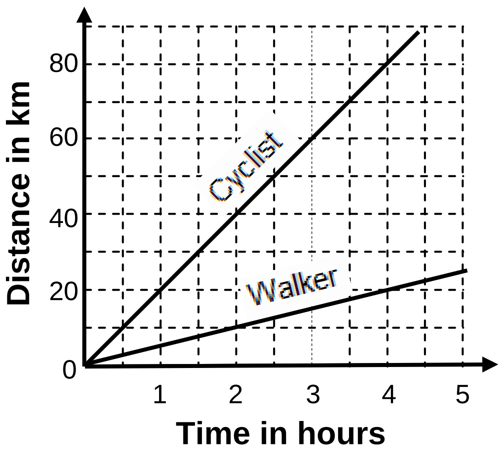
You learnt the meaning of a Cartesian plane and its features in the last lesson. You also learnt to locate and plot points on the coordinate plane.



In this lesson you will:

- define gradient
- find the gradient of a line

Let us consider travel graphs below showing the journeys of a cyclist and a walker.



(i) Which graph is steeper than the other?

(ii) Which way of travelling is faster than the other?

(iii) Name the graph that is less steep.

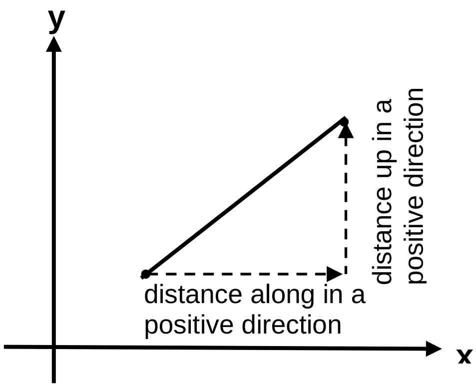
Notice that the faster the transport, the steeper the slope of a graph. In the graph above, the graph of a cyclist was steeper than the graph of a walker.

The slope of the graph is called the **gradient.**

The Gradient (also called <u>Slope</u>) of a straight line shows how steep a straight line is.

To find the gradient, we consider two points on the graph.

Then we divide the distance **up** the graph by the distance **along** the graph.



Thus, Gradient = $\frac{\text{Distance up}}{\text{Distance along}}$

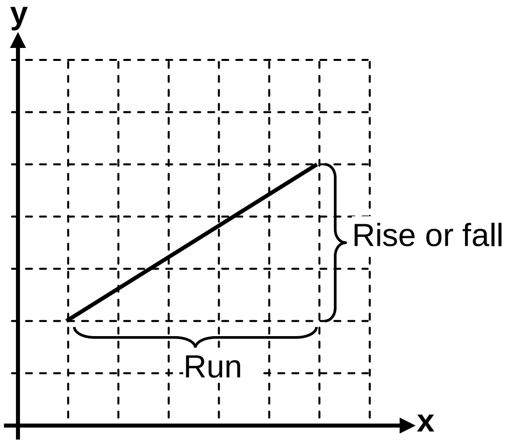
Notice that the **distance up** is the **change in y** coordinates and the **distance along** is the **change in x** coordinates.

Sometimes the horizontal change is called "**run**", and the vertical change is called "**rise**" or "**fall**":

Thus we also say that;

Gradient = $\frac{\text{Change in x}}{\text{Change in y}}$

Or Gradient = $\frac{\text{Rise}}{\text{Run}}$

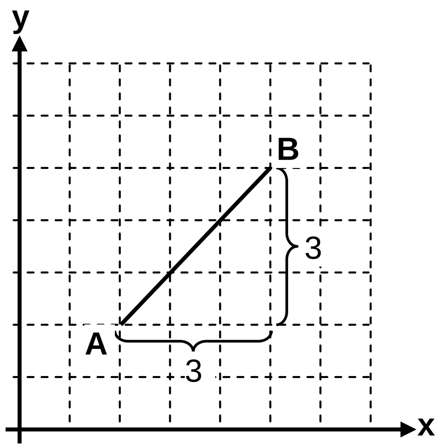


They are just different words, none of the calculations change.

Example 1

The gradient of line **AB** is $\frac{3}{3}$ = 1.

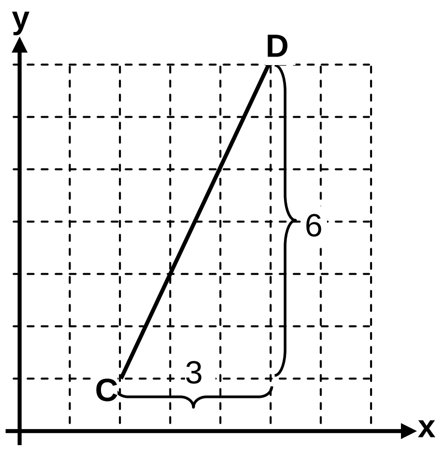
So, the gradient is equal to1.



Example 2

The gradient of line **CD** is $\frac{6}{3}$ = 2.

So, the gradient is equal to 2.

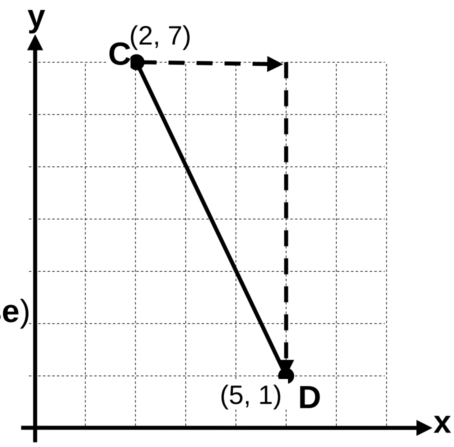


The line is steeper so it has a larger gradient or slope.

The gradient of a line has a value that can be given:

(a) as a whole number
(b) as a fraction (in its simplest form)
(c) as a decimal.

Example 3



The line **CD** leans or slopes downwards from left to right.

To find the gradient of line CD, use the points **C** and **D**.

- The change along the y-axis is 6 (**decrease**)
- The change in the x-axis is 3 (**increase**)

The gradient of line **CD** $= \frac{\text{Change in y}}{\text{Change in x}}$

$= \frac{-6}{3}$

$= -2$

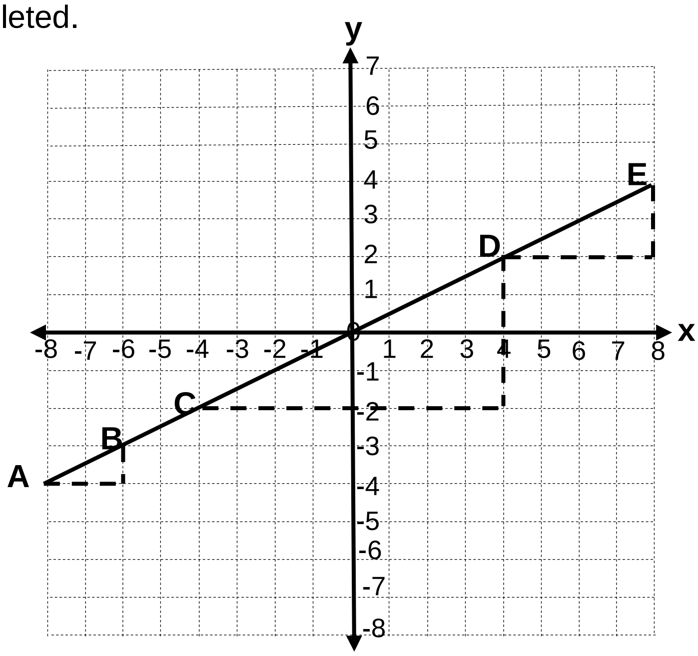
- When an **increase** along the x-axis gives a **decrease** along the y-axis we say the gradient is **negative.**

Example 4

Consider the line drawn on the Cartesian plane on the right.

Base on the graph, the table is completed.

Two Points	Distance Up (Rise)	Distance Along (Run)	$\frac{\text{Rise}}{\text{Run}}$
A & B	1	2	$\frac{1}{2}$
C & D	4	8	$\frac{1}{2}$
D & E	2	4	$\frac{1}{2}$
A & E	8	16	$\frac{1}{2}$



What can you say about the ratio $\frac{\text{Rise}}{\text{Run}}$ for any two points on the line?

What can you say about the gradient at each place on the line?

For a straight line graph, the gradient is the same for any part of the line wherever two points are considered.

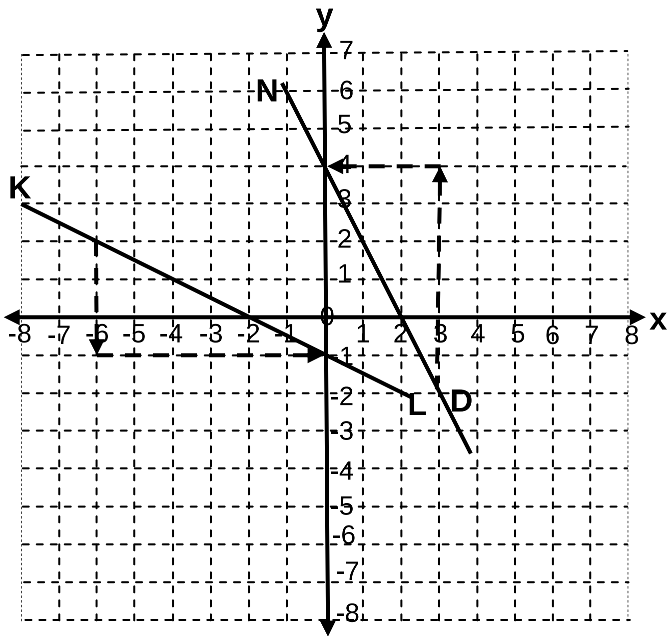
Example 5

Let us consider line **KL.**

Gradient = $\frac{-3}{6}$

Note that the run is negative, since it is opposite to the direction of distance up.

Gradient of line **KL** = $-\frac{1}{2}$



If you look at the line in example 4 and line **KL**, you will notice that the two graphs have the same steepness. A line leaning to the right has a **positive** gradient, while line that leans to the left has a **negative** gradient. The **sign** of the gradient only indicates where the graph is leaning or sloping.

What could be the sign of line **ND**? Why?

Let us check by computing the gradient:

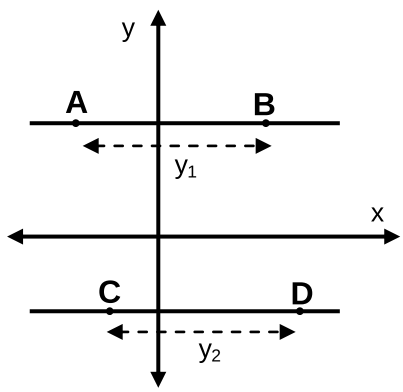
Gradient of line ND = $\frac{6}{-3} = -2$.

In summary, the gradient of a straight line can be described in this way:

- A line which **slopes upwards from left to right** has a **positive** gradient.
- A line which **slopes downwards from left to right** has a **negative** gradient.

Zero Gradient and Horizontal lines

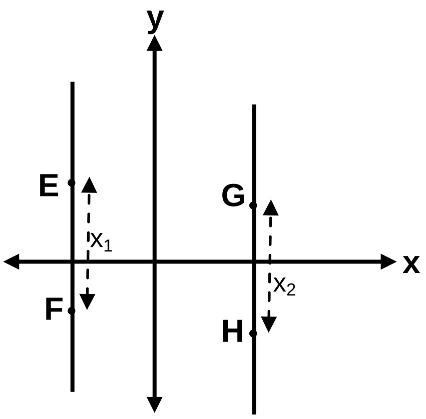
Considering the horizontal lines **AB** and **CD,** the gradient can be obtained by using the formulas



Gradient of **AB** = $\frac{\text{rise}}{\text{run}} = \frac{0}{y_1} = 0$

Gradient of **CD** = $\frac{\text{rise}}{\text{run}} = \frac{0}{y_2} = 0$

Undefined Gradient and Vertical Lines



Consider the vertical lines EF and GH. The gradient of each lines are:

$$\text{Gradient of EF} = \frac{\text{rise}}{\text{run}} = \frac{x_1}{0} = \text{undefined}$$

$$\text{Gradient of GH} = \frac{\text{rise}}{\text{run}} = \frac{x_2}{0} = \text{undefined}$$

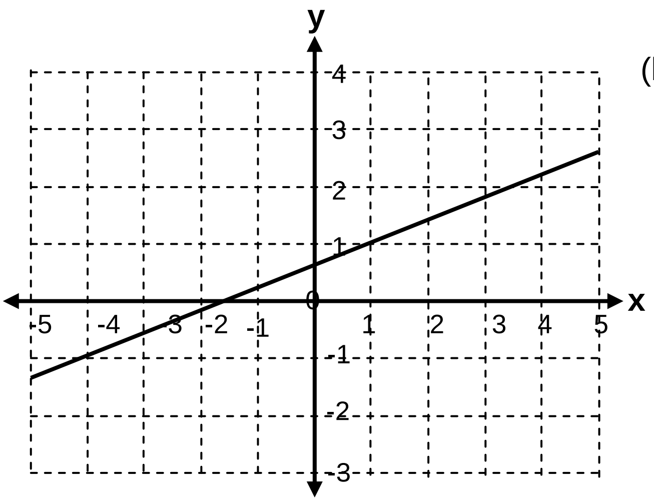
In summary**, the gradient of a horizontal line is 0** while **the gradient of a vertical line is undefined.**

NOW DO PRACTICE EXERCISE 7

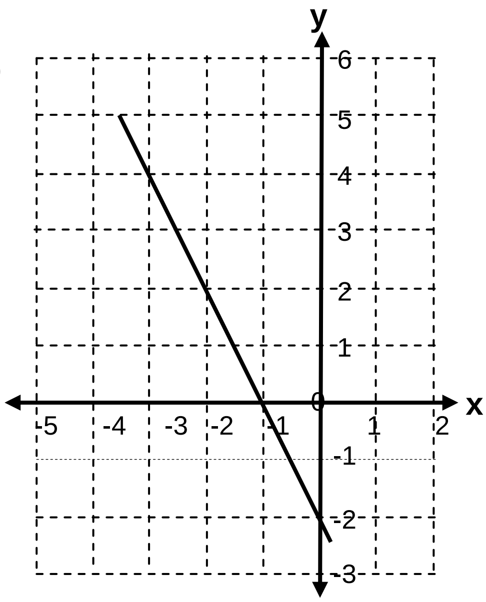
Practice Exercise 7

1. Find the gradient of each of the following lines.

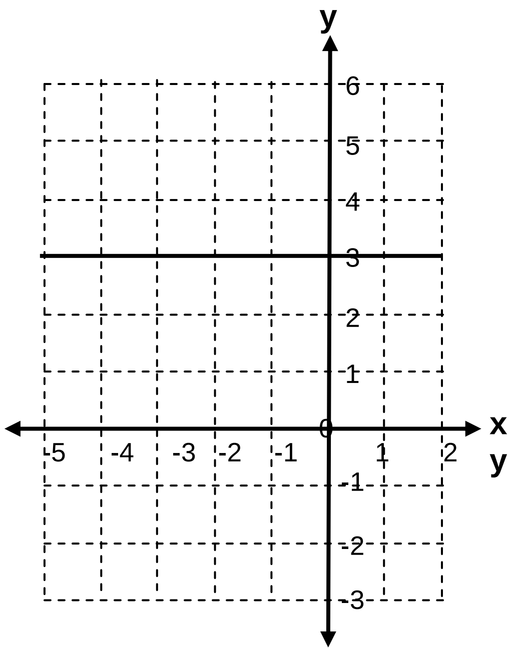
(a)



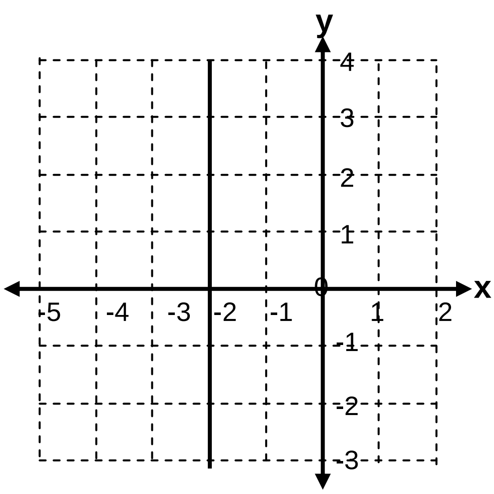
(b)



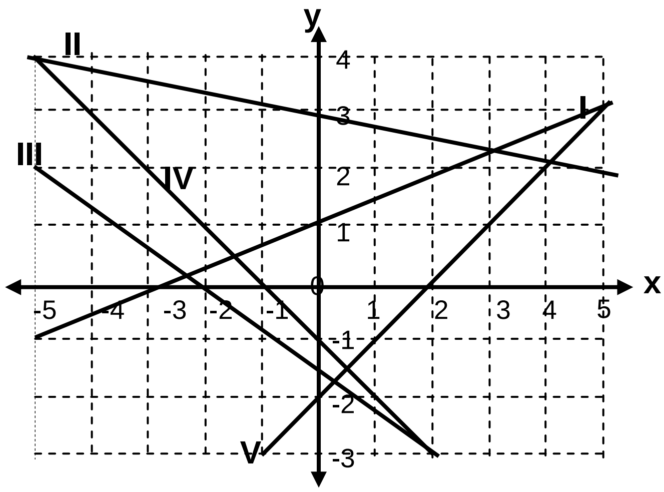
(c)



(d)



2. Shown below are five lines **I, II, III, IV** and **V**.



(a) Which of the lines from **I** to **V**, have negative gradient?

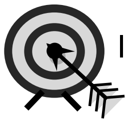
(b) Which of the lines is the steepest? What is its gradient?

(c) Which of the lines is the least steep? What is its gradient?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 8: Drawing Straight Line Graphs

You learnt the meaning of gradient in the last lesson. You also learnt to calculate the gradient of a straight line.



In this lesson you will:

- use table of values to draw a straight line graph.

A graph showing the linear relationship between two variables is a straight line and is called a **linear graph.**

To obtain a linear graph, we need to be given information to be able to draw it. One way is to use a linear equation and work out coordinate points to plot.

Let us consider the graph of y = 2x – 3.

First we would complete a table of corresponding x and y values which follow the rule

$$y = 2x - 3.$$

You need to choose any value of x as shown in the table of values.

x	-2	-1	0	1	2	3	4
y							

After choosing the value of x, substitute the x value and solve for y:

If x = - 2, y = 2 (**- 2**) - 3, then y = - 7

If x = - 1, y = 2 (**-1**) - 3, then y = - 5

If x = 0, y = 2 (**0**) - 3, then y = - 3

If x = 1, y = 2 (**1**) - 3, then y = - 1

If x = 2, y = 2 (**2**) - 3, then y = 1

If x = 3, y = 2 (**3**) - 3, then y = 3

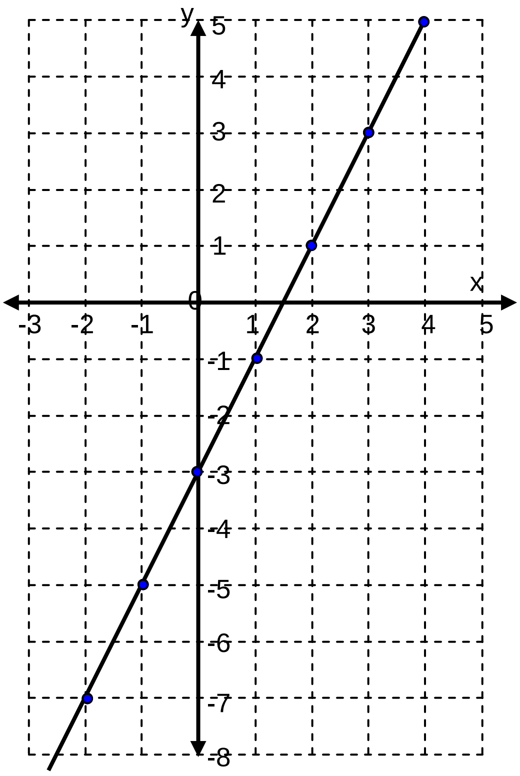
If x = 4, y = 2 (**4**) - 3, then y = 5

Completing the table, you will have

x	-2	-1	0	1	2	3	4
y	- 7	- 5	- 3	- 1	1	3	5

You can still continue adding values of x and compute for y if you want.

Then plot these points into a number plane, joining them with a straight line as shown on the next page.



Notice the 7 points plotted on the Cartesian plane and the straight line that joined them.

What is the value of x when the graph intersects the x-axis?

Yes, it's 1.5. This is called the **x-intercept**

What is the value of y when the straight line intersects the y-axis?

The line y = 2x –3 intersect the y-axis at –3. –3 is called the **y-intercept.**

Let us have another example.

Let us graph the straight line y = 1 – 2x.

Choose the values of x as shown on the table of values.

x	-4	-3	-2	-1	0	1	2
y							

We solve for the corresponding values of y.

If x = - 4, y = 1 - 2 (**- 4**), then y = 9

If x = - 3, y = 1 - 2 (**- 3**), then y = 7

If x = - 2, y = 1 - 2 (**- 2**), then y = 5

If x = - 1, y = 1 - 2 (**- 1**), then y = 3

If x = 0, y = 1 - 2 (**0**), then y = 1

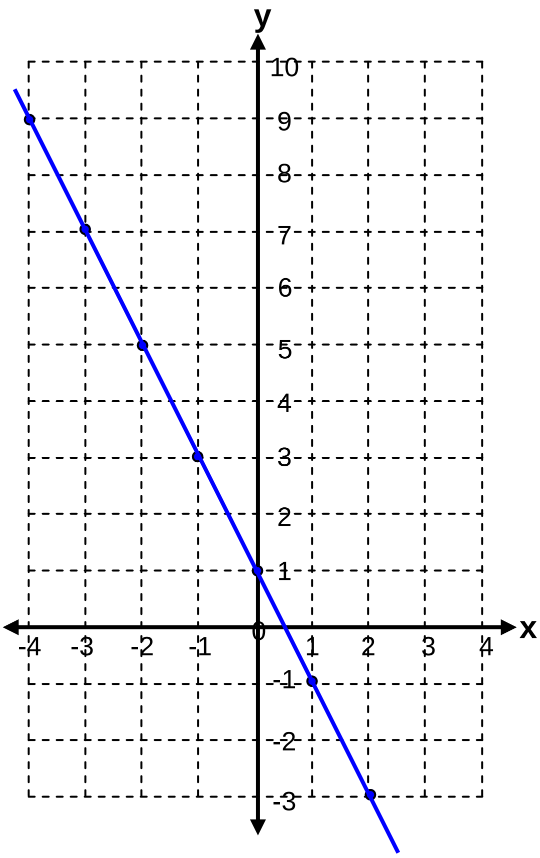
If x = 1, y = 1 - 2 (**1**), then y = - 1

If x = 2, y = 1 - 2 (**2**), then y = - 3

Completing the table of values, you will have

x	-4	-3	-2	-1	0	1	2
y	9	7	5	3	1	-1	-3

The graph of y = 1 - 2x is the one below.



In which direction does the line y = 1 - 2x leans? What could be the sign of the gradient of this line?

What is the x-intercept?

What is the y-intercept?

There are special lines that have definite form of an equation.

One of these has the equation **x = a**, where **a** is any number.

Example

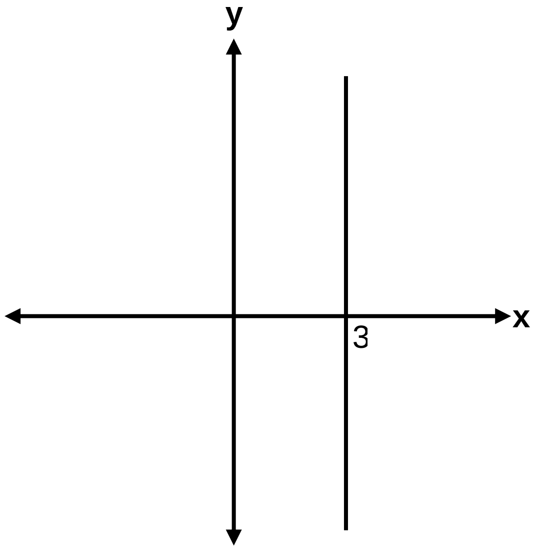
Consider the equation x = 3.

This would mean that 3 is always the value of x no matter what is the value of y.

Completing the table of values for x = 3, we will have

x	3	3	3	3	3	3	3
y	-3	-2	-1	0	1	2	3

The graph of x = 3 is shown below.



What kind of line is it?

What is the gradient of such kind of line?

The line x = 3 is parallel to the x-axis or the y - axis?

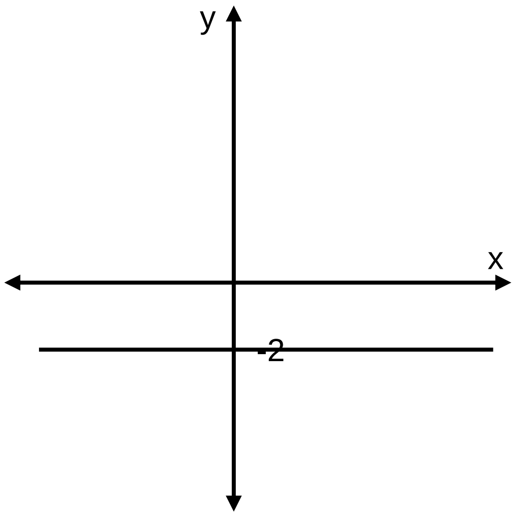
Another type of special line is **y = b**, where **b** is a number.

Consider the equation y = -2. This would mean that –2 is always the value of **y** and the value of x will be any number.

Completing the table of values for y = -2, we will have

x	-4	-3	-2	-1	0	1	2
y	- 2	- 2	- 2	- 2	- 2	- 2	- 2

The graph of y = -2 is shown below



What kind of line is it?

What is the gradient of such kind of line?

The line y = -2 is parallel to the x-axis or the y - axis?

To graph a straight line using table of values, you should

1. **Choose values of x and place them on table of values.**
2. **Using the equation, find the corresponding values of y for each value of x**
3. **Plot all the points on a Cartesian plane and connect all the points by a straight line.**

The graph of **x = a**, where **a** is a number, is a **vertical line** that cut the x-axis at the value of **a**.

The graph of **y = b**, where **b** is a number is a **horizontal line** that cuts the y - axis at the value of **b**.

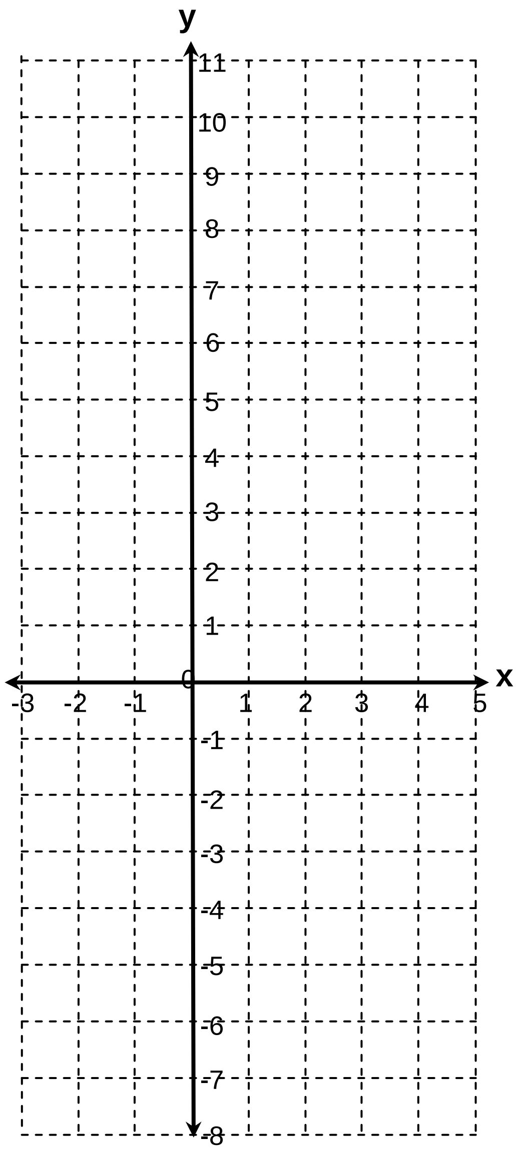
NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

1. (a) Complete the table of values of y = 3x + 2

x	-3	-2	-1	0	1	2	3
y							

(b) Plot all the points and connect by a straight line



(c) Find the gradient of the line.

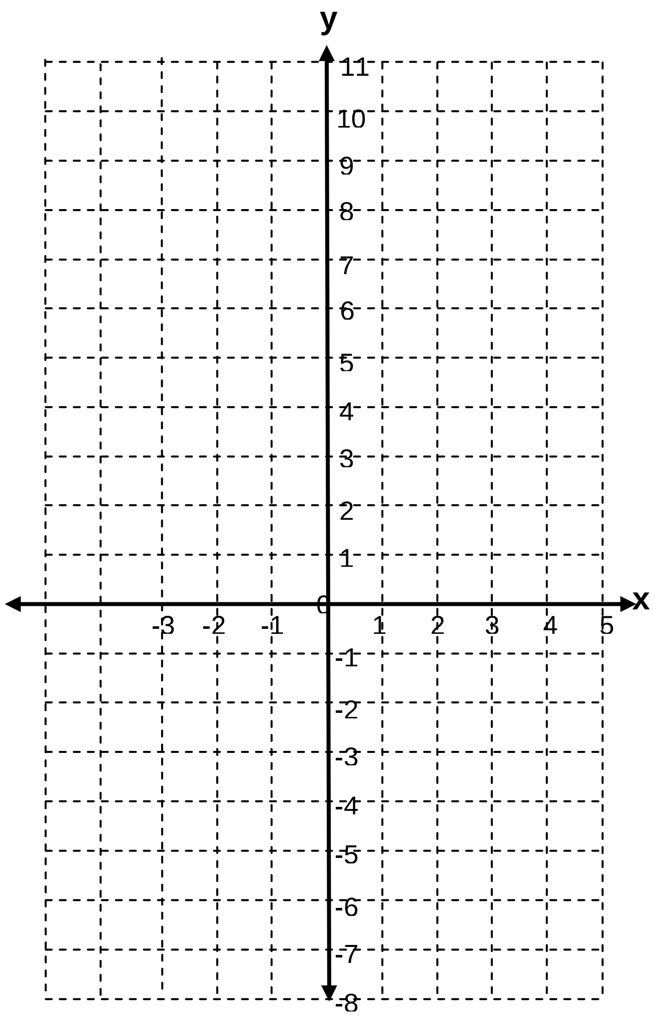
(d) At what value of x does the graph intersect the x-axis?

(e) At what value of y does the graph intersect the y-axis?

2. (a) Complete the table of values of y = -2x + 5

x	-3	-2	-1	0	1	2	3
y							

(b) Plot all the points and connect by a straight line.



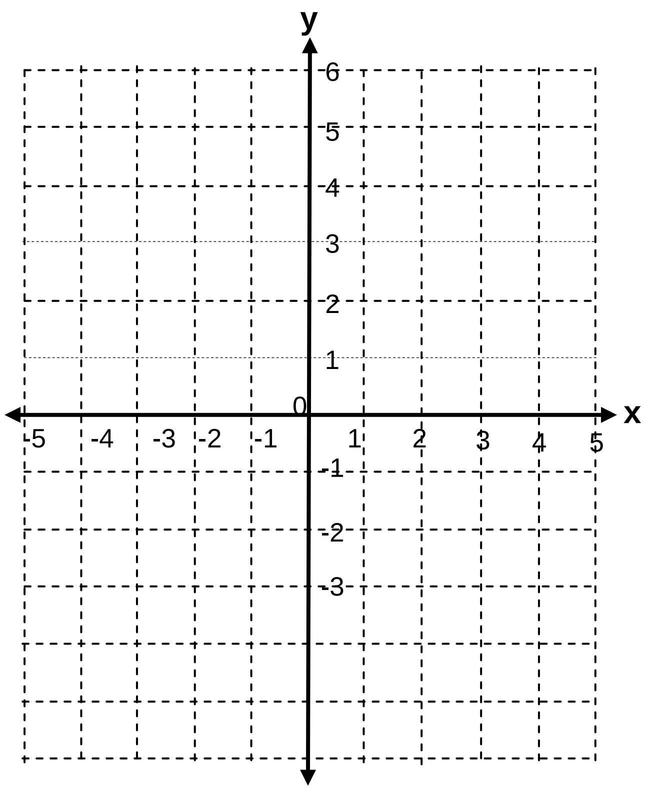
(c) Find the gradient of the line.

(d) At what value of x does the graph intersect the x- axis?

(e) At what value of y does the graph intersect the y- axis?

3. Write four points that lie on the following lines and graph them on the grid.

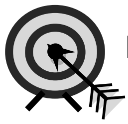
(a) y = 4 (b) x = - 1



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Equation of a Straight Line

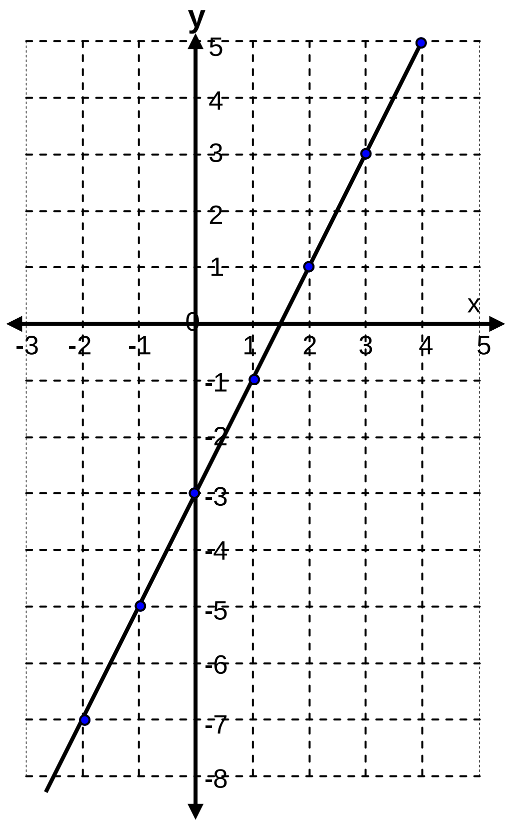
In the last lesson, you learnt to graph a straight line using a table of values.



In this lesson you will:

- identify the equation of a straight line
- find the equation of a straight line.

Let us recall the example that you have had in drawing straight line. The graph is shown on the right.



Let us compute the gradient of the line, considering the points (3, 3) and (1, -1).

Gradient = $\frac{\text{rise}}{\text{run}} = \frac{3-(-1)}{3-1} = \frac{4}{2} = 2$

The y-intercept as you would recall is the y-value of the intersection of the graph and the y-axis. The y-intercept is –3.

Recall too that the equation of the line is y = 2x—3.

Can you see the gradient of 2 and the y-intercept of –3 in the equation y = 2x – 3?

The gradient is the same as the number in front of x and the y-intercept is the same as the constant.

The equation of the line is in the form *y = mx + c*, where *m* and *c* are numbers. The number in front of x which is m is the gradient of the line, while the value of c is the y-intercept. This equation is called the *gradient - intercept form* of a straight line.

Another equation equivalent to y = 2x –3 is 2x - y - 3 = 0. This is in the **general form** of a straight line.

The general form of an equation of the line is Ax + By + C = 0. Where A, B and C are real numbers, A and B are not both 0.

You can change the gradient-intercept form of a straight line into the general form by transposing the expression on one side so that the other side would be zero.

Thus, from y = 2x –3, you can change it 2x –y – 3 = 0

Example 1

Write the equation $-\frac{2}{3}x + \frac{1}{2}y = 1$ in the general form.

Solution: $-\frac{2}{3}x + \frac{1}{2}y = 1$

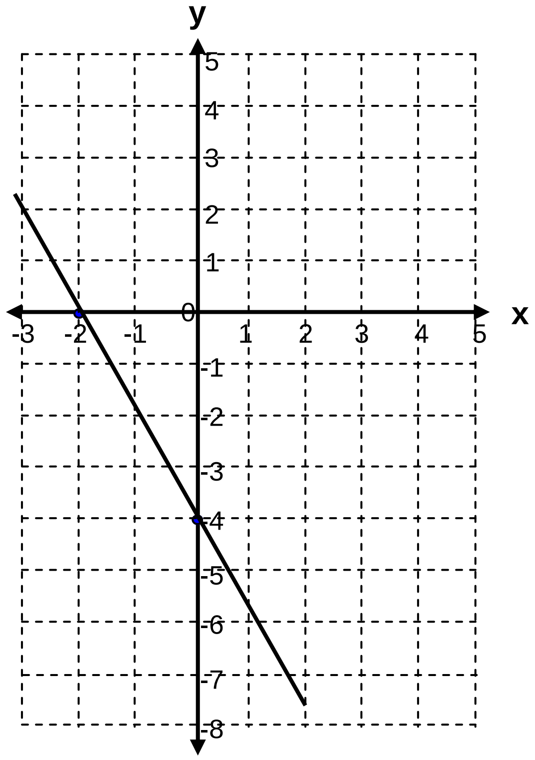
$$\frac{6}{1}\left(-\frac{2}{3}x + \frac{1}{2}y\right) = 1\left(\frac{6}{1}\right)$$

$$-4x + 3y = 6$$

$$\mathbf{4x - 3y + 6 = 0}$$

Example 2

Find the equation of the line.



(i) First, you need to find the gradient of the line. Considering the points given,

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = -\frac{4}{2} = -2$$

Note that a negative sign was place in front because the line leans to the left.

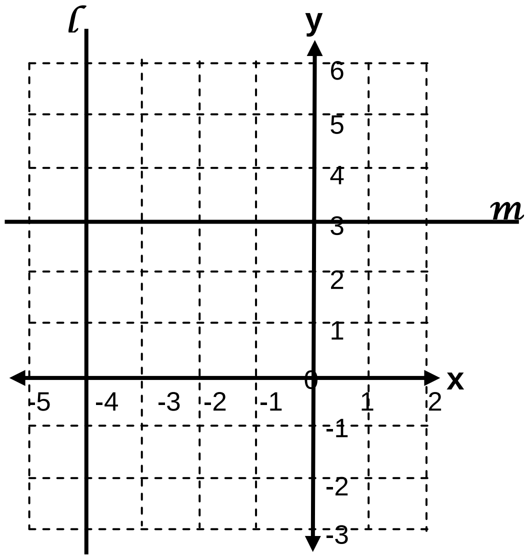
(ii) The y- intercept is –4.

(iii) Thus the equation of the line is y = - 2x – 4.

(v) Writing it in general form, 2x + y + 4 = 0.

Special Lines x = a and y = b

Let us consider lines that are parallel to the x-axis and y– axis.



(i) Lines that are parallel to the y-axis have the equation

x = a, where **a** is a number
a is also the x-intercept

(ii) Lines that are parallel to the x-axis have the equation

y = b, where **b** is a number
b is also the y- intercept.

Let us find the equation of line ℓ and m.

Line ℓ is parallel to the y– axis, and the x-intercept is –4.

Then the equation of line l is **x = -4.**

Line m is parallel to the x-axis, and the y-intercept is 3.

Thus, the equation of the line is **y = 3.**

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

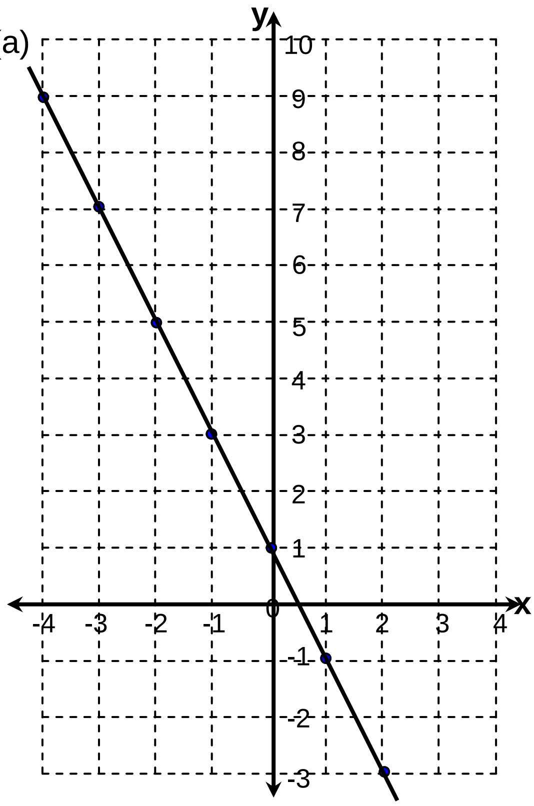
1. Complete the following table.

Equation of the Line	Gradient	y - intercept
i) y = –3x + 5		
ii) y = 4 – 2x	-2	4
iii)	$\frac{1}{3}$	–1
iv) y = –x		
v)	$-\frac{2}{3}$	5

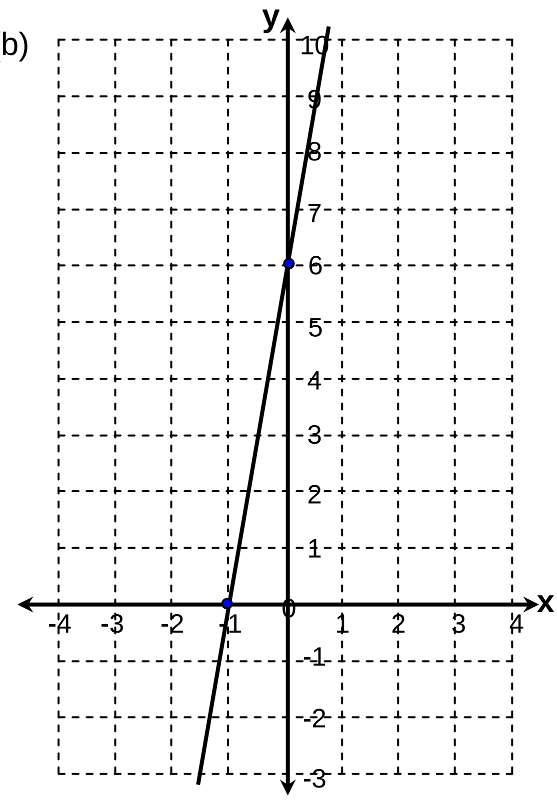
2. Write the equations in Question 1 (i-v) in general form.

3. Find the equation of each of the following line. Write the equation in gradient-intercept form and general form.

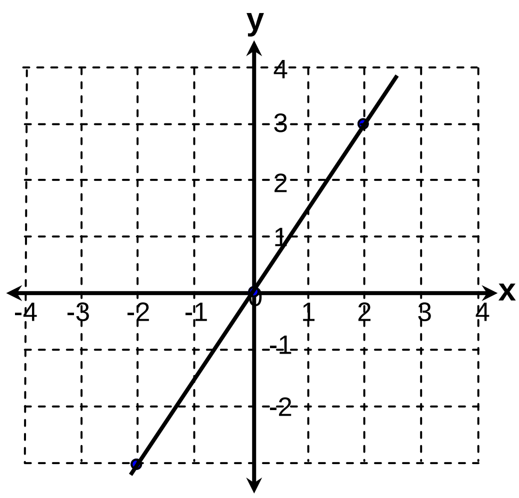
(a)



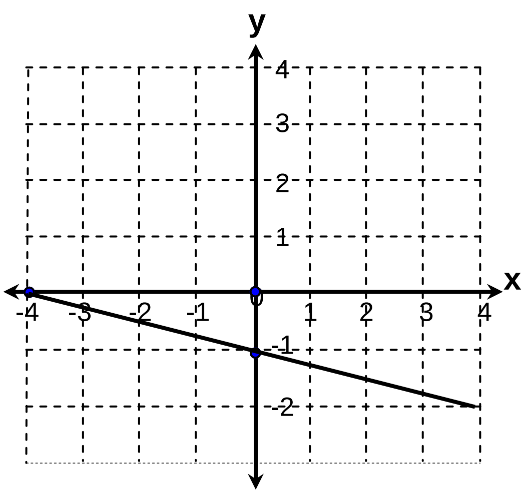
(b)



(c)



(d)



4. Match the line with its corresponding equation.
Write the letter of the correct name of the line

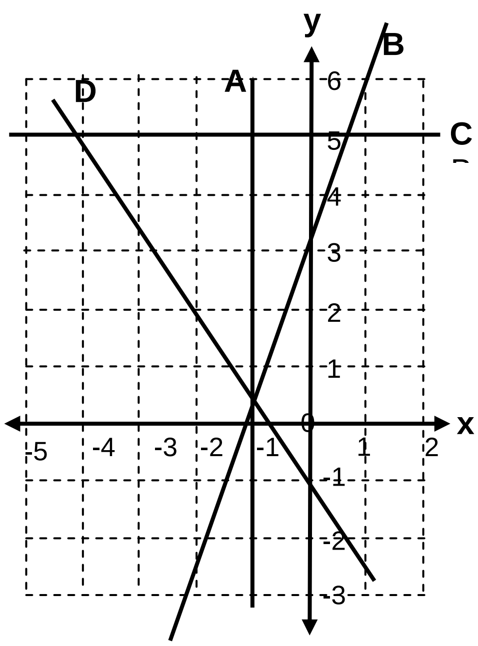
_____(a) $y = 5$

_____(b) $y = -\frac{3}{2}x - 1$

_____(c) $x = 0$

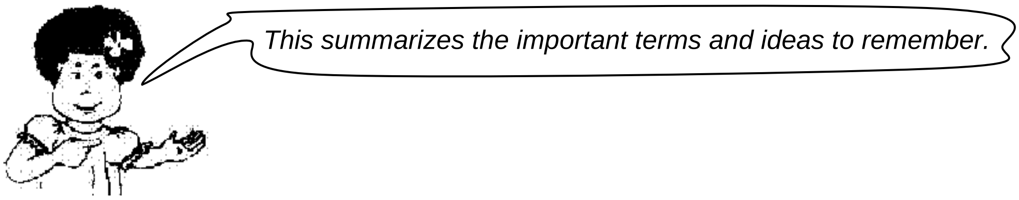
_____(d) $x + 1 = 0$

_____(e) $y = 3x + 3$



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

TOPIC 2: SUMMARY



- The **Cartesian plane** is a plane with rectangular coordinate system that associates each point in the plane with a pair of numbers

- **Gradient** of the line refers to a number associated with the steepness of the line. Another term used for gradient is slope

- For a straight line graph, the **gradien**t is the same wherever two points are considered. The gradient of the line can be obtained, by first getting two points that are exactly on the grid, then divide rise by run. Leave the ANSWERS without a sign, when the line leans to the right. Affix a negative sign if the line leans to the left.

- The gradient of **horizontal lines** is 0 while the gradient of **vertical line** is undefined.

- To **graph** a straight line using a table of values, you should:
  1. choose values of x and place them on table of values.
  2. using the equation, find the corresponding values of y for each value of x
  3. plot all the points on a Cartesian plane and connect all the points by straight line.
- The **graph of x = a**, where a is number is a vertical line that cut the x-axis at the value of a, while **the graph of y = b** , where b is a number is a horizontal line which cuts the y - axis at the value of b.

- The **equation of the line** is in the form of ***y = mx + c*,** where ***m*** and ***c*** are numbers. The number in front of **x** which is m is the gradient of the line, while the value of **c** is the y-intercept. This equation is called the **gradient-intercept form** of a straight line

- The **general form** of an equation of the line is **Ax + By + C = 0**. You can change a gradient-intercept form of a straight line into a general form by transposing the expression on one side so that the other side is equal to zero.

- **Lines that are parallel to the y - axis** have the equation **x = a**, where **a** is a number and **a** is the x-intercept. **Lines that are parallel to the x– axis** have the equation **y= b**, where b is a number and b is the y-intercept.

REVISE LESSONS 6-9 THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 4.

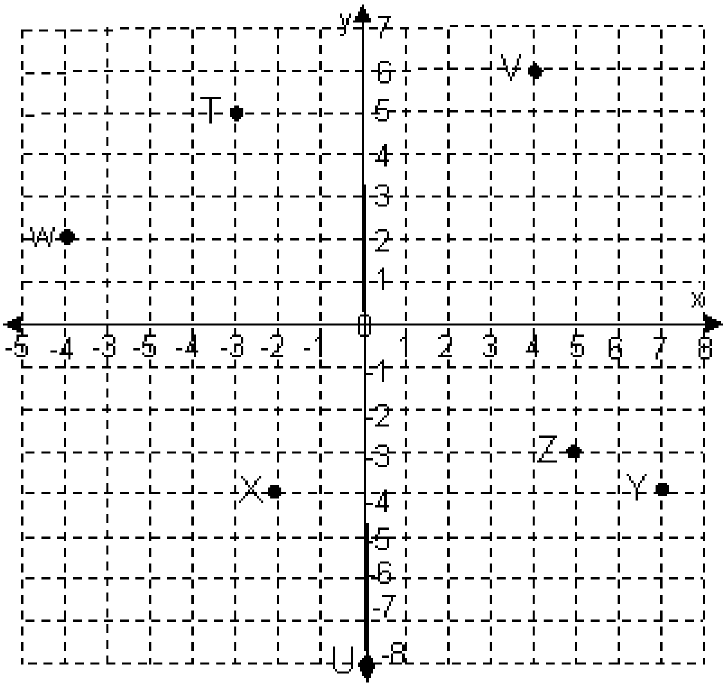
ANSWERS TO PRACTICE EXERCISES 6-9

Practice Exercise 6

1. (a) origin (b) x-axis (c) y-axis
   (d) Ordered pairs (e) second (f) (0, 0)

2. M (4,2) N (-5, 6) O (4, - 5)
   P (-4, 0) Q (0 , 6) R (3, -3)
   S (-5 , -6)

3.



4. Parallelogram TVZX

Practice Exercise 7

1. (a) $\frac{2}{5}$ (b) –2 (c) 0 (d) undefined

2 (a) II, III, IV

(b) IV and V with gradient –1 and 1, respectively

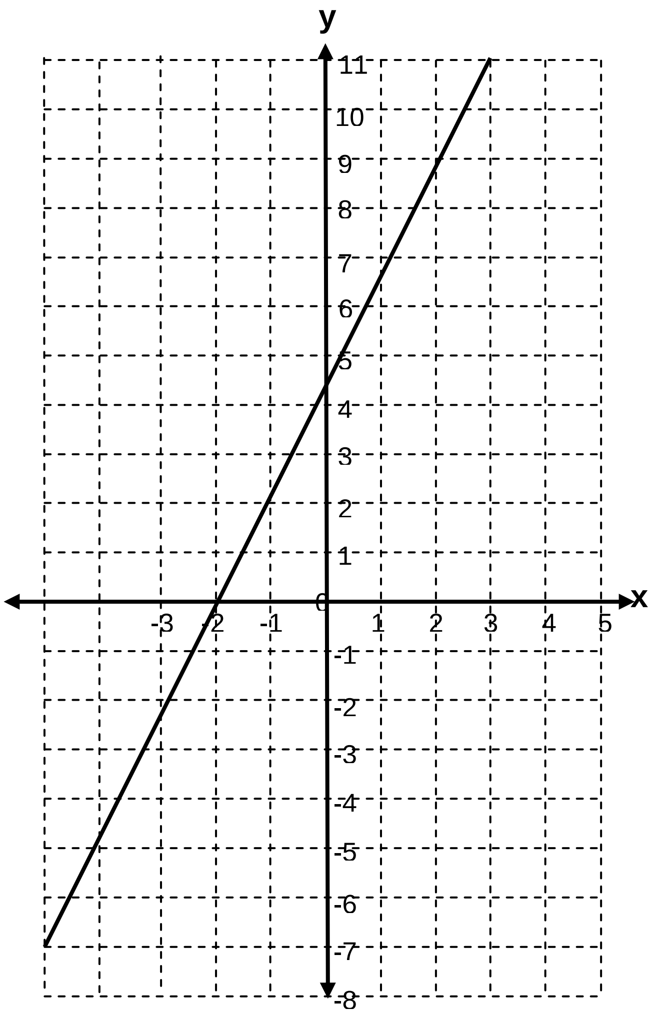
(c) II with gradient of 2.5

Practice Exercise 8

1. (a)

x	-3	-2	-1	0	1	2	3
y	-7	-4	-1	2	5	8	11

(b)



(c) 3

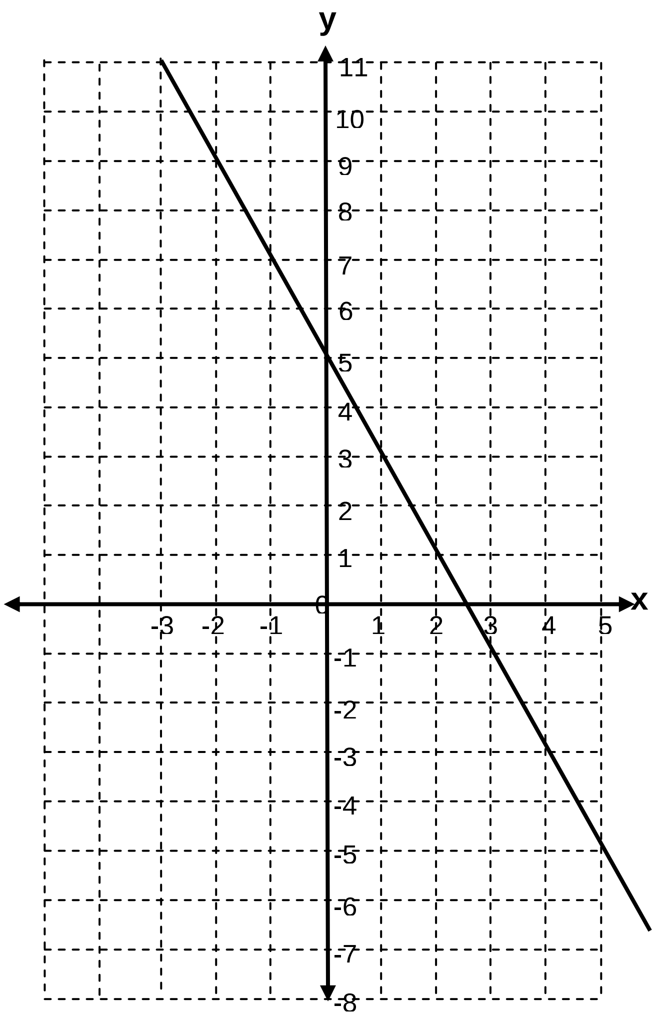
(d) $-\frac{2}{3}$ or 0.67

(e) 2

2.

(a)

x	-3	-2	-1	0	1	2	3
y	11	9	7	5	3	1	-1



(c) -2

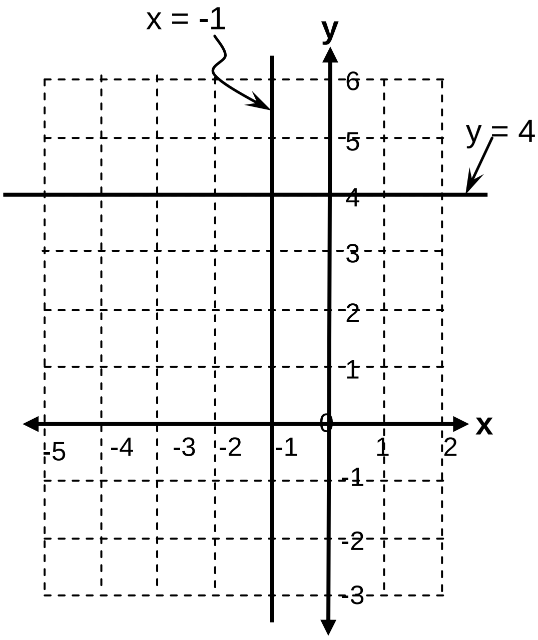
(d) 2.5

(e) 5

3. (a) (-2, 4), (-1, 4), (0, 4), (1, 4), (2, 4), (3, 4),…

(b) (-1, -2), (-1, -1), (-1, 0), (-1, 1), (-1, 2),…

Graphs



Practice Exercise 9

1.

Equation of the Line	Gradient	y - intercept
i). $y = -3x + 5$	-3	5
ii). $y = 4 - 2x$	-2	4
iii) $y = \frac{x}{3} - 1$	1/3	-1
iv). $y = -x$	-1	0
v). $y = \frac{2}{3}x + 5$	- 2/3	5

2. (i) $3x + y - 5 = 0$
   (ii) $2x + y - 4 = 0$
   (iii) $x - 3y - 3 = 0$
   (iv) $x + y = 0$
   (v) $2x - 3y + 15 = 0$

3. (a) $y = -2x + 1$; $2x + y - 1 = 0$ (b) $y = 6x + 6$; $6x - y + 6 = 0$

   (c) $y = \frac{3}{2}x$; $3x - 2y = 0$ (d) $y = -\frac{1}{4}x - 1$; $x + 4y + 1 = 0$

4. (a) C (b) D (c) y-axis

   (d) A (e) B

END OF TOPIC 2

UNIT 4

TOPIC 3

PROBABILITIES

Lesson 10:	**Basic Concept**
Lesson 11:	**Theoretical Probability**
Lesson 12	**Probability of Complementary Events**
Lesson 13:	**Adding Probabilities**
Lesson 14:	**Multiplying Probabilities**
Lesson 15:	**Union and Intersection of Two Events**
Lesson 16:	**Mixed Probability Problems**

TOPIC 3: PROBABILITY

Introduction

In many ordinary everyday situation, you would hear the expressions “50-50”, certain, and impossible, which are terms that are associated with the chance of something happening.

At times, through observation, one can predict things. If for instance in the past you found out that 25 out of 45 students are left-handed, then you could say that there is a big chance that some left-handed students would enrol this year.

These are situations in which you are applying the idea of **probability.**

Probability is the likelihood of something happening. When one mentions 50-50, that is actually an even chance associated with 50% of it happening or 50% of it not happening.

Probability originated in the mid-17th century inspired by the enquiry of gamblers seeking information to help them win at cards and dice.

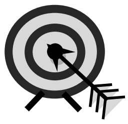
However, today probability has many important commercial and scientific applications. For example, businessmen carry out surveys to help predict the probable size at a given market. Life insurance people have tables of life expectancy to help them calculate their premiums. In graduate studies, researchers use the principle of probability as well as statistics to determine the validity of their studies.

The certainty of something happening implies the probability of 1, while impossibility has a probability of 0. Thus, the probability that the sun will rise tomorrow is 1 because it is certain that the sun will rise. The probability of getting a red 4 club in a deck of card is 0 for this is impossible

In this topic, you will explore the various basic terms of probability and its principles. Probability is associated with the union and intersection of sets.

Lesson 10: Basic Concepts of Probability

Welcome to the first lesson of Topic 3.



In this lesson you will:

- define the basic terms of probability
- find the probability of simple events.

Many events can't be predicted with total certainty. The best thing we can do is say how likely they are to happen, using the idea of **probability**.

What is probability?

- **Probability is the study of chance or the likelihood of an event happening**
- **Probability is defined as the chance or how likely something is to happen.**

Directly or indirectly, probability plays a role in every human activities.

For example, we may say that it will probably rain today because most of the days we have observed were rainy days.

The probability of an event can also be described in words and phrases, such as impossible, highly unlikely, very unlikely, less than even chance, even chance, better than even chance, very likely, highly likely, certain and so on. However, in mathematics, we would require a more accurate way of measuring probability.

For example:

1. **Tossing a Coin**

   When a coin is tossed, there are two possible outcomes:

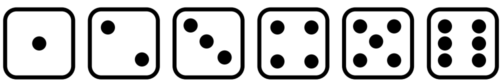
   - heads (H) or
   - tails (T)

   We say that the probability of the coin landing H is $\frac{1}{2}$.

2. **Throwing Dice**

   When a single dice is thrown, there are six possible outcomes.

   The probability of anyone of them is $\frac{1}{6}$.

There are some basic words or terms that have special meanings in Probability.

(i) **Experiment** is the process by which an observation or outcome is obtained.

Examples: tossing a coin, throwing a dice, seeing what colour people choose from a deck of card.

(ii) **Trials** refer to the number of times an experiment is performed.

(iii) **Sample Space** refers to the set **(S)** of all the **possible outcomes** of an experiment.

Example: Choosing a card from a deck. There are 52 cards in a deck (not including the Joker).

So, the **Sample space** is all the 52 cards:

The Sample Space is made up of Sample Points.

The **Sample Point** is just one of the possible outcomes.

Example: Deck of Cards

- The 6 of Diamonds is a sample point
- The Queen of Hearts is a sample point

A "Queen" is not a sample point as there are four (4) Queens that is 4 different sample points.

Same as "6" is not a sample point as there are four (4) 6 that is 4 different sample points.

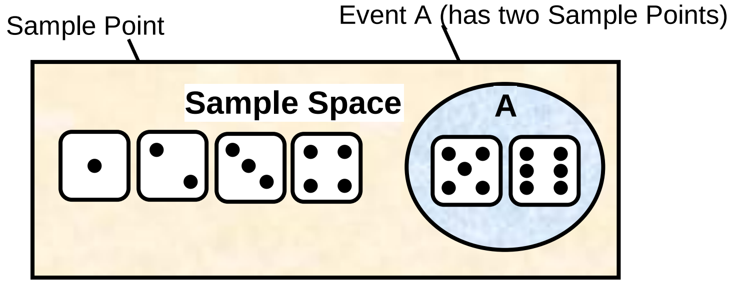
(iv) **Event** a single result of an experiment.

Example of events: Getting a tail when tossing a coin is an event

Getting a "6" when rolling a dice is an event

An event can include one or more possible outcomes.

- Choosing a "Jack" from a deck of cards (any of the 4 jacks) is an event.
- Rolling an "even number" (2,4 or 6) is also an event



The Sample Space includes all possible outcomes.

A Sample Point is just one possible outcome.

And an Event can be one or more of the possible outcomes.

Now let us use those words so you get used to them.

Example 1

Jetro wants to see how many times a “double” would come up when throwing two dice.

Throwing two dice is an **experiment.**
Each time Jetro throws the two dice is a **trial**.

The **Sample Space** is a set of all possible outcomes (**36 Sample Points**):

{1,1} {1,2} {1,3} {1,4} ... {6,3} {6,4} {6,5} {6,6}

The **Event** Jetro is looking for is a “double’, where both dice have the same number.

It is made up of these **6 sample space**.

{1, 1}, {2, 2}, {3, 3}, {4, 4}, {5, 5}, {6, 6},

Example 2

If a fair coin was tossed 20 times,

(i) **Experiment** = tossing of a fair coin

(ii) **Trial** = 20 times

(iii) **Possible Outcomes** = {getting a head, getting a tail}

(iv) **An Event** = Getting a head in tossing a coin

Now let us have examples of finding the probability of simple events.

In general,

$$\textbf{Probability of an event happening} = \frac{\textbf{Number of ways it can happen}}{\textbf{Total number of outcomes}}$$

In symbol, $P(A) = \dfrac{n(E)}{n(S)}$

Where n(E) = number of ways the event can happen
n(S) = total number of possible outcomes
P(A) = probability of an event

Example 1:

What are the chances of rolling a "5" with a dice?

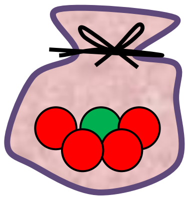
Number of ways it can happen = **1** (there is only one face with a "5" on it).

Total Number of Outcomes = **6** (there are six faces altogether)

So, the **probability** = $\frac{1}{6}$.

Example 2

There are 5 marbles in a bag. 4 are red, and 1 is green. What is the probability that red marble will be picked?

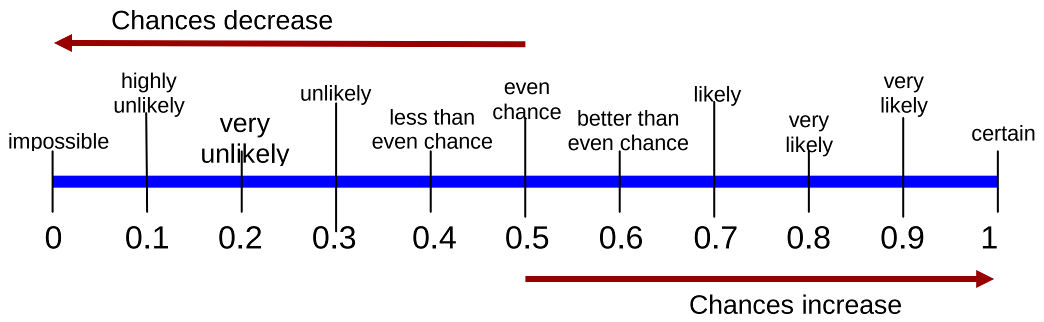


Number of ways it can happen = 4 (there are 4 reds)

Total number of outcomes = 5 (there are 5 marbles in all)

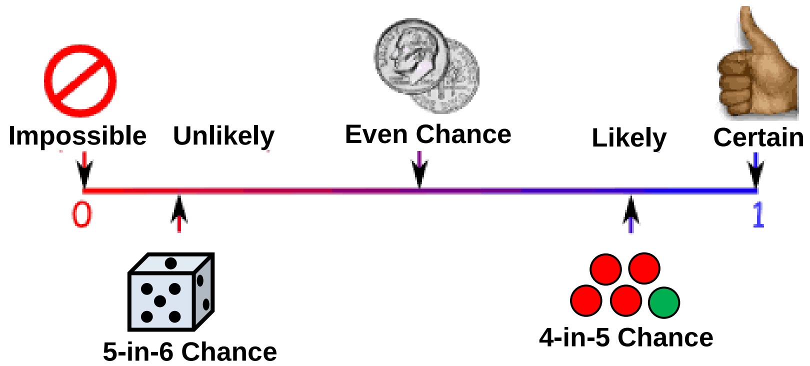
So the probability is $\frac{4}{5}$ = **0.8**

You can show probability on a **probability line or scale** between 0 and 1 inclusive as shown in the diagram below.



The probability of an event always from 0 (impossible) to 1 (certain) which can be expressed as a fraction, decimal or percentage.

For example, the probability in examples 1 and 2 in a probability line or scale can be shown below.



Probability does not tell us exactly what will happen, it is just a guide.

For example, toss a coin 100 times. How many heads will come up?

Probability says that heads have a $\frac{1}{2}$ chance, so we would expect **50 heads**.

But when you actually try it out you might get 48 heads or 55 heads… or anything really, but most cases it will be a number near 50.

Now look at the other examples.

Example 3

The weather forecast shows a 40% chance of rain. Give the probability of each outcome shown in the table.

Outcome	Rain	No Rain
Probability		

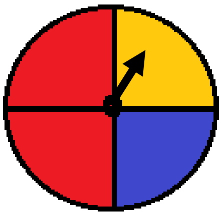
Solution: The probability of Rain is P(Rain) = 40% or 0.4.

The probabilities must add to 1.

So, the probability of No Rain is P(No Rain) = 1 – 0.4
= 0.6 or 60%.

Example 4

Use the Spinner below and find the probability for each outcome.



Outcome	Red	Yellow	Blue
Probability			

Solution: Half the spinner is red, so a reasonable estimate of the probability that the spinner lands on red is

$$P(\text{Red}) = \frac{1}{2}$$

One-fourth of the spinner is yellow, so a reasonable estimate of the probability that the spinner lands on yellow is

$$P(\text{Yellow}) = \frac{1}{4}$$

One-fourth of the spinner is blue, so a reasonable estimate of the probability that the spinner lands on yellow is

$$P(\text{Blue}) = \frac{1}{4}$$

Example 5

A quiz contains 5 multiple-choice questions. Suppose you guess randomly on every question. The table below gives the probability of each score.

Score	0	1	2	3	4	5
Probability	0.237	0.396	0.264	0.088	0.014	0.001

(a) What is the probability of guessing one or more correct?

(b) What is the probability of guessing fewer than 2 correct?

(c) What is the probability of passing the quiz (getting 4 0r 5 correct) by guessing?

Solution:

(a) The event “one or more correct” consists of the outcomes 1, 2, 3, 4, 5.

P(one or more correct) = 0.396 + 0.264 + 0.088 + 0.014 + 0.001

= 0.763 or 76.3%

(b) The event “fewer than 2 correct” consists of the outcomes 0 and 1.

P(fewer than 2 correct) = 0.237 + 0.396

= 0.633 or 63.3%

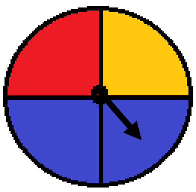
(c) The event “passing the quiz” consists of the outcomes 4 and 5.

P(passing the quiz) = 0.014 + 0.001

= 0.015 or 76.3%

Example 6

After 1000 times spins of the spinner, the following information was recorded.



Outcome	Red	Yellow	Blue
Spins	267	285	448

Estimate the probability of the spinner landing on red.

Solution: Probability of an event happening = $\frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$

$$P(\text{Red}) = \frac{267}{1000} = 0.267$$

The probability of landing on red is 0.267 or 26.7%.

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. A die is tossed. Name the outcomes that make up each of the following events.
   (i) an even numbers
   (ii) a number less than 3
   (iii) a number greater than or equal to 3
   (iv) a prime number
   (v) a number which is a multiple of 3

2. The weather forecast shows a 55% chance of snow. Give the probability of each outcome shown in the table.

Outcome	Snow	No Snow
Probability		

3. An experiment consists of drawing 4 marbles from a bag and counting the number of blue marbles. The table gives the probability of each outcome.

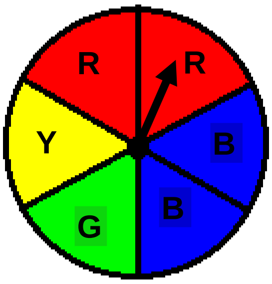
Number of blue marbles	0	1	2	3	4
Probability	0.024	0.238	0.476	0.238	0.024

   (a) What is the probability of drawing at least 3 blue marbles?

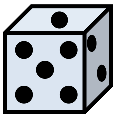
   (b) What is the probability of drawing fewer than three blue marbles?

4. Give the probability for each outcome

Outcome	Red (R)	Blue (B)	Yellow (Y)	Green (G)
Probability				



5. Given a standard die. Determine the probability for the following events when rolling the die on time.

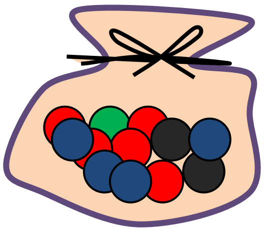


(a) Probability of getting 5

(b) Probability of getting an even number

(c) Probability of getting 7

6. There are 4 blue marbles, 5 red marbles, 1 green, and 2 black marbles in a bag. Suppose you select one marble at random. Find each probability below.



(a) P(black)

(b) P(Blue)

(c) P(Blue and Black)

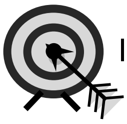
(d) P(not green)

(e) P(not purple)

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 11: Theoretical Probability

You learnt the meaning of probability and how to find the probability of simple events in the last lesson.



In this lesson, you will:

- define theoretical probability
- use the rule of theoretical probability to calculate probability of an event.

Let us recall the definition of Sample Space.

A sample space, **S**, is a list of all the possible outcomes obtained from an experiment and enclosed in a pair of braces { }.

Theoretical probability uses sample space to work out the chances of an event occurring.

What is Theoretical Probability?

Theoretical Probability is the possibility of an event happening based on all the possible outcomes or sample space.

The ratio for the probability of an event "P" occurring is

$$\textbf{P(Event)} = \frac{\textbf{number of favourable outcomes}}{\textbf{total number of possible outcomes}}$$

Example 1

From the letters A, E, I, O, U, what is the theoretical probability of selecting the letter O?

Solution: List of sample space: {A, E, I, O, U} there are 5 possible outcomes

Number of favourable outcomes = 1 (selecting the letter O).

$$\text{P(letter O)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$= \frac{1}{5}$ as fraction

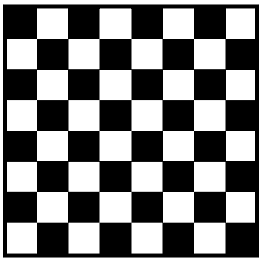
$= 0.2$ as decimal

$= 20\%$ as percent

So, the theoretical probability of selecting the letter O is $\frac{1}{5}$ or 0.2 or 20%.

Example 2

A coin is tossed on a standard 8 x 8 chessboard. What is the probability that the coin lands on a black square?



This is how an 8 x 8 chessboard look like.

Note that the diagram is not drawn to scale.

Solution:

$$\text{Theoretical Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Number of favourable outcomes = 32 (there are 32 black squares on a chess board)

Number of possible outcomes = 64 (there are 32 black and 32 white squares on an 8 x 8 chessboard)

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(A) = \frac{32}{64}$

$P(A) = \frac{1}{2} = 0.5$

Therefore, the theoretical probability that the coin lands on a black square is 0.5 or $\frac{1}{2}$.

Example 3

A bag contains 20 marbles, 15 of them are red and 5 of them are blue in colour. Find the probability of picking a red marble.

Solution:

Since there are 15 red marbles and just 5 blue marbles, it is obvious that there are three times as many red marbles than blue marbles.

So, the chances of picking a red marble is more than that of the blue one.

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$P(A) = \frac{15}{20}$

$P(A) = \frac{3}{4} = 0.75 \text{ or } 75\%$

Therefore the probability of picking a red marble is $\frac{3}{4}$ = 0.75 or 75%.

Example 4

A standard die is rolled.

(a) List the sample space for the experiment.
(b) Determine the probability of obtaining
(i) a 4
(ii) an odd number
(iii) 5 or less

Solution:

(a) Sample space are all the possible outcomes.= {1, 2, 3, 4, 5, 6}

(b) (i) The probability of obtaining 4 is 1 (there is only one 4 in the sample space).

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(4) = \frac{1}{6}$

Therefore the probability of obtaining a 4 is $\frac{1}{6}$.

(ii) There are three odd numbers in the sample space = {1, 3, 5}, so the number of favourable outcome is 3.

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(\text{odd number}) = \frac{3}{6}$

$= \frac{1}{2} = 0.5$ or 50%

Therefore the probability of obtaining an odd number is $\frac{1}{2}$ or 0.5 (50%)

(iii) The event of obtaining 5 or less consists of {5, 4, 3, 2, 1}

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(5 \text{ or less}) = \frac{5}{6}$

Therefore the probability of obtaining 5 or less is $\frac{5}{6}$ or 0.83 (83%).

Example 5

A card is drawn at random from a standard well-shuffled deck of cards.

(a) What is the probability of drawing a Heart?

(b) What is the probability of drawing a King or an Ace?

(c) What is the probability of drawing a black diamond?

Solution: A standard deck card consists of 52 cards.

(a) Number of possible outcome = 52

Number of favourable outcome of getting a Heart is 13.

So, $P(A) = \frac{\text{number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$P(\text{a Heart}) = \frac{13}{52} = \frac{1}{4} = 0.25$ or 25%

Therefore, the probability of drawing a heart is $\frac{1}{4}$ = 0.25 or 25%.

(b) The event of drawing a King or an Ace consists of 4 Kings and 4 Aces.

So, Number of favourable outcomes = 4 + 4 = 8

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(\text{a King or an Ace}) = \frac{8}{52} = \frac{2}{13}$

Therefore, the probability of drawing a King or an Ace is $\frac{2}{13}$.

(c) The event of drawing a black Diamond is impossible because all Diamonds on a deck of card are red.

So, Number of favourable outcomes = 0.

Hence, $P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$P(\text{a black diamond}) = \frac{0}{52} = 0.$

Therefore, the probability of drawing a King or an Ace is 0.

Note that a probability that is equal to zero means that an event is impossible to happen. To get a black diamond from a deck of well-shuffled card is impossible.

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. List the sample spaces for these experiments

   (a) Selecting a day of the week to go to town

   (b) Drawing a marble from a bag containing 3 reds, 2 whites, and1 black

   (c) Selecting an even numbers from the first 20 counting numbers

   (d) Winning a medal at the PNG Games

   (e) Drawing a face card from a standard pack of playing cards.

---

2. A card is drawn at random from a standard well-shuffled deck of cards.

   Find the probability of drawing:

   (a) the King of Hearts

   (b) a Jack or Queen

   (c) a Diamond

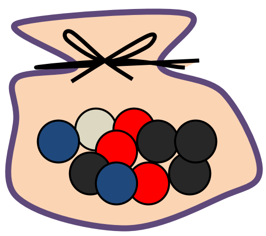
   (d) a black card

   (e) an Ace

   (f) a 4 or a Club

---

3. A bag contains 4 black, 3 red, 1 white and 2 blue marbles. If a marble is drawn at random, calculate the chance that it is



   (a) red

   (b) blue

   (c) not blue

   (d) red or black or white or blue

   (e) white

   (f) green

4. Find the probability of the following outcomes of experiments in which each of the outcomes is equally likely.

   (a) Selecting the Ace of hearts from a pack of 52 playing cards

   (b) Throwing a tail when tossing a fair coin

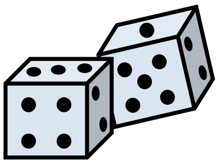
   (c) That the day you were born on was Monday.

   (d) That a new born baby is a boy

   (e) That a randomly selected person in your maths class will be you

---

5. Two dice are being rolled. The possible outcomes are as follows:



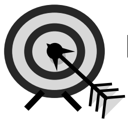
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(a) Find the probability of getting a sum of seven.

CORRECT YOUR WORK. ANSWER ARE AT THE END OF TOPIC 3

Lesson 12: Probability of Complementary Events

You learnt the meaning of theoretical probability in the last lesson. You also learnt to use the rule of theoretical probability to find the probability of an event based on all possible outcomes.



In this lesson, you will:

- define complementary events
- identify the notation used for complementary events
- identify the rule for calculating probability of complementary event
- compute the probability of complementary events

If a set was divided into two (A and B) such that A and B are combined you have the whole set, then A and B are **complementary events**.

What are complementary events?

Two events are said to be **complementary** if they are the only two possible outcomes.

Example 1

Imagine we were testing whether it rains on a particular day.

The two events **"it rains"** and "**it does not rain**" are complementary events.

Only one of the two events can occur. No other events can occur.

Therefore, these two events are complementary.

Example 2

Consider the rolling of a die to see whether the result is odd or even.

The events "odd" and "even" are complementary events because the result must be either "odd" or "even".

The result cannot be anything except "odd" or "even".

Therefore, these two events are complementary.

Example 3

If an event is throwing a 5 on a die, then the complement of that event is not throwing a 5, that is, throwing a 1, 2, 3, 4, and 6.

The complement of an event **taking place** is the event **NOT taking place.**

Example 4

Consider a bag only containing 6 red balls and 8 blue balls.

We are interested in whether a ball picked from the bag, is red or blue.

The events "red" and "blue" are complementary.

The probability of "red" is P(red) = $\frac{6}{14}$. The probability of "blue" is P(blue) = $\frac{8}{14}$.

Notice how P(red) + P(blue) = 1.

The **complement of an event A** is the collection of all the outcomes in the sample space that are **not** in **A**.

If A is an event, then not getting A is its complementary event. We write "Not A" as **A'** (read as A prime) and we call this A complement. So, if A is an event, and A' is the complementary even

P(A) + P(A') = 1

Because the sum of probabilities always equals 1, and because A and A' are mutually exclusive, it follows that

P(A') = 1 – P(A)

Mutually exclusive means you can't get both events at the same time. It is either one or the other, but **not both**.

Therefore, to find the probability an event NOT happening, we need to subtract the probability of that event from 1.

Example 6

If the probability of winning a certain game is $\frac{1}{4}$. Find the probability of losing the game.

Solution: P(A') = 1 – P(A)

P(A') = 1 - $\frac{1}{4}$

= $\frac{3}{4}$ = 0.75 or 75%.

Therefore the probability of losing the game is $\frac{3}{4}$ or 75%.

Example 7

The probability that Ross winning her tennis match is $\frac{5}{7}$. What is the probability of her not winning the match?

Solution: $P(A') = 1 - P(A)$

$$P(\text{does not win}) = 1 - P(\text{win})$$
$$= 1 - \frac{5}{7}$$
$$= \frac{2}{7}$$

Example 8

A spinner only has whole numbers as outcomes. The probability of getting an even number is 0.27.

What is the probability of getting an odd number?

Solution: $P(A') = 1 - P(A)$

$$P(\text{odd number}) = 1 - P(\text{even number})$$
$$= 1 - 0.27$$
$$= 0.73$$

Example 8

In rolling a die, the set of all possible outcomes is {1, 2, 3, 4, 5, 6}. Find the following probabilities:

(a) Event A = getting a 5 or 6?
(b) Complement of Event A.

Solution:

(a) Event (A) = {5, 6}
Number of favourable outcomes = 2.
Total number of outcomes = 6

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$
$$= \frac{2}{6} = \frac{1}{3}$$

Therefore, the probability of getting 5 or 6 is $\frac{1}{3}$.

(b) The complement of getting a 5 or 6 is not getting 5 or 6. We also say the complement of event A is Event A'.

Hence using the formula for finding the complement of an event

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Therefore, the probability of not getting 5 or 6 is $\frac{2}{3}$.

To check: $P(A) + P(A') = 1$

$$\frac{1}{3} + \frac{2}{3} = 1$$

$$1 = 1$$

Example 9

A card is drawn from a deck of 52 playing cards.

Find: (a) probability of drawing a Spade.
(b) probability of not drawing a spade.

Solution:

(a) Event A = getting a spade
Number of favourable outcomes = 13 (there13 spade on a deck of 52 cards)
Total number of outcomes = 52

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$= \frac{13}{52} = \frac{1}{4} = 0.25$$

(b) P(not a Spade) is the complement of P(A) let us denote it P(A').

Hence, $P(A') = 1 - P(A')$

$$= 1 - \frac{1}{4}$$

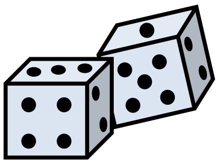
$$= \frac{3}{4} = 0.75$$

Sometimes it is easier to work out the complement first to find the probability of an event.

See example 10 on the next page.

Example 10

Throw two dice. What is the probability that the two scores are different?



Solution:

Different scores are like getting a 1 and 3, or a 4 and 5. It is quite a long list of sample space.

Event A = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 3), …, etc}

But the complement (which is when two scores are the same) is only 6 outcomes.

Event A’ = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

And the probability is easy to work out: P(A’) = 1 – P(A)

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

Knowing that P(A) and P(A’) together make 1. We can calculate the probability of event A (drawing two different scores

Hence, P(A) = 1 – P(A’)

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

So, in this example it is easier to work out P(A’) first, then P(A).

Remember:

Complementary events have no common elements and together make up the sample space.

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

1. A die is thrown. Write the complement of each event described below:

   (a) {2, 3, 4}
   (b) {3, 6}
   (c) a prime number
   (d) a score more than 2
   (e) a score of 3 or less.

2. The probability that Shawn will win his darts match is $\frac{5}{8}$.

   What is the probability that he will **not** win?

3. The probability that it will rain on Christmas Day is $\frac{1}{8}$.

   What is the probability that it will **not** rain on Christmas Day?

4. If you take a card at random from a deck of playing cards, the probability of getting a King is $\frac{1}{13}$.

   What is the probability that you do not get a King?

5. The probability that Simon gets all his spellings correct in his next test is 0.75.

   What is the probability that he does not get them all correct?

6. A number is chosen at random from a set of whole numbers from 1 to 50. Calculate the probability that the chosen number is not a perfect square.

---

7. A bag contains red and black balls. The probability of drawing a red ball is $\frac{2}{5}$.
What is the probability of drawing a black ball?

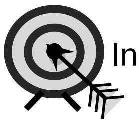
---

8. A box has slips numbered from 1 to 100. What is the probability of picking a number which is not divisible by 10?

CORRECT YOUR WORK. ANSWER ARE AT THE END OF TOPIC 3

Lesson 13: Adding Probabilities

You learnt the meaning of complementary events and how to find the probability of the complement of an event In the last lesson.



In this lesson, you will:

- define mutually exclusive events
- identify the addition law and works out the probabilities of mutually exclusive events
- use the concept of adding probabilities to calculate probabilities of events.

We learnt earlier that an **event** is a set of outcomes. It is a subset of the sample space for an activity or experiment.

Life is full of random events. We need to get a feel for them to be smart and successful people.

The tossing of a coin, throwing of a dice, lottery drawings and picking a ball are all examples of events.

When we say events, we mean one or more outcomes.

Events are of different types.

When an event corresponds to a single outcome of the activity, it is often called a **simple event.**

Examples of simple events

1. Drawing the queen of spades from a deck of standard cards.
2. Getting a tail when tossing a coin
3. Rolling a 5 in throwing a dice

An event can include several outcomes.

Examples

1. Choosing a King from a deck of cards (any of the 4 Kings) is also an event.
2. Rolling an even number (2, 4, 6) is an event.

When two events cannot happen at the same time they are called **mutually exclusive events.** This means that you cannot get both events at the same time. It is either one or the other, but not both.

A common example of a mutually exclusive event is a coin flip. Either the coin will come up heads or tails. Since the coin coming up heads means that it will not come up tails. That makes the coin flip a mutually exclusive event. It will either one side or the other, it cannot be both.

Here are other examples of mutually exclusive events.

1. Turning left or right
2. Heads and tails
3. Kings and Aces

Let's look at the probabilities of Mutually Exclusive events.

When two events let say Events A and B, are mutually exclusive it is **impossible** for them to happen together:

P(A and B) = 0

"The probability of A and B together equals 0 (impossible)"

But the probability of A **or** B is the sum of the individual probabilities:

P(A or B) = P(A) + P(B)

"The probability of A **or** B equals the probability of A **plus** the probability of B"

Example 1: A Deck of Cards

In a Deck of 52 Cards:

- the probability of a King is $\frac{1}{13}$, so **P(King)=**$\frac{1}{13}$
- the probability of an Ace is also $\frac{1}{13}$, so **P(Ace)=** $\frac{1}{13}$

When we combine those two Events:

- The probability of a card being a King **and** an Ace is **0** (Impossible).
- The probability of a card being a King **or** an Ace is $(\frac{1}{13}) + (\frac{1}{13}) = \frac{2}{13}$

They can be written like this:

P(King and Ace) = 0

P(King or Ace) = $(\frac{1}{13}) + (\frac{1}{13}) = \frac{2}{13}$

A special notation is used to denote the word "and" and "or" in the rule.

Instead of "and" you will often see the symbol ∩ (which is the "Intersection" symbol used in Venn Diagrams)

So, P(A and B) is written as P(A∩B)

Instead of "or" you will often see the symbol ∪ (the "Union" symbol)

So, P(A or B) is written as P(AUB)

Example 3: Scoring Goals

If the probability of:

- scoring no goals (Event "A") is **20%**
- scoring exactly 1 goal (Event "B") is **15%**

Then:

- The probability of scoring no goals **and** 1 goal is **0** (Impossible)
- The probability of scoring no goals **or** 1 goal is 20% + 15% = **35%**

They can be written like this:

$$P(A \cap B) = 0$$

$$P(A \cup B) = 20\% + 15\% = 35\%$$

So, to find the probability that one or the other of two mutually exclusive events will occur, add their individual probabilities.

This is called the Addition Rule.

Now let us have other examples.

Example 3

If two dice are tossed, the event A of rolling a total of 6 and the event B of rolling a total of 9 are mutually exclusive.

Find the probability of A or B occurring.

Solution: $P(A \text{ or } B) = P(A) + P(B)$

$$= \frac{5}{36} + \frac{4}{36}$$

$$= \frac{9}{36}$$

$$= \frac{1}{4} = 0.25 \text{ or } 25\%$$

Example 4

Which of the following represents a pair of mutually exclusive events when a die is rolled?

(a) obtaining an even number or obtaining a 4

(b) obtaining an odd number or obtaining a 3

(c) obtaining a number less than 3 or obtaining a number more than 5

(d) obtaining a multiple of 2 or obtaining a multiple of 3

(e) obtaining a factor of 6 or obtaining a multiple of 6

Solution:

(a) No. A = {2, 4, 6}, B = {4}. There is a common element

(b) No. A = {1, 3, 5}, B = {3}. There is a common element

(c) Yes. A = {1, 2}, B = {6}. No common element

(d) No. A = {2, 4, 6}, {3, 6}. There is a common element

(e) No. A = {1, 2, 3, 6}, B = {6}. There is a common element

Example 5

In a certain parking lot, there are 4 Toyota, 3 Isuzu and 1 Hyundai.
What are the chances of either a Toyota or an Isuzu car will leave the parking lot?

Solution:

A Toyota cannot be included with Isuzu. A given vehicle cannot be a Toyota and an Isuzu at the same time. Thus the two vehicles are mutually exclusive.

Thus,

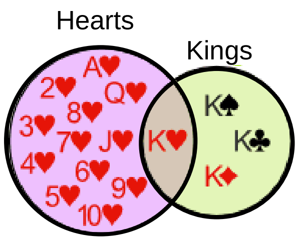
P(Toyota or Isuzu) = P(Toyota) + P(Isuzu)

$$= \frac{4}{8} + \frac{3}{8}$$

$$= \frac{7}{8}$$

Now let's see what happens when events are **not Mutually Exclusive**.

Example: Hearts and Kings



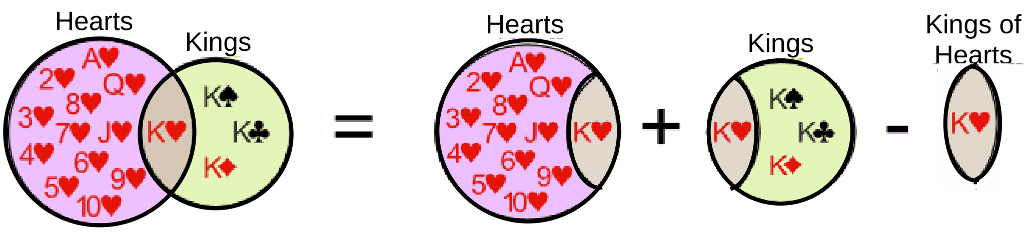
Hearts **and** Kings together is only the King of Hearts: 

But Hearts **or** Kings is:

- all the Hearts (13 of them)
- all the Kings (4 of them)

But that counts the King of Hearts twice!

So we correct our answer, by subtracting the extra "and" part:



16 Cards = 13 Hearts + 4 Kings - the 1 extra King of Hearts

You can count them to make sure this works.

As a formula this is:

P(A or B) = P(A) + P(B) - P(A and B)

"The probability of A **or** B equals the probability of A **plus** the probability of B **minus** the probability of A **and** B"

Here is the **same formula**, by using the symbol U and ∩.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Now look at Example 6.

16 people study French, 21 study Spanish and there are 30 altogether. Work out the probabilities!

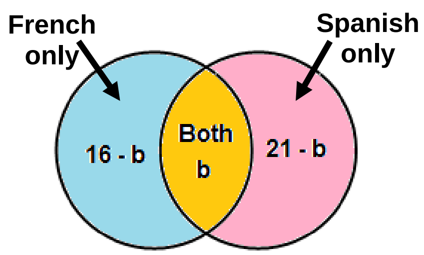
Solution:

This is definitely a case of **not** Mutually Exclusive (you can study French and Spanish).

Let's say **b** is how many study both languages:

- people studying French Only must be 16-b
- people studying Spanish Only must be 21-b

And we get:

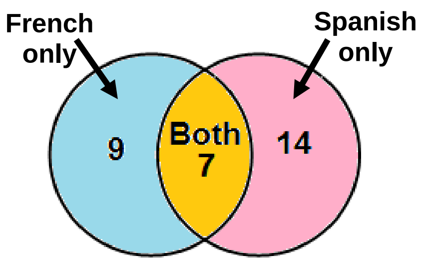


And we know there are **30** people, so: (16-b) + b + (21-b) = 30

37 - b = 30

b = 7

And we can put in the correct numbers:



So we know all this now:

- P(French) = 16/30
- P(Spanish) = 21/30
- P(French Only) = 9/30
- P(Spanish Only) = 14/30
- P(French or Spanish) = 30/30 = 1
- P(French and Spanish) = 7/30

Lastly, let's check with our formula:

P(A or B) = P(A) + P(B) – P(A and B)

Put the values in:

$$\frac{30}{30} = \frac{16}{30} + \frac{21}{30} - \frac{7}{30}$$

$$\frac{30}{30} = \frac{37}{30} - \frac{7}{30}$$

$$\frac{30}{30} = \frac{30}{30}$$

Yes, it works!

Remember:

Mutually Exclusive

- A **and** B together is impossible: **P(A and B) = 0**
- A **or** B is the sum of A and B: **P(A or B) = P(A) + P(B)**

Non- Mutually Exclusive

- A **or** B is the sum of A and B minus A **and** B: **P(A or B) = P(A) + P(B) - P(A and B)**

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

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1. For the rolling of a die, decide whether the following pairs of events are mutually exclusive or not:

   (a) A = {3, 5} ; B = {2, 4}

   (b) A = factor of 3 ; B = factor of 4

   (c) A = less than 3 ; B = more than 3

   (d) A = even number ; B = factors of 6

---

2. For a standard pack of cards, do the following pairs of events overlap?

   (a) A = king ; B = jack

   (b) A = a 6 ; B = a 3

   (c) A = heart ; B=: queen

   (d) A = black ; B = a 5

---

3. A die is rolled. Find the probability of the scores being:

   (a) either a 3 or a 5

   (b) an odd or even

   (c) a multiple of 3 or a multiple of 4

   (d) divisible by 4 or 6

   (e) multiple of 5 or factor of 6

   (f) multiples of 2 or multiples of 4

   (g) at least 3.

---

4. From a standard pack of cards, find the probability of drawing:

   (a) a club or a spade

   (b) a red or a black

   (c) a card which is not black

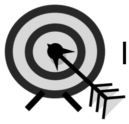
   (d) a 2 or a club

   (e) a 4 or a card less than 7 (excluding ace)

CORRECT YOUR WORK, ANSWER ARE AT THE END OF TOPIC 2

Lesson 14: Multiplying Probabilities

You learnt about mutually exclusive events and how to work out their probabilities using the addition rule.



In this lesson, you will:

- define independent events
- multiply probabilities of independent events
- find the probability of independent events using the multiplication rule.

Let us start this lesson by considering the following situation.

It is critical that the engine of a single-engine plane does not fail during a flight. These planes often have two **independent** electrical systems. In the event that one fails, for example, due to a faulty spark plug, the second system will still be able to keep the plane in flight.

Now let us consider another situation which involves an experiment where a jar contains one red, one black and one white marble. One marble is withdrawn and then **replaced**. A second withdrawal is made. What is the probability of drawing a white, then a red?

There are a couple of things to take note about this experiment. Withdrawal of one marble from the jar, replacing it, then, withdrawing again from the same jar is a **compound event**. Since the first marble was replaced, withdrawing a red marble on the first try has no effect on the probability of withdrawing a red pair on the second try. No matter what colour was taken in the first draw the second draw could be of any colours, because the first marble was **replaced.** The occurrence of one event does not affect the occurrence of the other. Thus the two draws are **independent** of each other.

Two events **A** and **B** are **independent events** if the fact that the occurrence of one event **A** does not affect the probability of the other **B**.

Here are some other examples of independent events.

(a) The events that a coin landing heads on one toss **AND** tails on another toss. The result of one toss does not affect the result of the other, so the events are independent.

(b) Landing on heads after tossing a coin **AND** rolling a 5 on a single 6-sided die.

(c) Choosing a marble from a jar **AND** landing on heads after tossing a coin.

(d) Choosing a 3 from a deck of cards, replacing it, **AND** then choosing an ace as the second card.

(e) Rolling a 4 on a single 6-sided die, **AND** then rolling a 1 on a second roll of the die.

Finding the Probability of independent Events

To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. This multiplication rule is defined symbolically below. Note that multiplication is represented by AND.

Multiplication Rule: When two events, A and B, are independent, the Probability of both events occurring is:

$$\mathbf{P(A \text{ and } B) = P(A) \bullet P(B)}$$

Example 1

A coin is tossed and a single sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 4 on the die.

Solution: First, find the Probability of each individual event.

$$P(\text{Head}) = \frac{1}{2}$$

$$P(4) = \frac{1}{6}$$

$$P(\text{Head and } 4) = P(\text{Head}) \bullet P(4)$$

$$= \frac{1}{2} \bullet \frac{1}{6}$$

$$= \frac{1}{12}$$

Example 2

A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a Queen and then a nine?

Solution: First, find the Probability of each individual event.

$$P(\text{Queen}) = \frac{4}{52}$$

$$P(9) = \frac{4}{52}$$

$$P(\text{Queen and } 9) = P(\text{Queen}) \bullet P(9)$$

$$= \frac{4}{52} \bullet \frac{4}{52}$$

$$= \frac{16}{2704}$$

$$= \frac{1}{169}$$

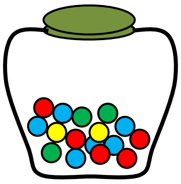
Example 3

A jar contains 3 green, 5 red, 2 yellow and 6 blue marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a red and then a blue marble?

Solution: First, find the Probability of each individual event.

$$P(\text{Red}) = \frac{5}{16}$$

$$P(\text{Blue}) = \frac{6}{16}$$



$$P(\text{Red and Blue}) = P(\text{Red}) \bullet P(\text{Blue})$$

$$= \frac{5}{16} \bullet \frac{6}{16}$$

$$= \frac{30}{256}$$

$$= \frac{15}{128}$$

Example 4

An experiment consists of spinning the spinner below three times. For a spin, all outcomes are equally likely.

(a) What is the probability of spinning an odd number three times?
(b) What is the probability of spinning a 5 at least once?

Solution:

(a) For each spin, the probability is: $P(\text{odd}) = \frac{3}{5}$

So, the probability of spinning an odd number three times is:

$$P(\text{odd, odd, odd}) = P(\text{odd}) \bullet P(\text{odd}) \bullet P(\text{odd})$$

$$= \frac{3}{5} \bullet \frac{3}{5} \bullet \frac{3}{5}$$

$$= \frac{27}{125}$$

(b) Think of this: P(at least one 5) + P(not 5, not 5, not 5) = 1

For each spin, $P(\text{not } 5) = \frac{4}{5}$

$$P(\text{not 5, not 5, not 5}) = \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5} = \frac{64}{125} = 0.512$$

Subtract from 1 to find the probability of spinning at least one 5.

So, P(at least one 5) = 1 – 0.512

= 0.488

Example 5

Charles drawer contains one pair of socks with each of the following colours: red, blue, green, black yellow and blue. Each pair is folded together in a matching set. Charles goes into the sock drawer and picked a pair of socks without looking. He replaced this pair and then picked another pair of socks. What is the probability that Charles will pick the red pair of socks both times?

Solution: First, find the Probability of each individual event.

$$P(\text{Red}) = \frac{1}{5}$$

$$P(\text{Red and Red}) = P(\text{Red}) \bullet P(\text{Red})$$

$$= \frac{1}{5} \bullet \frac{1}{5}$$

$$= \frac{1}{25}$$

Example 6

A nationwide survey found that 72% of people in Port Moresby like pizza. If 3 people are selected at random, what is the probability that all three like pizza? Round off your answer to the nearest per cent.

Solution: Let x represent the event of randomly choosing a person who likes pizza from Port Moresby.

$$P(x) \bullet P(x) \bullet P(x) = (0.72)(0.72)(0.72) = 0.37 = 37\%$$

Therefore, the probability that all three like pizza is 0.37 or 37%.

Sometimes some events can be "dependent".

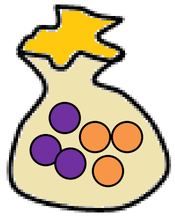
Two events **A** and **B** are **dependent events** if the fact that the occurrence of one event **A** does affect the occurrence of the other event **B** so that the probability is changed.

To calculate the probability of dependent events occurring, do the following:

1. Calculate the probability of the first event.
2. Calculate the probability that the second event would occur if the first event had already occurred.
3. Multiply the probabilities.

If **A** and **B** are **dependent events,** then **P(A and B)= P(A) • P(B after A)**

Suppose you draw 2 marbles without replacement from a bag that contains 3 purple and 3 orange marbles. On the first draw,

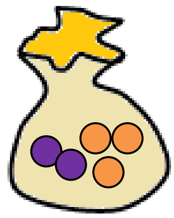


Before first draw

$$P(\text{purple}) = \frac{3}{6} = \frac{1}{2}$$

The sample space for the second draw depends on the first draw.

If the first draw was purple, then the probability of the second draw being purple is



After first draw

$$P(\text{purple}) = \frac{2}{5}$$

So the probability of drawing two purple is

$$P(\text{purple, purple}) = \frac{1}{2} \bullet \frac{2}{5} = \frac{2}{10} \text{ or } \frac{1}{5}$$

Now let us solve problems on the probability of dependent events.

Example 7

A bag contains one blue, two white and three red marbles. Find the probability of drawing a blue and then a red, if the first marble drawn is **not replaced**.

Solution:

The first draw P(blue) = $\frac{1}{6}$

The marbles available for the second draw **depend** on what was selected in the first.

Draw. With the blue not available in the bag, the number of possible outcomes reduces to 5. Hence, in draw 2, P(Red) = $\frac{3}{5}$.

Thus,

$$P(\text{B and then R}) = P(B) \bullet (P(R)$$

$$= \frac{1}{6} \bullet \frac{3}{5}$$

$$= \frac{1}{10}$$

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Four cards are chosen from a standard deck of 52 playing cards with replacement.

   What is the probability of choosing 4 hearts in a row?

---

2. A jar contains 6 red balls, 3 green balls, 5 white balls and 7 yellow balls. Two balls are chosen from the jar, with replacement.

   (a) What is the probability that both balls chosen are green?

   (b) What is the probability of choosing a red and a yellow ball?

---

3. Spin a spinner numbered 1 to 7, and toss a coin.

   What is the probability of getting an odd number on the spinner and a tail on the coin?

4. A survey showed that 65% of all the students dislikes eating vegetables. If 4 children are chosen at random, what is the probability that all 4 children dislike eating vegetables? Round off your answer to the nearest per cent.

---

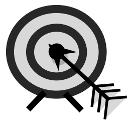
5. Two cards are chosen from a deck of 52 cards without replacement.

   What is the probability of choosing two kings?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 15: Union and Intersection of Two Events

You learnt to identify independent and dependent events in lesson 14. You also learnt to find the probability of independent and dependent events using the multiplication rule.



In this lesson you will:

- review the concepts of union and intersection of two events
- find the probability of the intersection and union of two events.

Let us start by understanding the concepts of **union** and **intersection.**

Suppose the experiment is throwing a die, then the sample space is {1, 2, 3, 4, 5, 6}.

From the sample space, we can have events such as:

Event A = {1, 3}

Event B = {4, 5, 6}

To find the **union** of A and B, in symbol AUB (read as A union B), we need to find a set whose elements are elements that is in A **or** element that is in B.

Thus AUB = {1, 3, 4, 5, 6}

In another case, if suppose Event C = {1, 2, 3, 6} and Event D = {2, 4, 6}

Then CUD = {1. 2. 3. 4. 6}

Another concept that you need to learn is **intersection.**

To find the **intersection** of A and B, in symbol A∩B (read as A intersects B), C∩D you need to find the element that is in A **and** that is in B too.

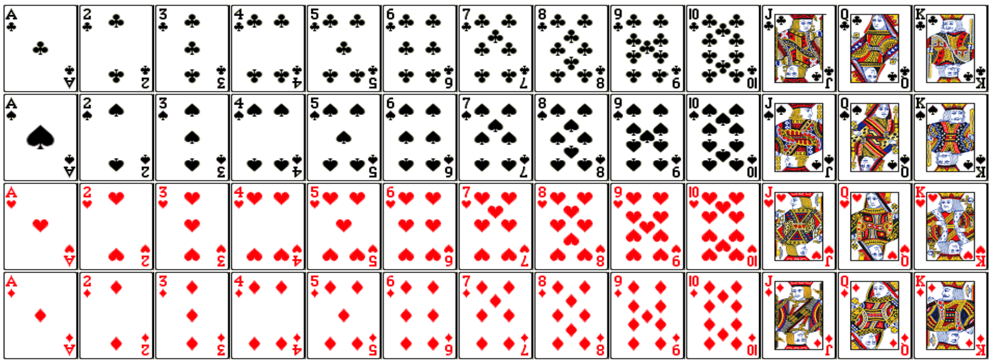
Thus A∩B = ϕ

Φ is symbol for null Set. It indicates that the two sets have no common Element.

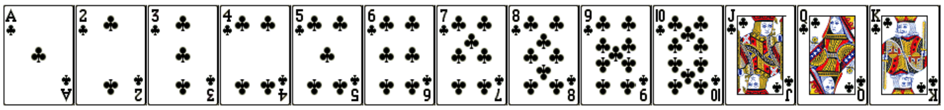
If you are asked to find C∩D, then C∩D = {2, 6}.

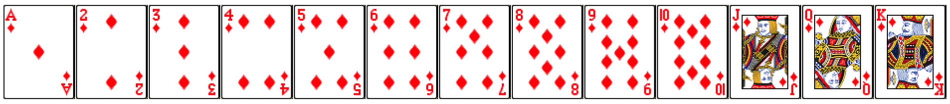
Since the elements 2 and 6 are both in C and D.

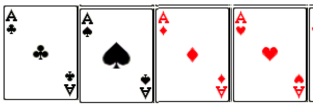
Let us have another example by considering the pack of cards below:



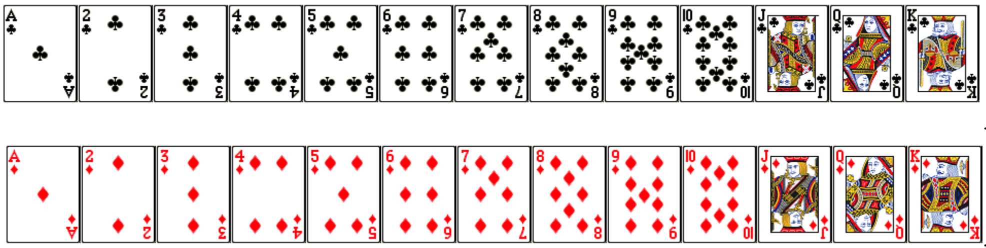
Let us have the following events:

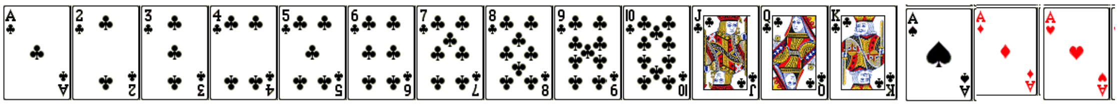
A = {  }

B = {  }

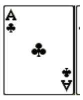
C = {  }

Thus,

AUB = {  }

AUC = {  }

A∩B = Ø

A∩C = {  }

Let us use the idea of union and intersection in probability.

Union of Events

The union of events includes not only the probability of A and B, but also the probability of only A and the probability of only B.

P(AUB) = P(A) + P(B) - P(A∩B)

The last term is important since it removed items that appear in both A and B, thereby avoiding double counting.

Consider the following example, meant to clarify the importance of the **- P(A∩B)** term in the formula for **P(AUB).**

Example 1

When rolling a die, what is the probability of rolling a number greater than 1 or a number that is even?

Solution: The presence of the word "**or**" is a clue that the question is asking about the union of events.

Let Event A = rolling a number greater than 1
Let Event B = rolling a number that is even

Since there are five possibilities and six total numbers that could be rolled,

The Probability of rolling a number greater than 1 is

$$P(A) = \frac{5}{6}$$

Since there are three possibilities (i.e., 2, 4, 6) and six total numbers that could be rolled (i.e., 1, 2, 3, 4, 5, 6)

The Probability of rolling a number that is even is

$$P(B) = \frac{3}{6}$$

$$P(AUB) \neq \frac{5}{6} + \frac{3}{6}$$

Since:

(i) this number is greater than one, which is impossible for a probability

(ii) this number double counts the even numbers greater than 1 (i.e., 2, 4, 6)

Since there are three possibilities (i.e., 2, 4, 6) and six total numbers that could be rolled (i.e., 1, 2, 3, 4, 5, 6)

$$P(A \cap B) = \text{rolling a number greater than 1 and even}$$

$$= \frac{3}{6}$$

Hence using the formula $P(AUB) = P(A) + P(B) - P(A \cap B)$

$$P(AUB) = \frac{5}{6} + \frac{3}{6} - \frac{3}{6}$$

$$= \frac{5}{6}$$

Therefore the probability of rolling a number greater than 1 or a number that is even is $\frac{5}{6}$.

Example 2

When drawing a card from a standard 52-card deck, what is the probability of drawing a red card or an ace?

Solution: The presence of the word "or" is a clue that the question is asking about the union of events.

Let Event A = drawing an ace

Let Event B = drawing a red card

Since there are four aces in each deck of 52 cards

$$P(A) = \frac{4}{52}$$

Since there are four suits and two of them are red (or 26 red cards in a deck of 52)

$$P(B) = \frac{26}{52} = \frac{1}{2}$$

Since there are 2 red aces in a deck of 52 cards)

$$P(A \cap B) = \text{the probability of drawing a red ace} = \frac{2}{52}$$

It is important to subtract P(A∩B), otherwise you would double count the red aces.

$$P(AUB) = P(A) + P(B) - P(A \cap B)$$

$$P(AUB) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$$

$$= \frac{28}{52} \text{ or } \frac{7}{13}$$

Therefore, the probability of drawing a red card or an ace is $\frac{28}{52}$ or $\frac{7}{13}$

Intersection of events

The intersection of events includes only the probability that both events A and B occur simultaneously. If event A occurs alone or if event B occurs alone, this does not fall within the intersection of events A and B.

Formula: P(A∩B) = probability of the intersection of events A and B

$$\mathbf{P(A \cap B) = P(A) \bullet P(B|A)}$$

or

$$\mathbf{P(A \cap B) = P(A) \bullet P(A|B)}$$

If events A and B are mutually exclusive (i.e., if one event occurs then the other cannot occur), the formula can be simplified:

$P(A \cap B)$ = probability of the intersection of events A and B

$$\mathbf{P(A \cap B) = P(A) \bullet P(B)}$$

Example 3

What is the probability of drawing a red card from a standard 52 card deck and rolling an even number on a fair die?

Solution: Since drawing a red card and rolling an even number are entirely unrelated and have no bearing on each other, the two events are independent and you can use the simplified formula for the intersection of events.

Let Event A = drawing a red card

Let Event B = rolling an even number

The probability of drawing a red card is: $P(A) = \frac{1}{2}$

The probability of rolling an even number is: $P(B) = \frac{1}{2}$

The probability of drawing a red card and rolling an even number is:

$$P(A \cap B) = P(A) \bullet P(B)$$

$$P(A \cap B) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

To conclude this discussion, remember the following:

The union of two events combines the outcomes in both, written A or B (either can happen). The union happens if either A or B or both occur. The word *or*, *either*… or, *at least* is associated with UNION. It also implies Addition Rule.

The intersection of two events is the outcomes that are in both at the same time, written A and B (both must happen). The intersection of two events is the event that occur in both. The words *and*, *both… and*, *and then* is associated with INTERSECTION. It also implies Multiplication Rule.

NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. The table below shows the sample space of an experiment of rolling a pair of dice.

RED DIE

BLACK DIE	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1,4)	(1,5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2,4)	(2,5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3,4)	(3,5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4,4)	(4,5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5,4)	(5,5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6,4)	(6,5)	(6, 6)

List down the elements of each of the following events: Complete the given set, one is already given.

(i) A = the sum of the two upper faces is 4

= {(3, 1), ______, ______,……}

(ii) B = the sum is 11

B = {(6, 5),______, ______,……}

(iii) C = double (the same number on both die)

C = {(1, 1), ______, ______,……}

(iv) D = sum is greater than or equal to 10

D = {(6, 4), ______, ______,……}

(v) E = sum is at least 10

E = {(6, 4),______, ______,……}

(vi) F = sum is at most 5

F = {(1, 1), ______, ______,……}

2. Using the events given in number 1, find each of the following.

(i) AUB

(ii) CUD

(iii) AUF

(iv) A∩B

(v) A∩C

(vi) D∩E

---

3. Find each of the following from question 1.

(i) P(AUB)

(ii) P(CUD)

(iii) P(AUF)

(iv) P(A∩B)

(v) P(A∩C)

(vi) P(D∩E)

---

4. Ten cards are numbered 1 to 10. One is drawn at random. Find the probability that the number is…

(i) greater than 7 or less than 3

(ii) greater than 6 or odd

5. A bag contains 2 black and 1 blue balls. A ball is drawn, its color noted and then replaced. A second ball is then drawn. Find the probability of drawing:

(i) a black ball and another black ball.

(ii) a black and then a blue

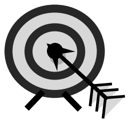
---

6. Rework number 5, if the first ball drawn is **not replaced**.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 16: Mixed Probability Problems

You learnt to identify the union and intersection of events as well as finding the probability of the union and intersection of events in the last lesson.



In this lesson, you will:

- solve mixed problems on probability in practical situations.

You learnt the different concepts of probability from Lesson 10 to 15. We can now apply those concepts in solving problems in practical situations.

Example 1

In a supermarket car park, there are spaces for 7 buses, 130 cars and 13 motorbike. If all vehicles have an equal chance of leaving at any time, find the probability that the next vehicle to leave will be:

(i) a bus

(ii) a car

(iii) a motorbike or a bus

(iv) not a car

Solution:

(i) P(a bus) = $\frac{7}{150}$

(ii) P(a car) = $\frac{130}{150} = \frac{13}{15}$

(iii) P(a motorbike or a bus) = P(a motorbike) + P(a bus)

$$= \frac{13}{150} + \frac{7}{150}$$

$$= \frac{20}{150} \text{ or } \frac{2}{15}$$

(iv) P(not a car) = 1– P(a car)

$$= 1 - \frac{130}{150}$$

$$= \frac{20}{150} \text{ or } \frac{2}{15}$$

Example 2

Consider a roulette wheel which is circular and is divided into 37 equal sectors numbered 0 to 36. The sector number 0 is coloured green. Half of the numbers 1 to 36 are coloured black and the other half are coloured red. A small ball is dropped onto the spinning roulette wheel and eventually lands on one of the numbers when the wheel stops spinning.
A gambler can bet on:

(i) any one of the numbers 0 to 36

(ii) even or odds

(iii) reds or blacks

(iv) groups of 2, 3, 4, … 18 numbers.

What is the probability that the ball will land on:

(a) red numbers?

(b) an even number?

(c) the green zero?

(d) an even or an odd number?

Solution:

(a) P(red number) $= \frac{18}{37}$

(b) P(even number) $= \frac{19}{37}$

(c) P(zero) $= \frac{1}{37}$

(d) P(even or odd) $= \frac{37}{37} = 1$

Example 3

A mal-functioning machine produces good articles and defective articles in the ratio 4:1. Three articles are selected at random from the production line.

Find the probability of selecting…

(a) 3 good ones

(b) 3 defective ones

(c) 1 good , then 2 defectives

Solution:

(a) P(G and G and G) = P(G) $\times$ P(G) $\times$ P(G)

$$= \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$

$$= \frac{64}{125}$$

(b) P(D and D and D) = P(D) x P(D) x P(D)

$$= \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$$

$$= \frac{1}{125}$$

(c) P(G and D and D) = P(G) x P(D) x P(D)

$$= \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}$$

$$= \frac{4}{125}$$

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. A carton of 12 eggs contains five brown eggs, three speckled eggs and four white eggs. If an egg is chosen at random, what is the probability that the egg is:

   (i) speckled?

   (ii) white?

   (iii) not speckled?

   (iv) brown or speckled?

---

2. A group of students is made up of 4 girls and 6 boys. Two students are to be selected to represent the group on the student representative council. They decide to write all names on identical pieces of paper, put them in a hat and choose two names randomly. Find the probability of

   (i) two boys being selected.

   (ii) two girls being selected

   (iii) one girl and 1 boy.

3. If A and B are independent events and P(A) = 0.6 and P(B) = 0.4, find

(i) $P(A \cap B)$

(ii) $P(A' \cap B)$ where A' is the complement of A

(iii) $P(A \cap B')$ where B' is the complement of B

(iv) $P(A' \cap B')$

---

4. Anton is an archer. The experimental probability that Anton will hit the target is $\frac{3}{5}$.

(i) What is the probability that Anton will hit the target on two successive attempts?

(ii) What is the probability that Anton will hit the target on three successive attempts?

(iii) What is the probability that Anton will not hit the target on two successive attempts?

5. Ana has 3 pairs of Nike and 2 pairs of Adidas running shoes. She has two pairs of Nike, 3 pairs of Rio and a pair of Red Robin socks. Preparing for an early morning run, she grabs at random for a pair of socks and a pair of shoes.

   What is the probability that she chooses:

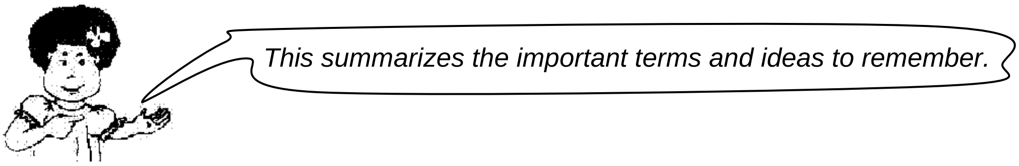
   (i) Nike shoes and Nike socks?

   (ii) Rio socks and Adidas shoes?

   (iii) Adidas shoes and socks that are not Rio?

CORRECT YOUR WORK. ANSWER ARE AT THE END OF TOPIC 2

TOPIC 3: SUMMARY



- An **experiment** is an activity in which results are observed. Each observation is called a **trial**, and each result is an **outcome**.
- The **sample space** is the set of all possible outcomes of an experiment.
- An **event** is any set of one or more outcomes.
- The **probability** of an event, written P(event) is a number from 0(or 0%) to 1 (or 100%) that tells you how likely the event is to happen.
- A probability of 0 means the event is **impossible,** or can never happen.
- A probability of 1 means the event is **certain**, or has to happen.
- The probabilities of all the outcomes in the sample space add up to 1.
- **Theoretical probability** is the **probability** of an event when all outcomes are equally likely. It is given by the formula:

  Probability (Events) = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$ or **P(A) =** $\frac{\mathbf{n(E)}}{\mathbf{n(S)}}$

- The complement of an event **A** is the collection of all outcomes in the sample space that are **not** in **A**. It is denoted by **A'.** If **A** and **A'** are complementary events then **P(A) + P(A') = 1**.
- When two events cannot happen at the same time they are called **mutually exclusive events.**
- To find the probability of mutually exclusive events we add the individual probabilities. The formula is: **P(A or B) = P(A) + P(B).** This is called **the Addition Rule.**
- **Independent events** are two events in which the occurrence of one has no effect on the occurrence of the other.
- To find the probability of two independent events, multiply their individual probabilities. The formula is: **P(A and B) = P(A) • P(B).** This is called the **Multiplication Rule**.
- **Dependent events** are two events in which the occurrence of one affects the occurrence of the other so that the probability is changed. The multiplication rule still holds.
- The **union of two events** combines the outcomes in both, written **A or B** (either can happen). The union happens if either A or B or both occur. The word ***or*****,** ***either*****… or,** ***at least*** is associated with UNION. It also implies **Addition Rule.**
- The **intersection of two events** is the outcomes that are only in both at the same time, written **A and B** (both must happen). The intersection of two events is the event that both occur. The word ***and*****,** ***both*****…** ***and*****,** ***and then*** is associated with INTERSECTION. It also implies **Multiplication Rule.**

REVISE LESSONS 10-16. THEN DO TOPIC TEST 3 IN ASSIGNMENT 4

ANSWERS TO PRACTICE EXERCISES 10-16

Practice Exercise 10

1. (i) {2, 4, 6} (iii) {3, 4, 5, 6} (v) {3, 6}
   (ii) {1, 2} (iv) {2, 3, 5}

2.

Outcome	Snow	No Snow
Probability	0.55	0.45

3. (a) 0.262 (b) 0.738

4 Red = $\frac{1}{3}$; Blue = $\frac{1}{3}$; Yellow = $\frac{1}{6}$; Green = $\frac{1}{6}$

5 (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) 0

6. (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{11}{12}$ (e) 1

Practice Exercise 11

1. a) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
   b) {Red, Red, Red, White, White, Blue}
   c) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
   d) {Gold, Silver, Bronze}
   e) {Jacks, Queens, Kings}

2. (a) $\frac{1}{52}$ (b) $\frac{8}{52}$ or $\frac{2}{13}$ (c) $\frac{1}{4}$
   (d) $\frac{1}{2}$ (e) $\frac{1}{13}$ (f) $\frac{4}{13}$

3. (a) $\frac{3}{10}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$
   (d) 1 (e) $\frac{1}{10}$ (f) 0

4. (a) $\frac{1}{52}$ (b) $\frac{1}{2}$ (c) $\frac{1}{7}$ (d) $\frac{1}{2}$
   (e) $\frac{1}{n}$, where n is the number of students in your class

5. $\frac{6}{36}$ or $\frac{1}{6}$

Practice Exercise 12

1. (a) {1, 5, 6}
   (b) {1, 2, 4, 5}
   (c) {1, 4, 6}
   (d) {1, 2}
   (e) {4, 5, 6}

2. $\frac{3}{8}$ 3. $\frac{7}{8}$ 4. $\frac{12}{13}$ 5. 0.25

6. $\frac{43}{50}$ 7. $\frac{3}{5}$ 8. $\frac{9}{10}$

Practice Exercise 13

1. (a) Mutually Exclusive (b) Non-mutually Exclusive
   (c) Mutually Exclusive (d) Non-mutually Exclusive

2. (a) No (b) No (c) Yes (d) Yes

3. (a) $\frac{1}{3}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{5}{6}$ (f) $\frac{1}{3}$
   (g) $\frac{2}{3}$

4. (a) $\frac{2}{13}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{4}{13}$ (e) $\frac{5}{13}$

Practice Exercise 14

1. $\frac{1}{256}$

2. (a) $\frac{1}{49}$ (b) $\frac{2}{21}$

3. $\frac{2}{7}$ 4. 18% 5. $\frac{1}{221}$

Practice Exercise 15

1. (i) A = {(3, 1), (2, 2), (1, 3)}
   (ii) B = {(6, 5), (5, 6)}
   (iii) C = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}
   (iv) D = {(6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)}
   (v) E = {(6, 4), (5, 5), (4, 6), (6, 5), (5,6), (6, 6)}

(vi) F = {(1,1), (2, 1), (1, 2), (1, 3), (2, 2), (3, 1, (3,2), (2, 3), (1, 4), (4, 1)}

2. (i) {(3, 1), (2, 2), (1, 3), (6, 5), (5, 6)}
   (ii) {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (6, 4), (4, 6), (6, 5), (5, 6)}
   (iii) {(3, 1), (2, 2), (1, 3), (1,1), (2, 1), (1, 2), (3,2), (2, 3), (1, 4), (4, 1)}
   (iv) ϕ
   (v) {(2, 2)}
   (vi) {(4, 6), (5, 5), (6, 5), (5,6), (6, 6)}

3. (i) $\frac{5}{36}$ (ii) $\frac{10}{36}$ or $\frac{5}{18}$ (iii)) $\frac{10}{36}$ or $\frac{5}{18}$
   (iv) 0 (v) $\frac{1}{36}$ (vi) $\frac{3}{36}$ or $\frac{1}{12}$

4. (i) $\frac{1}{2}$ (ii) $\frac{7}{10}$

5. (i) $\frac{4}{9}$ (ii) $\frac{2}{9}$

6. (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$

Practice Exercise 16

1. (i) $\frac{1}{4}$ (ii) $\frac{1}{3}$ (iii) $\frac{3}{4}$ (iv) $\frac{2}{3}$

2. (i) $\frac{1}{3}$ (ii) $\frac{2}{15}$ (iii) $\frac{4}{15}$

3. (i) 0.24 (ii) 0.16 (iii) 0.36 (iv) 0.24

4. (i) $\frac{9}{25}$ (ii) $\frac{27}{125}$ (iii) $\frac{4}{25}$

5. (i) $\frac{1}{5}$ (ii) $\frac{1}{5}$ (iii) $\frac{1}{5}$

END OF TOPIC 3

TOPIC 4

STATISTICAL ESTIMATION

TOPIC 4: STATISTICAL ESTIMATION

Introduction

Statistic is a quantity that is calculated from a sample of data. It is used to give information about unknown values in the corresponding population. For example, the average of the data in a sample is used to give information about the overall average in the population from which that sample was drawn.

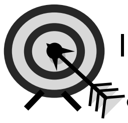
An estimate is an indication of the value of an unknown quantity based on observed data.

Suppose the manager of a shop wanted to know the mean expenditure of customers in her shop in the last year. She could calculate the average expenditure of the hundreds or thousands of customers who bought goods in her shop, that is, the population mean. Instead she could use an estimate of this population mean in calculating the mean of a representative sample of customers. If this value was found to be K25 then K25 would be her estimate.

In the lessons that follow, you will learn terms associated with statistical estimate, sampling techniques and survey.

Lesson 17: Population and Sample

Welcome to the first lesson of Topic 4.



In this lesson, you will:

- define the terms population, sample and parameter
- calculate mean of the population
- use the sample of a population to make an estimate

The study of statistics revolves around the study of data sets. This lesson describes two important types of data sets - **populations** and **samples.**

First let us define what population means.in stayistics.

When we think of the term "**population**," we usually think of people in our town, region, state or country and their respective characteristics such as gender, age, marital status, ethnic membership, religion and so forth.

In statistics the term "**population**" takes on a slightly different meaning.

The "*population*" in statistics includes all members of a defined group that we are studying or collecting information on for data driven decisions.

A population includes each element from the set of observations that can be made.

The term often refers to a group of people, as in the following examples.

- All registered voters in Kokopo Town
- All members of the Secondary Mathematics Club Teachers
- All Papua New Guineans who played Soccer at least once in the past years

But a population can refer to things as well as people:

- All Physics books sold last Monday by Oxford Book Supply
- All daily maximum temperatures in July for the Islands
- All basal ganglia cells from a particular rhesus monkey

Often when one wants to know things about a population; it is impractical for one to interview or collect data of all the members of the population. Instead, one might select a part or portion of the population.

A part of the population is called a **sample**. It is a proportion of the population, a slice of it, a part of it and all its characteristics. A sample is a scientifically drawn group that actually **possesses the same characteristics** as the population – if it is drawn randomly

A sample is a smaller group of members of a population selected to represent the population.

Example

A researcher, for instance, may want to determine the reading deficiencies of the grade school children in his school. However, he may not probably be able to test all the school children on account of their big numbers. He satisfies himself, therefore, in selecting representative sample from each of the grade levels through the random sampling method. He, then, administers the diagnostic test in reading and he makes use of the test results to report the reading deficiencies of the entire group of children in his school.

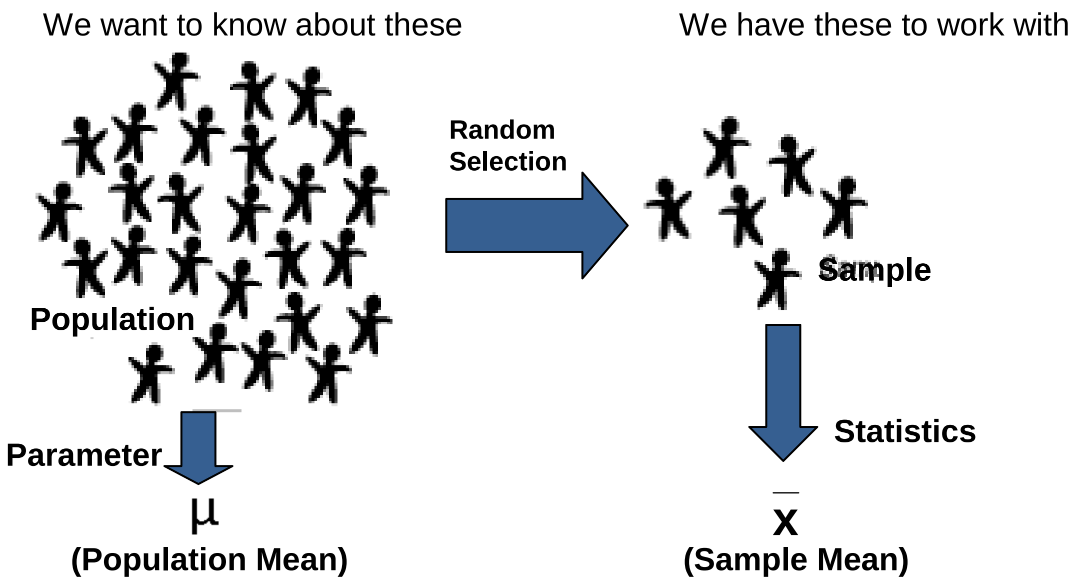
Imagine, for instance, that the population of the school was 8000 distributed among six grade levels. It would be very laborious for the researcher to involve all the children. He can make his work easier by drawing representative samples from each level, say 100 subjects per level. This means that he is going to test 600 children only. And yet he can report about the results as reflective of the reading deficiencies of the whole school population.

Parameter and Statistics

If the researcher, in the above example, had tested all the 8000 school children and computed measures of central tendency and variability he would have obtained what are referred to as **parameters.** These are measures of the population. Parameters exist whether we compute or not. The population parameters, **Mean** and **Standard Deviation** are symbolised by μ (read as mu) and σ (Greek sigma).

A **statistic** is a characteristic of a sample. Statistics enables you to make an educated guess about a population parameter based on a statistic computed from a sample random drawn from a population.

The diagram below illustrates the relationships between samples and populations.



Estimating the Population Mean

The population mean is the average of all the items in the population. Because a population is usually very large or unknown, the population mean is usually an unknown constant. An estimate of the population mean is **the sample mean**.

To calculate the population mean, we use the formula:

$$\mu = \frac{\sum x_i}{N} = \frac{x_1 + x_2 + x_3 + ... + x_n}{N}$$

Where: μ = Population Mean
$\sum X_i$ = sum of all the numbers in the grouping
N = number of population

The **sample mean** is represented by $\overline{X}$. It is given by the formula:

$$\overline{X} = \frac{\sum x_i}{N} = \frac{x_1 + x_2 + x_3 + ... + x_n}{N}$$

Where: $\overline{X}$ = the sample mean
$\sum X_i$ = sum of all the terms in the sample
N = is the number of terms in the sample.

The **Standard Deviation** is a measure of how spreads out numbers are. Its symbol is σ (the Greek letter sigma) or sometimes denoted by the capital letter **S**. It is calculated as the squared differences from the mean called **variance.**

In finding the standard deviation (**S**) of a population the formula used is:

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{N}} = \sqrt{\frac{\sum X^2}{N}}$$

Where: **S** = standard deviation
$\sum X^2$ = sum of the squared differences from the mean
N = total number of terms or scores

But when you use the sample as an estimate of the population, the standard deviation's formula changes to

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{N - 1}}$$

When you used this formula, you are actually correcting the sample standard deviation for bias. The sample standard deviation tends to be smaller than the population standard deviation.

Here are the steps for the calculation of the population standard deviation.

1. Calculate the mean.
2. For each number, subtract the mean. Square the result.
3. Add up all the squared results.
4. Divide the sum by the number of data points (**N**). This gives you the sample variance.
5. Take the square root of this value to obtain the sample standard deviation.

Example 1

Michelle has 20 potted roses. The number of flowers on each pot is as follows:

4	9	6	9	10	4	5	12	4	7
9	2	5	4	12	7	8	11	9	3

Work out the standard deviation.

Solution: The formula for the standard deviation of a population is

$$S = \sqrt{\frac{\sum(\mathbf{x} - \bar{\mathbf{x}})^2}{\mathbf{N}}}$$

Hence, to find the standard deviation, follow the following steps:

Step 1: Find the mean; $(\bar{X}) = \frac{\sum x_i}{N}$

$$\bar{\mathbf{X}} = \frac{4+9+6+9+10+4+5+12+4+7+9+2+5+4+12+7+8+11+9+3}{20}$$

$$= \frac{140}{20}$$

$$= 7$$

Step 2: Find $(x - \bar{X})^2$. Subtract the mean from each data points and square the results.

$(4 – 7)^2 = (-3)^2 = 9$
$(9 – 7)^2 = (2)^2 = 2$
$(6 – 7)^2 = (-1)^2 = 1$
$(9 – 7)^2 = (2)^2 = 4$
$(10 – 7)^2 = (3)^2 = 9$
$(4 – 7)^2 = (-3)^2 = 9$
$(5 – 7)^2 = (-2)^2 = 4$
$(12– 7)^2 = (5)^2 = 25$
$(4 – 7)^2 = (-3)^2 = 9$
$(7 – 7)^2 = (0)^2 = 0$
$(9 – 7)^2 = (2)^2 = 4$
$(2 – 7)^2 = (-5)^2 = 25$
$(5 – 7)^2 = (-2)^2 = 4$
$(4 – 7)^2 = (-3)^2 = 9$
$(12 – 7)^2 = (5)^2 = 25$
$(7 – 7)^2 = (0)^2 = 0$
$(8 – 7)^2 = (1)^2 = 1$
$(11 – 7)^2 = (4)^2 = 16$
$(9 – 7)^2 = (2)^2 = 4$
$(3 – 7)^2 = (-3)^2 = 9$

Step 3: Add up all the results.

$\sum (x - \bar{X})^2 = (9 + 2 + 1 + 4 + 9 + 9 + 4 + 25 + 9 + 0 + 4 + 25 + \ldots + 9)$
$=169$

Step 4: Calculate the mean of the squared differences or variance.

$$\text{Variance} = \frac{169}{20} = 8.45 = 8.5$$

Step 4: Take the square root of the variance to find the standard deviation.

$$S = \sqrt{8.5} = 2.907 = 2.9$$

Therefore, the standard deviation of the number of flowers Michelle has in all her 20 potted roses is 2.907 or 2.9.

Supposed Michelle only counted the flowers on 6 out of the 20 potted roses. The "**population**" is the 20 rose pots and the "**sample**" is the 6 she counted.

Let say, they were 5, 2, 12, 7, 9, 4

We can estimate the standard deviation. But when we use the sample as an estimate of the population, the standard deviation's formula changes to the one below

$$S = \sqrt{\frac{\sum(X - \overline{X})^2}{N - 1}}$$

Solution:

Step 1: Work out the mean. $(\overline{X}) = \frac{\sum x_i}{N}$

$$\overline{X} = \frac{5 + 2 + 12 + 7 + 9 + 4}{6}$$

$$= \frac{39}{6} = 6.5$$

Step 2: Calculate $\sum (x - \overline{x})^2$.

$\sum (x - \overline{x})^2 = (5 – 6.5)^2 + (2 – 6.5)^2 + (12 – 6.5)^2 + (7 – 6.5)^2 + (9 – 6.5)^2 + (4 – 6.5)^2$

$= (-1.5)^2 + (-4.5)^2 + (5.5)^2 + (0.5)^2 + (2.5)^2 + (-2.5)^2$

$= 2.25 + 20.25 + 30.25 + 0.25 + 6.25 + 6.25$

$= 65.5$

Step 3: Work out the mean of those squared differences or variance

$$\text{Variance} = \frac{\sum(X - \overline{X})^2}{N - 1} = \frac{65.5}{5} = 13.1$$

Step 4: Take the square root of the variance, to find the standard deviation.

$$S = \sqrt{\frac{\sum(X - \overline{X})^2}{N - 1}} = \sqrt{13.1} = 3.619 = 3.6$$

Therefore, the standard deviation for the number of flowers Michelle have is 3.6.

Comparing the two estimation of the standard deviation:

When we use the whole population, we get:

Population Mean = 7, standard deviation = 2.9

When we use the sample, we got:

Sample mean = 6.5; Standard Deviation = 3.6

Conclusion: The sample mean was wrong by 7%, and the sample standard deviation was wrong by 24%.

In calculating the mean of a population we need to consider the **standard error of the mean**.

Standard error of the mean is the standard deviation of those sample means over all possible samples (of a given size) drawn from the population.

The standard error of the mean can be viewed as the standard deviation of the error in the sample mean relative to the true mean, since the sample mean is an unbiased estimator.

It is usually estimated by the sample estimate of the population. Hence, the standard error of the mean is equals to sample standard deviation divided by the square root of the sample size. It is given by the formula:

$$SE_{\bar{x}} = \frac{S}{\sqrt{N}}$$

Where: $SE_{\bar{x}}$ = is the standard error of the mean

S = sample standard deviation

N = the size of the sample (number of observations)

It may be compared with the formula for a true standard deviation of the sample mean.

$$SD_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

Where: $SD_{\bar{x}}$ = is the true standard error of the mean

σ = standard deviation of the population

N = the size of the sample (number of observations)

In the case above, the standard error of the mean is

$$SE_{\bar{x}} = \frac{3.6}{\sqrt{6}} = \frac{3.6}{2.45} = 1.47$$

Comparing it to the true standard deviation of the sample mean, we have

$$\mathbf{SD}_{\bar{x}} = \frac{2.9}{\sqrt{20}} = \frac{2.9}{4.47} = 0.65$$

Based on the above comparisons, we can deduce that if the standard deviation is increased, the standard error of the mean is also increased. Conversely, increasing the sample size would decrease the standard error of the mean.

The normal curve rule states that the sampling distribution of the population mean will have almost all values between

$$\mu - 3 \times \frac{\sigma}{\sqrt{N}} \text{ and } \mu + 3 \times \frac{\sigma}{\sqrt{N}}$$

Therefore, the smallest and largest values of the population mean (μ) of the data Example 1 are:

$$7 - 3 \times 0.65 \text{ and } 7 + 3 \times 0.65$$

The estimated population mean is from 5.05 to 8.95.

Example 2

A test was given to a class of 50 students, and the test results are 10, 20, 30, 40 and 50.

(a) Calculate the mean.

(b) Calculate the sample standard deviation

(c) Find the standard error of the mean.

(d) Estimate the population mean.

Solution:

(a) Calculating the Mean:

$$(\bar{X}) = \frac{\sum x_i}{N}$$

$$\bar{X} = \frac{10 + 20 + 30 + 40 + 50}{5}$$

$$= \frac{150}{5}$$

$$\mathbf{= 30}$$

(b) Calculate the Sample Standard deviation.

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{N - 1}}$$

$$S = \sqrt{\frac{(10-30)^2 + (20-30)^2 + (30-30)^2 + (40-30)^2 + (50-30)^2}{4}}$$

$$S = \sqrt{\frac{(-20)^2 + (-10)^2 + (0)^2 + (10)^2 + (20)^2}{4}}$$

$$S = \sqrt{\frac{400 + 100 + 0 + 100 + 400}{4}}$$

$$S = \sqrt{\frac{1000}{4}}$$

$$S = \sqrt{250}$$

S = 15.81

(c) Standard Error of the mean: $\mathbf{SE_{\bar{x}} = \frac{S}{\sqrt{N}}}$

$$= \frac{15.81}{\sqrt{5}}$$

$$= \frac{15.81}{2.24}$$

= 7.07

(d) Estimated population mean:

Smallest value of the population mean is 30 – 3 x 7.07 =8.79
Largest value of the population mean is 30 + 3 x 7.07 = 51.21

Therefore, the estimated population mean is from 8.79 to 51.21.

Example 3

The Accountant wishes to obtain some information about all the invoices sent out to a supermarket's account customers. In order to estimate this information, a sample of twenty invoices is randomly selected from the whole population.

Value of Invoices (in Kina)

32.53	22.27	33.38	41.47	38.05
31.47	38.00	43.16	29.05	22.20
25.27	26.78	30.97	38.07	38.06
25. 11	24.11	43.48	32.93	42.04

(i) Find the sample mean.

(ii) Find the sample standard deviation.

(iii) Estimate the population mean.

Solutions:

(i) Sample Mean: $(\overline{X}) = \frac{\sum x_i}{N}$

$$= \frac{(32.53 + 31.47 + 25.27 + ... + 42.04)}{20}$$

$$= \frac{658.4}{20}$$

= K32.92

(ii) Sample Standard Deviation: $S = \sqrt{\frac{\sum(X - \overline{X})^2}{N - 1}}$

$$S = \sqrt{\frac{962.1344}{19}}$$

$$S = \sqrt{50.64}$$

S = K7.12

(iii) To find the estimated population mean, first compute the value of error of the mean

$$\text{Error of the Mean} = \frac{S}{\sqrt{N}} = \frac{7.12}{\sqrt{20}}$$

= 1.59

The smallest and largest values of the population mean (μ) are

K32.92 – 3 x1.59 and K32.92 + 3 x 1.59

Therefore, the estimated population mean is from K 28.15 to K37.49.

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

1. A chemist grows 20 crystals from a solution and measured the length of each crystal in millimetres.

   Here are the results.

10	3	6	5	13
8	12	10	4	9
8	5	13	6	5
11	10	7	10	5

   (a) Calculate the mean.

   (b) Calculate the standard deviation of the population size.

   (c) If this was the sample population, find the sample standard deviation of the data.

2. At the end of every school year, a province administers a reading test to a simple sample drawn without replacement, from a population of 20, 000 third graders. This, test was administered to 36 students selected via simple random sampling. The test score from each sampled students is shown below:

50	55	60	62	62	65	67	67	70
70	70	71	72	72	73	73	75	75
75	78	78	78	78	80	80	80	82
82	85	85	85	88	88	90	90	90

(i) Find the percentage of the population who got 75 and above.

(ii) Find the modal reading score of the population.

(iii) Estimate the mean reading score of the population.

---

3. A large hospital wants to estimate the average length of time that surgical patients remain in the hospital. To accomplish this, a random sample of 100 patient records is obtained from the records of all patients who have had surgery in recent years. For these 100 patients, the sample mean is found to be 7.8 days. The standard deviation of hospital stay for all surgical patients is 9.4 days.

Estimate the population mean length of stay, by forming an interval that almost certainly contains the value of the mean, μ.

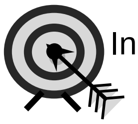
4. Suppose that the amount of time teenagers spend weekly at part-time jobs is normally distributed with a standard deviation of 20 minutes. A random sample of 100 observations is drawn, and the sample mean is computed as 125 minutes.

   Estimate the population mean by forming an interval that certainly contains the value of the mean.

CORRECT YOUR WORK, ANSWER ARE AT THE END OF TOPIC 4

Lesson 18: Sampling Methods

You learnt the meaning of population, sample and parameter in the last lesson. You also learnt how to estimate the mean of a population.



In this lesson, you will:

- define and identify the different sampling methods
- estimate the population mean from random sampling and use it to make conclusions.

Let us begin by covering a brief introduction to sampling. Usually researchers cannot make direct observations of every individual in the population they are studying. Instead, they collect data from a subset of individuals (a **sample**) and use those observations to make inferences about the entire population.

Sampling is a process of selecting units (e.g. people, organizations) from a population of interest so that by studying the sample we may generalize our results back to the population from which they were chosen.

The way that observations are selected from a population to be in the sample for a sample survey refers to **sampling method**.

The sampling process comprises several stages:

(i) Defining the population of concern

(ii) Specifying a sampling frame and a set of items or events that are possible to measure

(iii) Specifying a sampling method for selecting items or events from the frame

(iv) Finding the sample size

(v) Implementing the sampling plan

(vi) Sampling and data collecting

As a group, sampling methods fall into two categories. These are:

1. Probability Sampling Methods
2. Non – Probability Sampling Methods

- **Probability Sampling is a method wherein all elements of the population have a known (non-zero) chance of being chosen for the sample.**
- **Non-Probability Sampling is a method wherein we do not know the probability that each element of the population will be chosen, and/or we cannot be sure that each element has a non-zero chance of being chosen.**

PROBABILITY SAMPLING

The main types of Probability Sampling Methods are: Simple Random Sampling, Stratified Sampling, Cluster Sampling, Multistage Sampling and Systematic Random Sampling.

1. **Simple Random Sampling**

Simple random sampling refers to any sampling method that has the following properties:

- The population consists of N objects.
- The sample consists of n objects.
- If all possible samples of n objects are equally likely to occur, the sampling method is simple random sampling.

There are many ways to obtain a simple random sample. One way would be the **lottery method**. Each of the unit (N) of the target population is assigned a unique number. The numbers are placed in a bowl and thoroughly mixed. Then a blind folded researcher selects the numbers (n). Population members having the selected numbers are included in the sample.

For example, let's say you have a population of 1,000 people and you wish to choose a simple random sample of 50 people.

First, each person is numbered 1 through 1,000. Then, you generate a list of 50 random numbers (typically with a computer program) and those individuals assigned those numbers are the ones you include in the sample.

2. Stratified Sampling

With stratified sampling, the population is divided into groups, based on some characteristic. Then, within each group, a probability sample (often a simple random sample) is selected.

When the population is composed of several subgroups, it may be wiser to employ the stratified technique to ensure representation of each subgroup in the sample. In stratified sampling, the groups are called **strata**.

Example: Suppose you conduct a national survey. We might divide the population into groups or strata, based on geography—North, East, West and South. Then, within each stratum, we might randomly select sample survey respondents.

There are two ways of employing this technique.

2.1 **Simple Stratified Random Sampling**

This is done by separating the lists of subgroups in the population and simply drawing randomly the desired sample size from each subgroup. In drawing the subject, any of the simple random sampling techniques may be used.

Look at the example on the next page.

Population		**Sample**
N = 800		n = 200
Grade 10	185	50 Ss
Grade 9	200	50 Ss
Grade 8	215	50 Ss
Grade 7	200	50 Ss
Total	800	200 Ss

2.2 Stratified Proportional Random Sampling

In some cases, the characteristic of the population is such that the proportions of the subgroups are grossly unequal. The researcher may wish to maintain this characteristic in the sample. Therefore, the stratified Proportional technique may be used.

Study the example below.

Population N = 800		Proportion	**Sample** n = 200
Grade 10	120	0.15	30 Ss
Grade 9	200	0.25	50 Ss
Grade	220	0.275	55 Ss
Grade 7	260	0.325	65 Ss
Total	800	1.000	200Ss

The proportion of each stratum or subgroup is computed by dividing the number of members in each stratum or subgroup by the total population. The result is multiplied by the number of sample desired, thus $\frac{120}{800}$x 200 for the first stratum or subgroup is equal to 30.

3. Cluster Sampling

With Cluster Sampling, every member of the population is assigned to one, and only one group. Each group is called **cluster**. A sample of cluster is chosen, using a probability method (often simple random sampling). Only individuals within sampled clusters are included in the survey.

Cluster sampling may be used when it is either impossible or impractical to compile a complete list of the elements that make up the target population. Usually, however, the population elements are already grouped into subpopulations and lists of those subpopulations already exist or can be created.

For example, let’s say the target population in a study was church members in Papua New Guinea. There is no list of all church members in the country. The researcher could, however, create a list of churches in Papua New Guinea, choose a sample of churches, and then obtain lists of members from those churches.

4. Multistage Sampling

With multistage sampling, we select sample bu using combinations of the different sampling methods.

For example, in Stage 1, we might use cluster sampling to choose clusters from a population. Then, in stage 2, we might use a simple random sampling to select a subset of elements from each cluster for the final stage.

5. Systematic Random Sampling

With systematic random sampling, we create a list of every member of the population. From the list, we randomly select the first sample element from the first k^{th} elements on the population list. Thereafter, we select systematically every k^{th} element on the list for inclusion in the sample.

For example, if the population of study contains 2000 students at a high school and the researcher wanted a sample of 100 students, the students would be put into a list form and then every 20^{th} student would be selected for inclusion in the sample. To ensure against any human bias in this method, the researcher should select the first individual at random. This is technically called a systematic sampling with a random start.

This method is different from simple random sampling method since every possible sample of **(n)** elements is not equally likely.

NON- PROBABILITY SAMPLING

Non-Probability Sampling Method offers two potential advantages: convenience and cost. The main disadvantage is that non-probability sampling methods do not allow you to estimate the extent to which sample statistics are likely to differ from population parameters. Only probability sampling methods permit that kind of analysis. There are two main types of non-probability sampling methods.

1. Voluntary Sample
2. Convenience Sample

Voluntary Sample is made up of people who self-elect into the survey. Often folks have a strong interest in the main topic of the survey.

Example, a TV Talent show asks viewers to participate in an on-line poll. This would be a volunteer sample. The sample is chosen by the viewers, not only by the survey administrator.

Convenience Sample is made up of people who are easy to reach.

Example, a researcher interviews shoppers at a local supermarket. If the supermarket was chosen because it was a convenient site from which to solicit survey participants and/or because it was close to the researcher's home or business, this would be a convenience sample.

An important benefit of simple random sampling is that it allows researchers to use statistical methods to analyze sample results. For example, we can estimate the extent to which a sample statistic is likely to differ from a population parameter.

Let examine the example below:

At the end of every school year, the province administers a numerical test to a sample drawn without replacement, from a population of 1000 grade 9s. The test was administered to 20 students selected. The test score from each sampled students is shown below:

No.	Score	No.	Score	No.	Score	No.	Score	No.	Score
73	48	49	50	06	58	45	62	57	62
56	69	78	72	50	72	47	72	41	72
75	73	77	76	79	62	66	78	80	78
40	84	65	82	87	78	27	85	26	88

(i) What do you think is the kind of sampling technique used? Explain.

(ii) Find the sample mean

(iii) Determine the sample standard deviation.

(iv) Estimate the population mean.

Solution:

(i) Since the scores were randomly selected as given by the number, we can conclude that the sample was selected by random sampling.

(ii) Mean: $(\overline{X}) = \frac{\sum x_i}{N}$

$$= \frac{(48+69+73+84+50+72+76+82+\ldots+88)}{20}$$

$$= \frac{1421}{20}$$

$$= 71.05$$

(iii) Standard Deviation: $S = \sqrt{\frac{\sum(X-\overline{X})^2}{N-1}}$

$$= \sqrt{\frac{(48-71.05)^2+(69-71.05)^2+\ldots+(88-71.05)^2}{19}}$$

$$= \sqrt{\frac{2410.9725}{19}}$$

$$= \sqrt{126.90}$$

$$= 11.26$$

(iv) Error of the Mean = $\frac{11.26}{\sqrt{20}} = 2.52$

The smallest and largest values of the population mean μ are:

$71.05 - 3 \times 2.52$ and $71.05 + 3 \times 2.52$

$71.05 - 7.56$ and $71.05 + 7.56$

63.49 and 78.61

Therefore, the estimated population mean is from 63.49 to 78.61

Example 2

At the end of the year, a state administers a summative test to 36 Grade 9 students taken as simple random sample drawn without replacement from a population of 20,000 students. The test score from each sampled student is shown below.

50	55	60	62	62	65	67	67	70
70	70	70	72	72	73	73	75	75
75	78	78	78	78	80	80	80	82
82	85	85	85	88	88	90	90	90

(a) Find the sample mean

(b) Determine the sample standard deviation.

(c) Estimate the population mean.

Solution:

(a) Mean: $(\overline{X}) = \frac{\sum x_i}{N}$

$= \frac{(50 + 55 + 60 + ... + 90 + 90)}{36}$

$= 75$

(b) Standard Deviation: $S = \sqrt{\frac{\sum (X - \overline{X})^2}{N - 1}}$

$= \sqrt{\frac{(50 - 75)^2 + (55 - 75)^2 + ... + (90 - 75)^2}{35}}$

$= \sqrt{\frac{3528}{35}}$

$= \sqrt{100.8}$

$= 10.03$

(d) Error of the mean: $\frac{S}{\sqrt{N}} = \frac{10.03}{\sqrt{36}}$

$= \frac{10.03}{6}$

$= 1.67$

(e) The smallest and largest values of the population mean μ are:

$75 - 3 \times 1.67$ and $75 + 3 \times 1.67$

$75 - 5.01$ and $75 + 5.01$

69.9 and 80.01

Therefore, the estimated population mean is from 69.9 to 80.01

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Examine each of the following methods used in obtaining sample. Identify the type of sampling technique used in each case.

(a) Taking every tenth name for every fifth name.

(b) The relevant characteristics to be used are identified on the basis of the questions to be asked

e.g. membership or non-membership of an organisation, female or male members.

A random list is then drawn up for each subgroup and respondents chosen randomly within each

(c) The interviewer follows specific random instructions e.g. take the first road right, interview at the second house on your left, continue down the road, interview tenth household on your right etc. and interviews individuals as they are encountered.

(d) Take the list or map and give each unit a number, write the numbers on individual slips of paper, put them in a bag and mix the slips up thoroughly, and then draw out the number of slips required. Alternatively a random number table can be used. If no suitable list or map exists, it may be possible to use participatory methods to solve this problem.

(e) Group maybe geographical, for example villages or markets. They may also be for example microfinance groups or particular social categories within geographical locations e.g. all upper caste households.

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2. Shown below is the table of 20 samples of the number of minutes it took the Grade 9 to study Mathematics in a day.

No.	Score	No.	Score	No.	Score	No.	Score	No.	Score
5	45	25	123	45	34	65	33	85	44
10	50	30	100	50	56	70	30	90	77
15	34	35	80	55	78	75	56	95	38
20	67	40	56	60	45	80	112	100	45

(i) What is the probable sample technique used?

(ii) Estimate the population mean.

(iii) From the population mean, is the number of minutes to study enough in studying Mathematics?

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3. The 30 samples is taken from 1000 students in a school showing the scores in Mathematics 10 out of 30 marks

Takubar		Kokopo Town		Bitapaka		Palnakaur		Bitagalip	
No.	Score	No.	Score	No.	Score	No.	Score	No.	Score
29	23	34	24	07	23	23	23	12	19
45	21	112	28	78	17	45	12	51	23
134	19	47	22	44	15	78	23	97	12
56	14	90	20	23	10	122	14	45	11
78	10	56	18	17	09	24	15	23	14
12	17	12	29	04	12	21	16	18	22

(i) What is the type of sampling technique that was used?

(ii) Find the sample mean.

(iii) Find the sample population.

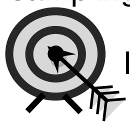
(iv) Estimate the population mean.

(v) Which place has the mean in the range of population mean? Which places did better?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: Survey

You learnt the different methods of sampling in the last lesson and how to estimate the population mean from the random sampling and used it to make conclusions.



In this lesson you will:

- define survey
- identify methods of survey
- carry out a simple survey
- present a report on the survey

To derive conclusions from data, we need to know how the data were collected; that is, we need to know the method(s) of data collection. Aside from experiments, we can use survey to collect data.

A **survey** is a method of collecting information about a human population. Pinsonneault and Kraemer (1993) defined a survey as a "means for gathering information about the characteristics, actions, or opinions of a large group of people"

There are various types of surveys that you can choose from. Basically, the types of surveys are broadly categorized into two: **according to instrumentation** and **according to the span of time involved**.

According to Instrumentation

In a survey, direct (or indirect) contact is made with the units of the study (e.g., individuals, organizations, communities) by using systematic methods of measurement such as **questionnaires** and **interviews**.

Questionnaire is a series of written questions a researcher supplies to subjects, requesting for response.

Usually the questionnaire is self-administered in that it is posted to the subjects, asking them to complete it and post it back.

The usual questions found in questionnaires are **closed-ended questions**, which are followed by response options. However, there are questionnaires that ask **open-ended questions** to explore the answers of the respondents.

Today, questionnaires are utilized in various survey methods, according to how they are given. These methods include the **self-administered**, the **group-administered,** and the **household drop-off**.

Among the three, the self-administered survey method is often used by researchers nowadays. The **self-administered questionnaires** are widely known as the **mail survey method**.

However, since the response rates related to mail surveys had gone low, questionnaires are now commonly administered online, as in the form of web surveys.

Advantages: Ideal for asking closed-ended questions; effective for market or consumer research

Disadvantages: Limit the researcher's understanding of the respondent's answers; requires budget for reproduction of survey questionnaires

Interview is a series of questions a researcher addresses personally to respondents.

An interview includes two persons - the researcher as the **interviewer**, and the respondent as the *interviewee*.

An interview may be **structured** (where you ask clearly defined questions) or **unstructured** (where you allow some of your questioning to be led by the responses of the interviewee.

There are several survey methods that utilize interviews. These are the personal or face-to-face interview, the phone interview, and more recently, the online interview.

Especially when using unstructured interviews, using tape recorder can be a good idea, if it does not affect the relationship with the person being interviewed.

Advantages: Follow-up questions can be asked; provide better understanding of the answers of the respondents

Disadvantages: Time-consuming; many target respondents have no public-listed phone numbers or no telephones at all studies, panel studies and trend studies.

Between the two broad types of surveys, interviews are more personal and probing. Questionnaires do not provide the freedom to ask follow-up questions to explore the answers of the respondents, but interviews do.

According to the Span of Time Involved

The span of time needed to complete the survey brings us to the two different types of surveys: **cross-sectional and longitudinal surveys.**

Cross-Sectional Survey is a survey where in data or information are collected at a single period of time. It usually utilise questionnaires to ask a particular topic at one point in time.

For instance, a researcher conducted a cross-sectional survey asking teenagers' views on cigarette smoking as of May 2010.

Sometimes, cross-sectional surveys are used to identify the relationship between two variables, as in a comparative study. An example of this is administering a cross-sectional survey about the relationship of peer pressure and cigarette smoking among teenagers as of May 2010.

Longitudinal Surveys

Longitudinal survey is a survey wherein the researcher attempts to collect or gather data or information over a period of time or from one point in time up to another.

The aim of longitudinal surveys is to collect data and examine the changes in the data gathered. Longitudinal surveys are used in cohort studies, panel studies and trend studies.

The sample questionnaire that follows is for illustrative purposes only.

Thank you for taking the time to complete the following Unit Evaluation. The information you provide will be used to help us improve the content of the Unit and monitor the quality of our program.

1. Which of the following best describes your reason for taking the module? (Please put a tick beside your choice)

_____ Upgrade my marks
_____ As a refresher's course
_____ Force to take up the subject
______Reinforcement purpose

1. Was the difficulty of the module in line with your expectations?

_____ Yes
_____ No
_____ Maybe

3. Would you recommend this unit to other students?

_____Yes
_____No

4. If not, why?

5. What is your gender?

_____ Male
_____ Female

6. What area of the module you would recommend for improvement ?

_____ Lesson Outcome
_____ Definitions and Main Concepts
_____ Examples
_____ Exercises

Thank You

Suppose after conducting the survey, you collate the data, the table below is just an example of data collected.

Question No.	First	Second	Third	Fourth
1	14	3	2	1
2	12	2	6	
3	18	2		
4	Needs for financial assistance ; difficult			
5	17	3		
6	1	2	6	11

A report can be made in that simple survey:

From the 20 students surveyed, 85% are male and 15% are female. The main reason for taking the course or doing the module is upgrading the marks, that is, 70% of the respondents. For them, the module, pass their expectations and would recommend to others. However, there is a need to improve on the exercises.

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. Make use of the illustrative example questionnaire and conduct a survey. Make a report on the outcome of the survey including the presentation of the result. If there is a need to include some more questions, please do so.

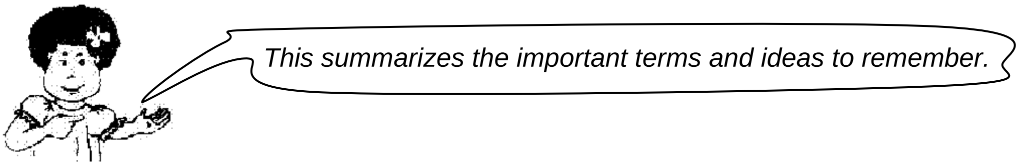
2. Make a questionnaire on the effect of shortage or inadequacy of each of the following in your school:

   (a) instructional materials (e.g. textbooks)
   (b) budget for supplies (e.g. paper, pencils)
   (c) school buildings and grounds
   (d) lighting system
   (e) classrooms
   (f) computers

Choices would be **none**, **a little**, **some**, **a lot**.

TO CORRECT YOUR WORK. PLEASE SEE YOUR DISTANCE TEACHER.

TOPIC 3: SUMMARY



- A **population** is a collection of data whose properties are analysed. The population is the complete collection to be studied. It contains all subjects of interest.
- A **sample** is a part of the population of interest, a sub-collection selected from a population.
- A **parameter** is a numerical measurement that describes a characteristic of a population, while a **sample** is a numerical measurement that describes a characteristic of a sample. In general, we will use a statistic to infer something about a parameter.
- The **population mean** is the average of all the items in the population.
- The **Standard Deviation** is a measure of how spreads out numbers are. Its symbol is σ (the Greek letter sigma) or sometimes denoted by the capital letter **S**. It is calculated as the squared differences from the mean called **variance.**
- **Standard error of the mean** is the standard deviation of those sample means over all possible samples (of a given size) drawn from the population.
- A **survey** is a method of collecting information about a human population.
- **Probability Sampling** is a method wherein all elements of the population have a known (non-zero) chance of being chosen for the sample.
- Simple random sampling refers to any sampling method that has the following properties:
  1. The population consists of N objects.
  2. The sample consists of n objects.
  3. If all possible samples of n objects are equally likely to occur, the sampling method is simple random sampling.
- **Non-Probability Sampling** is a method wherein we do not know the probability that each element of the population will be chosen, and/or we cannot be sure that each element has a non-zero chance of being chosen.
- **Questionnaire** is a series of written questions a researcher supplies to subjects, requesting for response.
- **Interview** is a series of questions a researcher addresses personally to respondents.
- **Cross-Sectional Survey** is a survey where in data or information are collected at a single period of time. It usually utilise questionnaires to ask a particular topic at one point in time.
- **Longitudinal Survey** is a survey wherein the researcher attempts to collect or gather data or information over a period of time or from one point in time up to another.

REVISE LESSONS 17-19. THEN DO TOPIC TEST 4 IN ASSIGNMENT 4

ANSWERS TO PRACTICE EXERCISES 17- 19

Practice Exercise 17

(1) (i) Mean = $\frac{160}{20}$ = 8

(ii) SD of the population = $\sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{N}} = \sqrt{\frac{178}{20}} = \sqrt{8.9} = 2.9$

(iii) Sample Standard deviation = $\sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{N-1}} = \sqrt{\frac{178}{19}} = \sqrt{9.3} = 3.06$

(2) (i) $55\frac{5}{9}\%$ or 55.56%

(ii) 78

(iii) Sample Mean = 75.03

(3) Sample Mean = 7.8 days
Standard Deviation = 9.4 days
Population Mean = 4.98 days to 10.62 days

(4) Sample Mean = 125 minutes
Standard Deviation = 20 minutes
Population Mean = 119 days to 131 days

Practice Exercise 18

1) (a) Systematic Sampling
(b) Stratified Sampling
(c) Reliance On Available Subjects
(d) Random Sampling
(e) Cluster Sampling

2. (i) Systematic Sampling
(ii) Sample Mean = 60.15
Sample Standard Deviation = 27.03
Population Mean = 42.03 to 78.27
(iii) Not Enough

3. (i) Stratified Sampling
   (ii) Sample Mean = 17.83
   Sample Standard Deviation = 5.47
   Population Mean = 15.1 to 20.56
   (iii) All except Takubar and Bitagalip ; Kokopo did better

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Practice Exercise 19

See you distance teacher or tutor.

END OF TOPIC 4

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