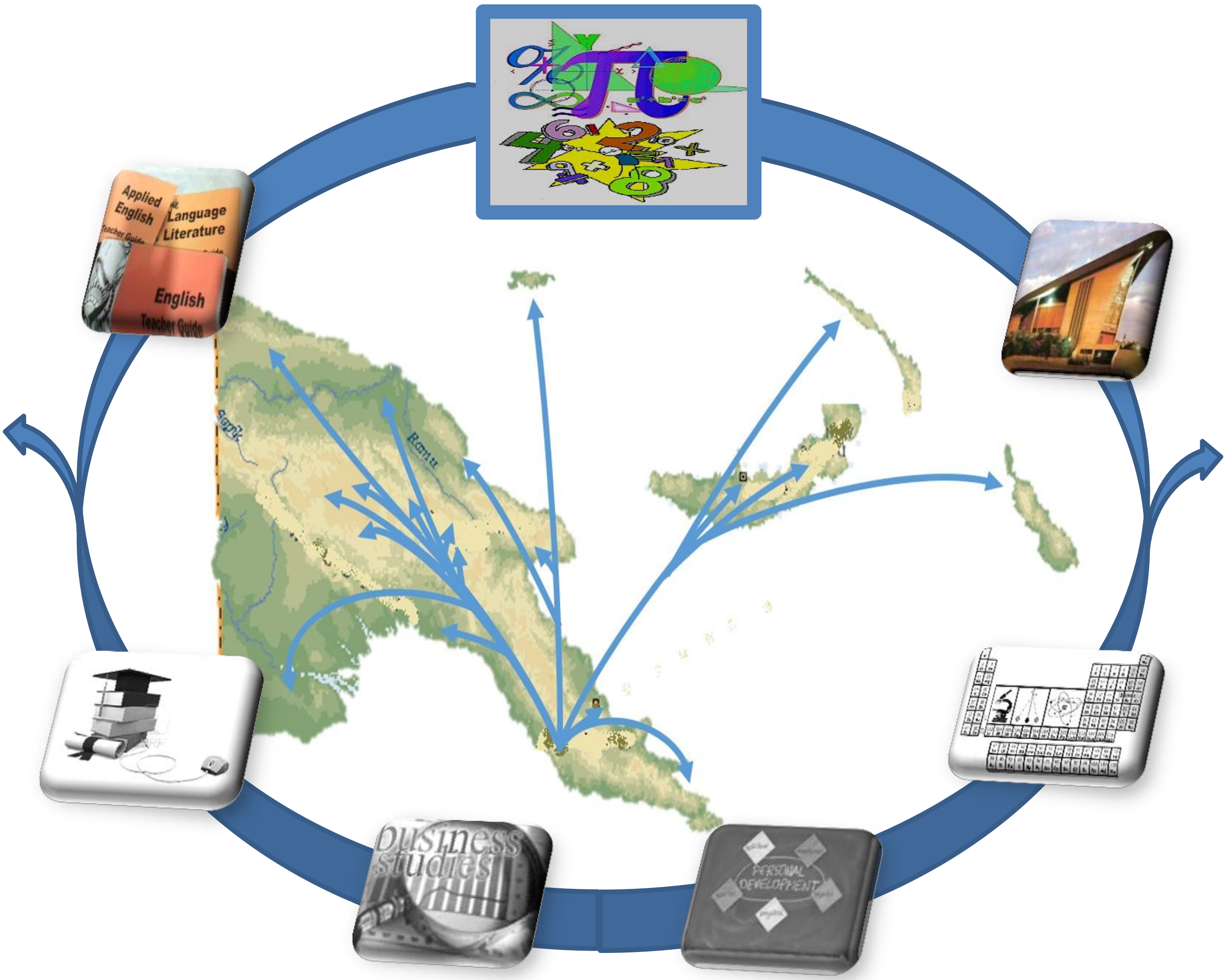
DEPARTMENT OF EDUCATION

GRADE 11 ADVANCE MATHEMATICS

11.1: NUMBER AND APPLICATION



FODE DISTANCE LEARNING

PUBLISHED BY FLEXIBLE OPEN AND DISTANCE EDUCATION
FOR THE DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA
2017

GRADE 11

ADVANCE MATHEMATICS

MODULE 1

NUMBER AND APPLICATION

ACKNOWLEDGEMENT

We acknowledge the contributions of all Secondary Teachers who in one way or another have helped to develop this Course.

Our profound gratitude goes to the former Principal of FODE, Mr. Demas Tongogo for leading FODE team towards this great achievement. Special thanks to the Staff of the English Department of FODE who played an active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers of Central Province and NCD.

We also acknowledge the professional guidance provided by Curriculum and Development Assessment Division throughout the processes of writing, and the services given by member of the English Review and Academic Committees.

The development of this book was Co-funded by GoPNG and World Bank.

DIANA TEIT AKIS

PRINCIPAL

Flexible Open and Distance Education

Papua New Guinea

Published in 2017

ISBN 978-9980-89-329-1

National Library Services of Papua New Guinea

Compiled and finalised by: Mathematics Department-FODE

Printed by the Flexible, Open and Distance Education

CONTENTS

SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is a part of the new Flexible, Open and Distance Education curriculum. The learning outcomes are student-centred and allows for them to be demonstrated and assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution, Government Policies and Reports. It is developed in line with the National Education Plan (2005 -2014) and addresses an increase in the number of school leavers affected by the lack of access into secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education's Mission which is fivefold:

- To facilitate and promote the integral development of every individual
- To develop and encourage an education system satisfies the requirements of Papua New Guinea and its people
- To establish, preserve and improve standards of education throughout Papua New Guinea
- To make the benefits of such education available as widely as possible to all of the people
- To make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced to provide alternative and comparable pathways for students and adults to complete their education through a one system, many pathways and same outcomes.

It is our vision that Papua New Guineans' harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers, university lecturers and many others who have contributed in developing this course.

UKE KOMBRA, PhD

Secretary for Education

UNIT INTRODUCTION

"It is simply inexcusable for anyone to say 'I can't do maths.'"

Chris Humphries, Chairman of National Numeracy

This Unit focuses on the mathematics we use every day in our communities to measure, compare and present information numerically. It covers the development of real number system and their application in real life. The content develops from the "Number and Application" strand.

11.1.1: BASIC NUMERACY

This topic discusses real numbers in a complex number field; recurring and finite decimals, significant figures and estimation and error. It expands further to discussing standard numbers; indices and logarithms; and scientific calculator.

11.1.2: UNITS OF MEASUREMENT

This topic connects metric and imperial units of measurement through conversion; it further explains devices used and scales of devices.

11.1.3: RATIO AND PROPORTION

The topic defines ratio and proportion and explains difference in ratio and proportion. It explains forms of proportion and their application and discusses direct and indirect proportion and their applications.

11.1.4: BASIC ALGEBRA

The topic discusses factorizing, expanding and simplifying algebraic expressions. Expressions include algebraic fractions. It also discusses methods of quadratic solutions and plotting and sketching quadratic graphs. Then it expands to inequalities and systems of inequalities.

The summary gives the key points of each unit covered. There is no glossary provided since new words or key words are defined within the text.

Student Learning Outcomes

On successful completion of this module, students will be able to:

- discuss the historical development of real numbers
- classify and relate symbols to all real numbers
- plot real numbers on the real number line
- apply the properties of real numbers
- state the laws or properties of surds.
- apply properties of surds
- simplify and rationalize surds
- state the number of significant figures
- round off significant figures
- apply the laws of indices
- express index numbers in surds form
- write metric measurements of length, mass and capacity
- convert metric measurement to imperial or vice versa
- apply scales on actual lengths on the ground
- communicate mathematical processes and results verbally
- measure and use appropriate techniques and instruments to estimate and calculate physical quantities
- apply knowledge of numbers and their relationships to investigate a range of different context
- plan, organize and carry out mathematical activities individually and cooperatively

Time Frame

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Solve all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular question right in the first attempt, you should not get discouraged but instead, go back and attempt it again.

If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not pass any question without solving it first.

11.1.1 BASIC NUMERACY

Basic Numeracy skills count. It is not just for teachers, scientists, accountants and engineers, many professions require at least basic level of understanding when it comes to numeracy.

The numbers and their symbols that we use today have advanced over many years ago. People began to use mathematics in their lives when they first started to use the numbers in counting objects. Later on, they became farmers and builders and the system and the way of writing numbers became more sophisticated. Its adaptation in the civilization developed trade, science, arts, ownership, structures, insurance, technology, sports and sense of time.

11.1.1.1 Real Numbers

History of Real Numbers

The natural numbers were developed or understood by Greek mathematicians. The Indian and Chinese mathematicians developed the science of the acceptance of zero, negative, real, integral and fractional numbers. Due to this development and the development of Algebra, the irrational numbers are treated as algebraic objects by Arabic mathematicians in the middle ages. Abu Kamil Shuja ibn Aslam an Egyptian mathematician first considered irrational numbers as solution of quadratic equations or coefficients to square roots and cube roots in an equation and so on. Around 500 BC, Greek mathematicians realized the need for irrational numbers particularly the square root of two. In this way the **real number system** was developed.

Real Numbers

All numbers on the number line are known as real numbers. It includes positives, negatives, integers, rational numbers, whole numbers, square roots, cube roots, π, etc. A real number is a value that represents a quantity along a continuous line (called the number line) and is denoted by $\mathbf{R}$. Because they lie on a number line, their size can be compared. You can say one is greater or less than another, and do arithmetic with them.

Types of Real Numbers ($\mathbf{R}$)

- **Natural numbers (Counting numbers):**
  All the numbers starting from '1' to 'infinity' are natural numbers. The set of numbers 1, 2, 3, 4, 5, … is called the set of natural numbers. They are also known as counting numbers or positive integers and is denoted by $\mathbf{N}$.

- **Whole numbers ($\mathbf{W}$)**
  The need of representing "nothing" was also realized and so the number zero was added to the set of counting numbers forming the set of whole numbers. Whole numbers include zero and the set of natural numbers. The set will be like {0, 1, 2, 3, 4, 5, ...} and is denoted by $\mathbf{W}$.

- **Integers (J)**
  Later on, the need of representing something "owed", such as money borrowed or temperature "below zero" led to the development of the negative numbers. The negative numbers are created by putting a negative sign before each natural number. Integers are the set of negative numbers and whole numbers. An integer is a number that has no fractional part, and no digits after the decimal point. An integer can be positive, negative or zero. It is denoted by **J**. The set will be like {..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...}

- **Fractions and decimals**
  The concept of "part of a whole" led to the development of fractions. A fraction is two quantities written one above the other that shows how much of a whole thing we have. Later on, these fractions have their decimal representations.

Rational Numbers

For many centuries whole numbers and fractions were thought to be the only types of numbers that we would ever have to deal with. They were collectively called the set of **rational numbers.**

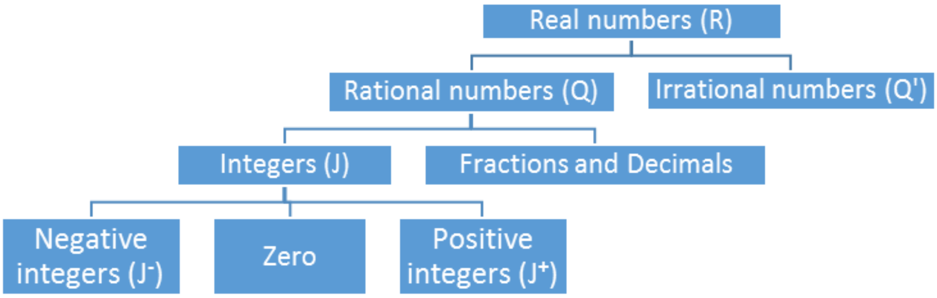
Rational numbers:

- can be written as a **ratio** of two integers
- called **ratio**nal numbers because the first five letters spell the word 'ratio'.
- can be expressed in the form Q = $\frac{a}{b}$, b ≠ 0; where, a and b are integers.
- can be positive or negative.
- can have a zero as numerator, e.g $\frac{0}{6}$ since this number is equal to zero.
- cannot have a zero as denominator , e.g $\frac{6}{0}$ since this number is **undefined**.

Irrational numbers (I)

It became clear about 2,500 years ago that there existed another type of number that could not be written as a ratio. This was in the time of the Greek philosopher and mathematician Pythagoras (580-500 BC) who belonged to a powerful brotherhood whose motto was "All is number". They believed that all things in the universe could be explained in terms of rational numbers. However, using a right-angled triangle and Pythagoras' rule they discovered and proved that there existed another type of number that could not be written as a ratio. These were numbers such as $\sqrt{2}$, $\sqrt{7.69}$ and π which appear as **infinite, non-recurring decimals.** Today these numbers are called **irrational numbers.** Pythagoras's brotherhood kept the existence of these irrational numbers secret for about 400 years as it ruined their "All is number" theory.

Today, the set of **rational numbers** together with the set of **irrational numbers** make up the set of **real numbers.** The set of real numbers is displayed on the following diagram. The capital letter next to the number headings represents commonly used letters for that set of numbers.



Showing that numbers are rational by changing them into a ratio of integers

i. **Whole numbers** and **integers** can be expressed as a ratio by using the whole number in the numerator and 1 in the denominator.

For example:

The number 8 can be written as $\frac{8}{1}$ which is a ratio of the whole number 8 and 1.

The integer –2 can be written as the ratio $\frac{-2}{1}$.

ii. **Proper fractions** and **improper fractions** are also given as a ratio; it is usual to give these in their simplest form.

For example:

The proper fraction $\frac{3}{5}$ is the ratio of 3 and 5.

The improper fraction $\frac{14}{6}$ is a ratio but should be written as $\frac{7}{3}$ since it is the simplest form of $\frac{14}{6}$.

iii. **Mixed numbers** need to be changed to improper fractions.

For example:

$1\frac{3}{5}$ can be written as $\frac{8}{5}$; the numerator of the improper fraction is found by multiplying the whole part (1) by the denominator (5) and adding the numerator (3) to give 8, then copy the denominator (5).

iv. Decimals

In the denominator, place a 1 below the decimal point and add as many zeroes as needed to cover the decimal parts (the number of zeroes to be added must be equal to the number of digits at the right of the decimal point). Remove the decimal point from the numerator.

For example:

a. 0.6 can be written as $\frac{0\ 6}{10} = \frac{6}{10}$.

b. 0.14 can be written as $\frac{0\ 14}{100} = \frac{7}{50}$.

c. 6.1403 can be written as $\frac{6\ 1403}{10000} = \frac{61\ 403}{10\ 000}$.

d. 0.0011 can be written as $\frac{0\ 0011}{10000} = \frac{11}{10\ 000}$.

v. Recurring decimals

Decimal numbers that have digits that repeat forever after the decimal point. The part that repeats is usually shown by placing dots over the first and last digits of the repeating pattern, or sometimes a line over the pattern.

For example:

a. 1.7 = 1.777777… the 7 is the only repeated number.

b. $3.\dot{1}40\dot{6}$ = 3.140614061406… a dot above the beginning and the end of the repeated digits.

c. $\overline{0.1253}$ = 0.125312531253… a bar above the digits that are repeated.

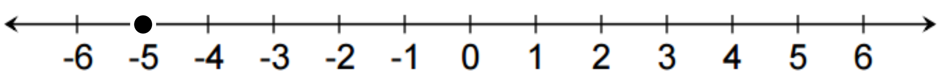
Plotting real numbers on a Real Number Line

An integer is any number from the set {…, –4, –3, –2, –1, 0, 1, 2, 3, 4, …}. The three dots (…) are called an **ellipsis** which means it goes on forever (infinite). Integers greater than 0 are positive integers while integers less than zero are negative integers. Zero itself is neither positive nor negative. Positive integers are usually written without the + sign, e.g +8 and 8 are the same.

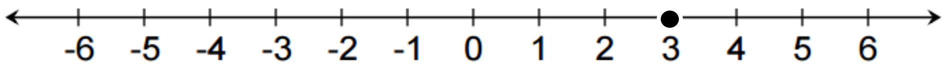
Integers can be represented using a **number line**. A number line is usually represented as being horizontal. Positive integers are represented as **points** to the right of 0 and negative integers are represented as **points** to the left of 0. An **arrowhead** on either end of the drawing means that the line continues indefinitely in the positive and negative real numbers.

Other real numbers can also be plotted on a number line.

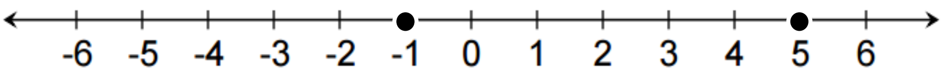
Examples: Plot the point –5 on the number line below.



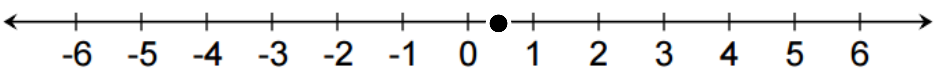
Plot the point 3 on the number line below.



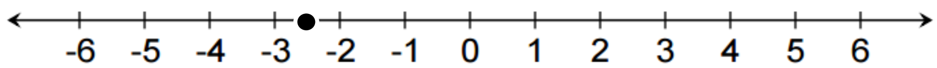
Plot the points –1 and 5 on the number line below.



Plot the point $\frac{1}{2}$ on the number line below.



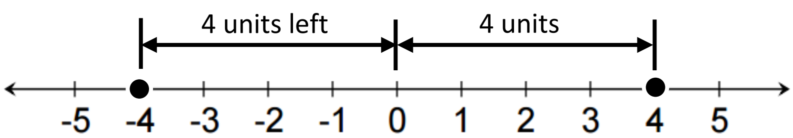
Plot the point $-\frac{5}{2}$ on the number line below.



Plot the point $-\frac{12}{3}$ on the number line below.

Opposites and Absolute Value

Observe the two points on the number line below.



What is the distance of –4 from zero, or of 4 from zero?
How will you compare their distances?

Two numbers are said to be **opposites** if they are represented by points that are at the same distance from zero, but on different directions.

So, what can you say about –4 and 4? Are they opposites?

The **absolute value** of an integer tells how far the point that represents it is from zero. Absolute value can be found by just removing the + or – sign before the number. The absolute value of *n* is written as $|n|$. So $|-4| = 4$ and $|4| = 4$.

For example:

a. The absolute value of –15 = |-15| = 15.

b. The absolute value of 23 = |23| = 23.

c. The absolute value of $-\frac{2}{3}$ = $|-\frac{2}{3}|$ = $\frac{2}{3}$.

Properties of Real Numbers

In performing fundamental operations on rational numbers, a set of mathematical laws called **Properties of Real Numbers** is necessary. These properties are called **axioms** which are applicable to addition and multiplication only. An axiom is a rule or a statement that is accepted to be true even without proof.

Axiom 1: Closure property of Addition

The law states that when adding two real numbers the sum is another real number. It can be shown algebraically as *a* + *b* = c, where *a*, *b,* and *c* are all real numbers.

For example:

a. 6 + 14 = 20

b. $7 + \frac{1}{2} = 7\frac{1}{2}$

c. 2.5 + 3.2 = 5.7

Axiom 2: Closure property of Multiplication

The law states that when multiplying two real numbers the product is another real number. It can be shown algebraically as *a* x *b* = c, where *a*, *b,* and *c* are all real numbers.

For example:

a. 2 x 6 = 12

b. $(7)\left(\frac{1}{2}\right) = \frac{7}{2}$

c. 11•3 = 33

Note: The symbols "x", "( )", and "•" are all used as symbols for multiplication.

Closure properties shows an assurance that the sum or product of any two real numbers is also a real number.

Axiom 3: Commutative property of Addition

The law states that real numbers can be added in any order but their sum will be the same. It can be shown algebraically as $a + b = b + a$.

For example:

a. $2 + 3 = 3 + 2$

$5 = 5$

b. $\frac{3}{5} + \frac{1}{5} = \frac{1}{5} + \frac{3}{5}$

$\frac{4}{5} = \frac{4}{5}$

Axiom 4: Commutative property of Multiplication

The law states that real numbers can be multiplied in any order but their product is unchanged. It can be shown algebraically as $a \text{ x } b = b \text{ x } a$.

For example:

a. $2 \text{ x } 3 = 3 \text{ x } 2$

$6 = 6$

b. $\frac{3}{5} \text{ x } \frac{1}{5} = \frac{1}{5} \text{ x } \frac{3}{5}$

$\frac{3}{25} = \frac{3}{25}$

Commutative properties state that the order of the addends or the factors does not change the sum or product, respectively.

Axiom 5: Associative property of Addition

The law states that the order of association of real numbers will not affect the sum. It can be shown algebraically as $(a + b) + c = a + (b + c)$.

For example:

a. $(7 + 6) + 3 = 7 + (6 + 3)$

$13 + 3 = 7 + 9$
$16 = 16$

b. $\left(\frac{2}{9} + \frac{1}{9}\right) + \frac{3}{9} = \frac{2}{9} + \left(\frac{1}{9} + \frac{3}{9}\right)$

$\frac{3}{9} + \frac{3}{9} = \frac{2}{9} + \frac{4}{9}$

$\frac{6}{9} = \frac{6}{9}$

$\frac{2}{3} = \frac{2}{3}$

Axiom 6: Associative property of Multiplication

The law states that the order in which real numbers are associated in multiplication will not alter the product. It can be shown algebraically as (a x b) x c = a x (b x c).

For example:

a. (3 x 8) x 2 = 3 x (8 x 2)
24 x 2 = 3 x 16
48 = 48

b. $\left(\frac{2}{4} \times \frac{1}{4}\right) \times \frac{3}{4} = \frac{2}{4} \times \left(\frac{1}{4} \times \frac{3}{4}\right)$

$\frac{2}{16} \times \frac{3}{4} = \frac{2}{4} \times \frac{3}{16}$

$\frac{6}{64} = \frac{6}{64}$

$\frac{3}{32} = \frac{3}{32}$

Associative properties state that the grouping of the addends or the factors does not change the sum or product, respectively.

Axiom 7: Identity property of Addition

The law states that when adding zero to any real number the sum is that real number. The number did not lose its identity so zero is the additive identity element. It can be shown algebraically as $a + 0 = 0 + a = a$.

For example:

a. $4 + 0 = 0 + 4 = 4$
b. $7 + 0 = 0 + 7 = 7$

Axiom 8: Identity property of Multiplication

The law states that when multiplying one to any real number the product is that real number. The number did not lose its identity so one is the multiplicative identity element. It can be shown algebraically as $a + 1 = 1 + a = a$.

For example:

a. $4 \times 1 = 1 \times 4 = 4$
b. $7 \bullet 1 = 1 \bullet 7 = 7$
c. $(2)(1) = (1)(2) = 2$

Identity laws imply that the sum or product of any real number and 0 or 1, respectively, will result to that real number.

Axiom 9: Inverse property of Addition

The law states that for every real number there exists another real number which when added will result to a sum of 0. It is shown algebraically as $a + (-a) = 0$.

For example:

a. $4 + (-4) = 0$
b. $7 + (-7) = 0$

If the sum of two real numbers is 0, the real numbers are said to be **additive inverse or opposites.**

Axiom 10: Inverse property of Multiplication

The law states that for every real number there exists another real number which when multiplied will result to a product of 1. It is shown algebraically as $a \bullet \frac{1}{a} = 1$.

For example:

a. $5 \bullet \frac{1}{5} = \frac{5}{5} = 1$

b. $(3)(\frac{1}{3}) = \frac{3}{3} = 1$

c. $(\frac{2}{3})(\frac{3}{2}) = \frac{6}{6} = 1$

If the product of two nonzero real numbers is 1, then the real numbers are said to be **multiplicative inverses** or **reciprocals.**

Axiom 11: Zero property of Multiplication

The law states that any real number when multiplied by 0 the product is always 0. It is shown algebraically as $a \times 0 = 0$.

For example:

a. $4 \times 0 = 0$
b. $-7 \times 0 = 0$

Axiom 12: Distributive property of Multiplication over Addition

The distributive property lets you multiply a sum by multiplying each addend separately and then add the products. It is shown algebraically as $a(b + c) = ab + ac$

For example:

a. $4(2 + 3) = 4(2) + 4(3)$
$= 8 + 12$
$= 20$

b. $-2(6 + 3) = -2(6) + -2(3)$
$= 12 - 6$
$= 6$

LEARNING ACTIVITY 11.1.1.1

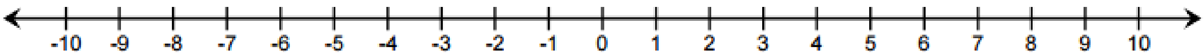
20 minutes

Write an integer for each situation.

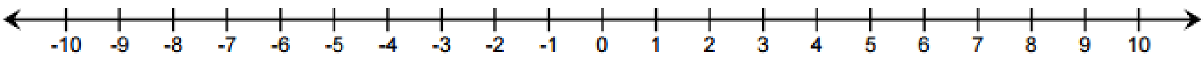
1. a profit of Php 100.00 ___________
2. a withdrawal of Php 500.00 ___________
3. 9 meters increase ___________
4. a gain of 5 pounds ___________
5. a loss of 7 points ___________
6. 3 °F above zero ___________
7. 7 °F below zero ___________

Plot the following real numbers on the number line.

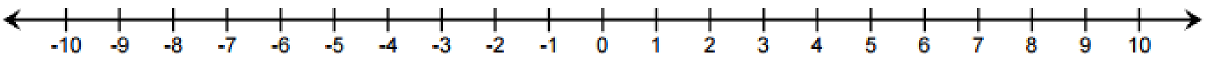
8. 9



9. -8



10. $-\frac{7}{2}$



Find the opposite and the absolute value of the following numbers.

numbers	opposite number	absolute value
11. 3	_____	_____
12. -5	_____	_____
13. 23	_____	_____
14. $-\frac{6}{5}$	_____	_____
15. $\frac{10}{3}$	_____	_____

Complete each statement using the given property.

16. $21 + 32$ = ____________________ Commutative property
17. $2 + (3 + 5)$ = ____________________ Associative property
18. $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)$ = ____________________ Inverse property

Identify the property illustrated in each statement.

19. $(82 + 67) + 9 = 82 + (67 + 9)$ = ______________________
20. $(\frac{2}{3})(0) = 0$ = ______________________
21. $(\frac{2}{3} \bullet \frac{1}{2}) \bullet \frac{1}{3} = \frac{2}{3} \bullet (\frac{1}{2} \bullet \frac{1}{3})$ = ______________________

11.1.1.2 Surds

Surds are numerical expressions which involve irrational numbers. In some references, they call surds as radicals.

In the previous topics, indices were discussed. In addition to the integer indices you have learnt, you should also know that fractions can also be used as exponents.

If you are asked to get the value of $2^{\frac{1}{3}}$, how will you be able to solve for it? How will you multiply 2, $\frac{1}{3}$ times itself? This is where knowledge of surds is needed.

In the expression $2^{\frac{1}{3}}$, the base is 2 and the exponent is $\frac{1}{3}$. Now, observe that $2^{\frac{1}{3}}$ can be written or expressed as $\sqrt[3]{2^1}$.

In the expression, note the following:

index $\sqrt[3]{2^1}$ exponent

radicand

Using the expression above:

2 is still called as the base or radicand.
3 is called as the index, taken from the denominator of the fractional exponent.
1 is the exponent of the radicand, taken from the numerator of the fractional exponent.

Note that the calculator value given as decimals are approximate value of surds.

Example 1 Write the following surds and complete the table below:

Given	Surd / Radical Form	Radicand (Base)	Index (Denominator of the fractional exponent)	Exponent of the radicand (Numerator of the fractional exponent)
"sixth root of 8 squared"	$\sqrt[6]{8^2}$	8	6	2
"square root of x raised to the 7^{th} power"	$\sqrt[2]{x^7}$	x	2	7
"cube root of 3 squared"	$\sqrt[3]{3^2}$	3	3	2

Laws of Surds

In dealing with surds, the following Laws or Rules must be followed to facilitate ease and accuracy in computing.

Law 1 states that the factors of a radicand can be expressed as separate surds.

Law 1: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Example 1 Simplify $\sqrt{48}$

Solution:

Since the $\sqrt{48}$ is not a perfect square number, we can get its estimated value without using the calculator by just simplifying it. Now, think of a factor of 48 where one is a perfect square number since the surd has an index of 2. (When an index is not indicated, it means it is 2, just like when no exponent of the radicand is indicated, I means its 1).

Knowing that factors of 48 are 16 and 3, we can re write $\sqrt{48}$ as: $= \sqrt{(16x3)}$

Using Law 1: $= \sqrt{16} \times \sqrt{3}$

Simplifying the $\sqrt{16}$ and leaving $\sqrt{3}$ as $\sqrt{3}$ is: $= 4\sqrt{3}$

Therefore, $\sqrt{48} = 4\sqrt{3}$

Example 2 Simplify $\sqrt[3]{24}$

Solution:

Since the $\sqrt[3]{24}$ is not a perfect cube number, we can get its estimated value without using the calculator by just simplifying it. Now, think of a factor of 24 where one is a perfect cube number since the surd has an index of 3.

Knowing that factors of 24 are 8 and 3, we can re write $\sqrt[3]{24}$ i as: $\sqrt[3]{8x3}$

Using Law 1: $\sqrt[3]{8}$ x $\sqrt[3]{3}$

Simplifying the $\sqrt[3]{8}$ and leaving $\sqrt[3]{3}$ as is: $2\sqrt[3]{3}$

Therefore, $\sqrt[3]{24}$ **=** $2\sqrt[3]{3}$

Law 2: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Law 2 states that the numerator and denominator of a fractional radicand can be expressed as separate surds.

Example 3 Simplify $\sqrt{\frac{3}{4}}$

Solution:

Since the radicand of $\sqrt{\frac{3}{4}}$ is not a perfect square number, we can get its estimated value without using the calculator by just simplifying it. Since the numerator 3 and itself as its factors, we can no longer find a perfect square factor, so we will just leave it as is. But notice that the denominator 4 is a perfect square number and it is possible to get its square root.

Using Law 2, we write the $\sqrt{\frac{3}{4}}$ as: $\frac{\sqrt{3}}{\sqrt{4}}$

Simplifying the denominator $\sqrt{4}$ and leaving the numerator $\sqrt{3}$ as is: $\frac{\sqrt{3}}{2}$

Therefore, $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Law 3: $(\sqrt{a}\,)^2 = a$

Law 3 states that if a surd is raised to a power same with its index, the result is the radicand itself.

Example 4 Simplify $(\sqrt{3})^2$

Solution:

To fully understand this law, let us first transform $\sqrt{3}$ as a fractional exponent before we raise it to the second power: $(3^{\frac{1}{2}})^2$.

Now we use Law 2 of Indices (power raised to a power) $3^{\frac{2}{2}}$

Simplifying the exponent, we get: $3^1 = 3$

Therefore, $(\sqrt{3})^2 = 3$.

Example 5 Simplify $(\sqrt[3]{5x})^3$

Solution:

By just using Law 3, we may simply cancel the exponent and power $(\sqrt[3]{5x})^3$ leaving us with 5x.

Therefore, $(\sqrt[3]{5x})^3 = 5x$.

Addition and Subtraction Surds

Only similar surds can be added or subtracted. This means that when adding or subtracting surds, you must use the same concept as that of adding or subtracting "like" terms in algebra.

The following are similar surds: $\sqrt{3}$, $2\sqrt{3}$, $-4\sqrt{3}$. It is like treating surds as variables like x, 2x and -4x in algebra.

The following are not similar surds: $\sqrt{3}$, $\sqrt[3]{3}$, $\sqrt[4]{3}$ and $\sqrt[5]{3}$.

Why not? Although the surds have similar radicands, they differ with their indices which make them not similar.

How about $\sqrt[3]{3}$, $\sqrt[3]{13}$, $\sqrt[3]{23}$ and $\sqrt[3]{33}$? Are they similar?

No, $\sqrt[3]{3}$, $\sqrt[3]{13}$, $\sqrt[3]{23}$ and $\sqrt[3]{33}$ are not similar surds. Although they have the same index, their radicands are different.

Since the idea of similar surds is already clear, let us try adding and subtracting surds.

Example 1 Find the sum of the following:

a) $3\sqrt{6} + 4\sqrt{6}$

b) $-\sqrt{12} + 3\sqrt{3}$

c) $3\sqrt{3} + \sqrt{27}$

Solution:

a) Since $3\sqrt{6}$ and $4\sqrt{6}$ are similar surds, just add the whole numbers and copy the common surds: $(3+4)\ \sqrt{6}\ = 7\sqrt{6}$

b) Since $-\sqrt{12}$ and $3\sqrt{3}$ are not similar surds, let us first try to simplify $-\sqrt{12}$.
Applying Law 1 for surds, we simplify $-\sqrt{12}$ as : $-\sqrt{4} \times \sqrt{3} = -2\sqrt{3}$
We replace $-\sqrt{12}$ by $-2\sqrt{3}$: $-2\sqrt{3} + 3\sqrt{3}$
We add the integers and copy the common radicand: $(-2+3)\sqrt{3}$
Simplify: $1\sqrt{3}$
This can also be written as $\sqrt{3}$. **Therefore, the sum of** $-\sqrt{12}$ **and** $3\sqrt{3}$ **is** $\sqrt{3}$ **.**

c) Since $3\sqrt{3}$ and $\sqrt{27}$ are not similar surds, let us first try to simplify $\sqrt{27}$. Applying Law 1 for surds, we simplify $\sqrt{27}$ as: $\sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
We replace $\sqrt{27}$ by $3\sqrt{3}$: $3\sqrt{3} + 3\sqrt{3}$
We add the integers and copy the common radicand: $(3+3)\sqrt{3}$
Simplify: $6\sqrt{3}$
Therefore, the sum of $3\sqrt{3}$ and $\sqrt{27}$ **is** $6\sqrt{3}$ **.**

Example 2 Find the difference of the following pairs of surds:

a) $4\sqrt{3}\ - (-2)\sqrt{3}$
b) $2\sqrt{45}\ - 2\sqrt{5}$

Solution:

a) Since $4\sqrt{3}$ and $-2\sqrt{3}$ are similar surds, just get the difference of the integer and copy the common surds: $(4 - -2)\ \sqrt{3}\ = 6\sqrt{3}$.
Therefore, $4\sqrt{3}\ - (-2)\sqrt{3}\ = 6\sqrt{3}$ **.**

d) Since $2\sqrt{45}$ and $2\sqrt{5}$ are not similar surds, let us first try to simplify $2\sqrt{45}$.
Applying Law 1 for surds, we simplify $2\sqrt{45}$ as: $(2)\sqrt{9} \times \sqrt{5}$
Get the $\sqrt{9}$: $(2)(3)\sqrt{5} = 6\sqrt{5}$
We replace $2\sqrt{45}$ by $6\sqrt{5}$: $6\sqrt{5} - 2\sqrt{5}$
We get the difference of the integers and copy the common radicand:
$(6-2)\sqrt{5}$
Simplify: $4\sqrt{5}$
Therefore, $2\sqrt{45}\ - 2\sqrt{5} = 4\sqrt{5}$ **.**

Adding and subtracting surds is similar to adding and subtracting like terms in algebra. You can try making the terms similar by simplifying the surds using the laws/rules for surds.

Multiplication of Surds

You can multiply surds by using the Distributive Property or the FOIL Method. In both procedures, you can also make use of the Multiplication Property of Surds. Recall that the product of two surds is given by

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

Where **a** and **b** are real numbers whose **nth roots** are also **real numbers**.

Example 1 Find the product and simplify.

a) $\sqrt{5} \times \sqrt{10}$

b) $\sqrt{6}$ x $\sqrt{6}$

c) $\sqrt{2}\,(3 + \sqrt{2}\,)$

d) $\sqrt{5}\,(\sqrt{8} - \sqrt{2}\,)$

Solution:

a) Applying the rule in multiplying surds we say: $\sqrt{5} \times \sqrt{10} = \sqrt{50}$

Simplify $\sqrt{50}$: $\sqrt{25}$ x $\sqrt{2} = 5\sqrt{2}$

Therefore, $\sqrt{5} \times \sqrt{10}$ **=** $\mathbf{5}\sqrt{2}$

b) Applying the rule in multiplying surds we say: $\sqrt{6}$ x $\sqrt{6} = \sqrt{36}$

Since $\sqrt{36}$ is a perfect square: $\sqrt{36} = 6$

Therefore, $\sqrt{6}$ x $\sqrt{6}$ **= 6.**

c) Use distributive property of multiplication: $3\sqrt{2} + \sqrt{2}$ x $\sqrt{2}$

Simplify: $3\sqrt{2} + \sqrt{4}$

But $\sqrt{4}$ can be simplified as 2: $3\sqrt{2} + 2$

Notice that $3\sqrt{2}$ and 2 are not similar surds, therefore we leave them as the final answer and conclude that $\sqrt{2}\,(3 + \sqrt{2}\,) = 3\sqrt{2} + 2$.

d) Use distributive property of multiplication: $\sqrt{5}\,\sqrt{8} - \sqrt{5}\,\sqrt{2}$

Multiply: $\sqrt{40} - \sqrt{10}$

Simplify $\sqrt{40}$ as $\sqrt{4}$ x $\sqrt{10} = 2\sqrt{10}$ and replace: $2\sqrt{10} - \sqrt{10}$

Get the difference of the integers: $(2-1)\sqrt{10}$

Simplify: $1\sqrt{10}$

Therefore $\sqrt{5}$ **(** $\sqrt{8}$ **-** $\sqrt{2}$ **) =** $\sqrt{10}$ **.**

Conjugates

Multiplying surds and simplifying surds at times involve conjugates. The expressions $3 + \sqrt{10}$ and $3 - \sqrt{10}$ are called conjugates of each other. Notice that they differ only in the sign between the terms. The product of two conjugates is the difference of two squares, which is given by the special product formula $(a + b)(a - b) = a^2 - b^2$.

Example 2 Find the conjugate of the given expressions on the first column and find their products.

Expression	Conjugate	Product
$1 - \sqrt{3}$	$1 + \sqrt{3}$	$= (1)^2 – (\sqrt{3})^2$ $= 1 – 3$ $= -2$
$\sqrt{5} + \sqrt{2}$	$\sqrt{5} - \sqrt{2}$	$= (\sqrt{5})^2 – (\sqrt{2})^2$ $= 5 – 2$ $= 3$
$\sqrt{10} -3$	$\sqrt{10} + 3$	$=(\sqrt{10})^2 –(3)^2$ $= 10 – 9$ $= 1$
$\sqrt{x} + 2$	$\sqrt{x} - 2$	$= (\sqrt{x})^2 – (2)^2$ $= x – 4$

Did you see how simple it is to find the conjugate of a surd?

Comparing the expressions in column 1 with their conjugates in column 2, you will notice that the only difference is their signs. If the expression is a sum (+)**,** its conjugate is a difference (-) or vice versa.

Now, we observe the products in column 3. Note that when the expression and its conjugate were multiplied, they result to a difference of two squares like in the algebraic expression $x^2 – y^2$. Squaring both terms of the surd is easy by just following Law 3 for surds where $(\sqrt{a})^2 = a$.

Now let us have more examples.

Example 3 Find the conjugate of the expression $2 - \sqrt{5}$ and multiply the expression by its conjugate.

Solution:

Since the given is a difference $2 - \sqrt{5}$, its conjugate is a sum $2 + \sqrt{5}$.

Find the product of the two: $(2 - \sqrt{5})(2 + \sqrt{5})$

Following the rules in getting the difference of two squares: $(2)^2 – (\sqrt{5})^2$

Simplify: $4 – 5 = -1$

Therefore, the product of 2 - $\sqrt{5}$ and its conjugate 2 + $\sqrt{5}$ is -1.

Division of Surds

To simplify a **quotient** involving surds, you have to **rationalize the denominator**.

Rationalizing the denominator is a way of removing or eliminating surds in the denominator. An expression is simplified or rationalized when there are no more surds found in the denominator. Usually, this process is used for single term denominator. To rationalize a denominator involving two terms, multiply both the numerator and denominator by the conjugate of the denominator. The skills you have learnt in the previous lessons will help you in dividing and simplifying surds.

Example 1 Find the quotient of the following:

a) $\sqrt{20} \div \sqrt{100}$

b) $\sqrt{3} \div \sqrt{36}$

Solution:

a) The given can be written as: $\frac{\sqrt{20}}{\sqrt{100}}$

Simplifying both numerator and denominator gives: $\frac{\sqrt{4}x\sqrt{5}}{10} = \frac{2\sqrt{5}}{10}$

Factor out 2 on both numerator and denominator: $\frac{\sqrt{5}}{5}$

Therefore, $\frac{\sqrt{20}}{\sqrt{100}} = \frac{\sqrt{5}}{5}$.

b) The given can be written as: $\frac{\sqrt{3}}{\sqrt{36}}$

Simplifying both numerator and denominator gives: $\frac{\sqrt{3}}{6}$

Since there is no more surd in the denominator and no common factor between the numerator and denominator, we consider $\frac{\sqrt{3}}{6}$ as the quotient of $\frac{\sqrt{3}}{\sqrt{36}}$.

Example 1 shows the direct way of finding the quotient and simplifying them when there is no more surd in the denominator.

The next examples involve dividing surds with single term denominator.

Example 2 Find the quotient of the following:

a) $2 \div \sqrt{3}$

b) $\sqrt{3} \div \sqrt{5}$

Solution:

a) We all know that $2 \div \sqrt{3}$ can be written as $\frac{2}{\sqrt{3}}$. This expression has a surd in the denominator. To find its simplified quotient, we simply use rationalizing the denominator.

First, multiply the numerator and the denominator by the given denominator (dividend) $\sqrt{3}$: $\frac{2}{\sqrt{3}} \frac{(\sqrt{3})}{(\sqrt{3})}$

Get the product of the numerators: $2(\sqrt{3}) = 2\sqrt{3}$

Get the product of the denominators: $(\sqrt{3})(\sqrt{3}) = (\sqrt{9}) = 3$

Since the product of the numerators is $2\sqrt{3}$ and the product of the denominators is 3, we write our quotient as $\frac{2\sqrt{3}}{3}$.

Therefore, $2 \div \sqrt{3} = \frac{2\sqrt{3}}{3}$.

Since we already had the detailed explanation of the solution, in example a, let us try a shorter way of rationalizing surds of single term denominator.

b) Since the given $\sqrt{3} \div \sqrt{5}$ can be written as $\frac{\sqrt{3}}{\sqrt{5}}$,we rationalize the denominator by the following simple steps:

Step 1: Multiply the numerator and the denominator by the denominator. The product of the denominator becomes a perfect square.

$$\frac{\sqrt{3}}{\sqrt{5}} x \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{\sqrt{25}}$$

Step 2: Simplify the denominator by expressing the denominator as square root of its perfect square.

$$\frac{\sqrt{15}}{\sqrt{25}} = \frac{\sqrt{15}}{5}$$

Therefore, $\frac{\sqrt{3}}{\sqrt{5}}$ is rationalized as $\frac{\sqrt{15}}{5}$.

For surds with two-term denominators, we rationalize them by multiplying both the numerator and denominator by the conjugate of the denominator. When we multiply the conjugate, the product results in the form (a - b)(a + b) = $a^2 - b^2$, the product of sum and difference of two terms.

Example 3 Find the quotient of the following surds.

a) $\frac{\sqrt{3}}{1-\sqrt{5}}$

b) $\frac{4}{2-\sqrt{3}}$

Solution:

a) **Step 1**: Multiply the numerator and the denominator by the conjugate of the denominator

$$\frac{\sqrt{3}}{1-\sqrt{5}} \text{ X } \frac{1+\sqrt{5}}{1+\sqrt{5}}$$

Step 2: Express the numerators and denominators as products.

$$\frac{(\sqrt{3})(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$$

Step 3: Get the product of the numerators by applying Distributive Law in multiplying surds and get the product of the denominator by multiplying conjugates.

$$\frac{(\sqrt{3})(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{\sqrt{3}+\sqrt{15}}{1^2-(\sqrt{5})^2} = \frac{\sqrt{3}+\sqrt{15}}{1-5} = \frac{\sqrt{3}+\sqrt{15}}{-4}$$

Factor out $(1+\sqrt{5})$ yields $\frac{\sqrt{3}}{1-\sqrt{5}}$

Therefore, $\frac{\sqrt{3}+\sqrt{15}}{-4}$ is equal to $\frac{\sqrt{3}}{1-\sqrt{5}}$

But if we make the denominator positive, since we have multiplied and simplified, we obtain $\frac{\sqrt{3}+\sqrt{15}}{-4} = \frac{-1(\sqrt{3}+\sqrt{15})}{-1(4)} = \frac{-\sqrt{3}-\sqrt{15}}{4}$

b) **Step 1**: Multiply the numerator and the denominator by the conjugate of the denominator.

$$\frac{4}{2-\sqrt{3}} \text{ X } \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

Step 2: Express the numerators and denominators as products.

$$\frac{(4)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

Step 3: Get the product of the numerators by applying Distributive Law in multiplying surds and get the product of the denominator by multiplying conjugates.

$$\frac{(4)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{8+4\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{8+4\sqrt{3}}{4-3} = \frac{8+4\sqrt{3}}{1} = 8+4\sqrt{3}$$

Revise the lesson you have just learned and improve your skills by answering the following learning activity.

LEARNING ACTIVITY 11.1.1.2

20 minutes

Perform the indicated operation(s) involving surds.

1) $-5\sqrt{5} + 12\ \sqrt{5}$ = ______________________________

2) $\sqrt{40} \times \sqrt{8}$ = ______________________________

3) $\sqrt{5}\ (3\sqrt{5} - 2\sqrt{5}\,)$ = ______________________________

4) $\frac{\sqrt{16}}{\sqrt{6}}$ = ______________________________

5) $\sqrt{3}\,(4 + \sqrt{25}\,)$ = ______________________________

6) $-6\sqrt{20} + 2\sqrt{45} + 7\sqrt{5}$ = ______________________________

7) $-4\sqrt{20} + 7\sqrt{5} + 3\sqrt{45}$ = ______________________________

8) $13\sqrt{3} + 18\sqrt{81}$ = ______________________________

9) $\frac{2}{\sqrt{8}}$ = ______________________________

10) $-\sqrt{2}\ (\sqrt{3} + \sqrt{27}\,)$ = ______________________________

11) $(-8 - \sqrt{2})(-8 + \sqrt{2})$ = ______________________________

12) $2\sqrt{24} \div \sqrt{144}$ = ______________________________

13) $\dfrac{\sqrt{8}}{\sqrt{5}}$ = ______________________________

14) $(\sqrt{2} + 3)(\sqrt{2} - 3)$ = ______________________________

15) Complete the table below. In the second column**,** write the conjugates of the given expression and write their products on the third column.

Expression	Conjugate	Product
a) $-7 + \sqrt{5}$		
b) $\sqrt{2} + \sqrt{3}$		
c) $\sqrt{2} - 5$		
d) $-\sqrt{6} + 4$		
e) $-2y^2 + \sqrt{y^4}$		

16) Simplify: $\dfrac{\sqrt{32}}{4 - \sqrt{3}}$

17) Find the quotient when $\sqrt{3}$ is divided by $3 + \sqrt{12}$.

18) Add the conjugate of $3 + \sqrt{5}$ to $\sqrt{180}$. Simplify the final answer.

11.1.1.3 Significant Figures

The significant figures (sf) of a number are those digits that carry meaning contributing to its precision. Every non-zero digit in a number is a significant figure. Also, any zero that is not at the end of a whole number or at the beginning of a decimal fraction is a significant digit.

Counting the number of significant figures in a number

Start counting (from left to right) at the first non-zero digit and continue to the end of the number excluding zeroes on the end of a whole number. All zeroes within the number and at the end of a decimal point are counted.

Here are the basic rules for significant digits:

1) All nonzero digits are significant.
2) All zeroes between significant digits are significant.
3) All zeroes which are both to the right of the decimal point and to the right of all non-zero significant digits are themselves significant.

Example 1 State the number of significant figures of the following numbers.

a) 143
b) 0.0032
c) 12300
d) 3.007

Solution:

a)	143	=	3 significant figures The first is 1, the second is 4, and the third is 3.
b)	0.0032	=	2 significant figures Count from 3 because the leading zeroes are not significant.
c)	12300	=	3 significant figures Zeroes at the end of a whole numbers are not counted.
d)	3.007	=	4 significant figures Zeroes within a number are significant.

Rounding to a particular Significant Figures

Rounding a number means writing the number with fewer significant figures. The rounded number will be approximately equal in size to the original number.

Here are the rules to consider when rounding a number to a particular number of significant figure:

1. In the number, start from the first significant figure and count to the right the number of significant figures required.

2. If the next digit is 5 or greater, increase the last significant figure by 1 and use zeroes where necessary as place holders.

3. If the next digit is less than 5, leave the last significant figure unchanged and use zeroes where necessary as place holders.

Example 2 Round off 152,396 to two (2sf), three (3sf), and four (4sf) significant figures.

Solution:

The underlined digits show the number of significant figures to be rounded off.
152,396 = 150,000 (2 sf)
152,396 = 152,000 (3 sf)
152,396 = 152,400 (4 sf)

Example 3 Round off 0.06284 to two, three, and four significant figures.

Solution:

The underlined digits show the number of significant figures to be rounded off.
0**.062**84 = 0.063 (2 sf)
0**.0628**4 = 0.0628 (3 sf)
0.**06284** = 0.06284 (4 sf)

Example 4 Round off 131.35 to two, three, and four significant figures.

Solution:

The underlined digits show the number of significant figures to be rounded off.
131.35 = 130 (2 sf)
131.35 = 131 (3 sf)
131.35 = 131.4 (4 sf)

Rounding to a particular number of Decimal Places (d.p.)

When rounding a number to a particular number of decimal places, the digits after the decimal point are counted.

Example 1 Round the following to decimal places indicated:

a) 4.346 (2 decimal places)
b) 3.0234 (3 decimal places)
c) 2.97 (1 decimal place)

Solution:

a) 14.**34**6 = 14.35 (2 decimal places)
The first two digits after the decimal point are 3 and 4. Followed by 6 which is greater than 5, so 1 is added to 4 in the second decimal place.

b)3.**023**4 = 3.023 (3 decimal places)
The first three digits after the decimal point are 0, 2, and 3. Followed by 4 which is less than 5, so .023 is left unchanged.

c) 2.**9**7 = 3.0 (1 decimal place)
The first digit after the decimal point is 9 and the next is 7, which is greater than 5, so 1 must be added to 9 and it is 3.0 It has a zero after the decimal point because what is required is a 1 decimal place]

LEARNING ACTIVITY 11.1.1.3

20 minutes

1. Complete the table below by identifying the number of significant figures on the second column and rounding off the given to 1sf (3rd column) and 2 sf (4th column).

Given	Number of Significant Figures	Round off to 1 sf	Round off to 2 sf
a) 0.00002530000			
b) $2.15x10^5$			
c) 120 000 000			
d) 12.00045			
e) 15.23000000			
f) $2.05x10^8$			
g) $3.4x10^{-3}$			
h) 2.000001			
i) 3 000 000 000			
j) $0.0025x10^{-4}$			

2) Round off the following to the nearest 2 and 3 decimal places

Given	Two decimal places	Three decimal places
a) 12.3428		
b) 0.89923		
c) 17.00297		

11.1.1.4 Estimation

Estimating is an important part of mathematics and a very handy tool for everyday life. There are situations in life that sometimes we do not really need an exact value or answer. An estimate may be sufficient. Estimation is finding a number that is close enough to the right answer.

Estimation (or estimating) is the process of finding an estimate, or approximation, which is a value that is usable for some purpose even if input data may be incomplete, uncertain, or unstable. Many companies use estimating techniques to find out the cost of buying materials or machineries, the cost involved in manufacturing goods, etc. The process of estimation gives them a fair idea of the amount to be invested.

How estimation is done

An estimate is made by rounding the number in a calculation usually to one significant figure (sometimes two significant figures). This depends on the kind of estimation needed. When a calculation involves division, the divisor is rounded to 1 significant figure, and the numerator is rounded to a multiple of the divisor (which may be 2 sf if necessary). An estimate can be used to check the accuracy of a calculated answer.

Example 1 Estimate the following calculations:

a) 19.2×235.4
b) $47.56 \div 6.34$
c) $(734 \times 43.7) \div 8.87$

Solution:

a) 19.2×235.4
$= 20 \times 200$ (rounding to 1 sf)
$= 4\ 000$

b) $47.56 \div 6.34 = 48 \div 6$ (6.34 is rounded to 6 and 47.56 is rounded to 48, which is the closest multiple of 6 to the figure 47.56)
$= 8$

c) $(734 \times 43.7) \div 8.87$ $= (700 \times 40) \div 9$ (rounding all to 1 sf)
$= 28\ 000 \div 9$ (simplifying)
$= (27\ 000) \div 9$ (rounding numerator to a multiple of 9)
$= 3\ 000$

LEARNING ACTIVITY 11.1.1.4

20 minutes

1. Write each of the following numbers to the number of significant figures indicated.
   a) 777 (2 sf) ____________________
   b) 2 036 (3 sf) ____________________
   c) 21 984 (4 sf) ____________________
   d) 98 747 (3 sf) ____________________
   e) 0.0029 (2 sf) ____________________

2. Write each of the following numbers to the number of decimal places indicated.
   a) 7.77 (2 decimal places) ____________________
   b) 20.36 (1 decimal places) ____________________
   c) 21.984 (3 decimal places) ____________________
   d) 9.8747 (3 decimal places) ____________________
   e) 9.0029 (2 decimal places) ____________________

3. Complete the table below by estimating the given by rounding off to a whole number (2^{nd} column) and 1 significant figure (3^{rd} column)

Given	Estimated value by rounding off to a whole number	Estimated value by rounding off to 1 significant figure
12.65		
9.42		
15.12		
18.52		
12.86		

4. Find the estimated sum or difference by rounding off to the nearest whole number or 2sf: (The first one is done for you.)

	Given	Estimates	Estimated Answer
	23.45 + 12.20 + 18.55 =	23 + 12 + 19	= **54**
a)	58.44 + 28.62 + 11.12 =	____________________	= ____________
b)	74.48 – 29.24 =	____________________	= ____________
c)	(12.82 – 6.24) + 15.67 =	____________________	= ____________

5. Find the estimated product or quotient by rounding off to 1sf.

	Given	**Estimates**	**Estimated Answer**
a)	12.55 x 89.20 =	____________________	= ____________
b)	52.48 $\div$ 47.20 =	____________________	= ____________
c)	18.42 x 64.08 =	____________________	= ____________
d)	121.42 $\div$ 4.50 =	____________________	= ____________

11.1.1.5 Indices and Logarithmic Laws

Indices are a useful way of more simply expressing large numbers. They also present us with many useful properties for manipulating them using what are called the **Law of Indices**.

The expression 2^5 is defined as $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. It means 2 is to be multiplied 5 times itself.

exponent

$$\mathbf{2^5}$$

base

2^5 is a power, in which the number 2 is called the base and the number 5 is called the index or exponent.

The **base** is the number to be multiplied while the **exponent** tells the number of times the base is to be multiplied by itself.

In the given example 2^5, it only means that 2 is to be multiplied 5 times itself giving the product 32.

Example 1 Simplify the following using index form:

a) 3x3x3x3x3x3x3x3x3x3
b) 2x2x2x4x4x4x4x4
c) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$
d) (2) (2) (2) (2) (2) aaaabbbbbbcc

Solution:

a) Since 3 is multiplied 10 times itself its base is 3 and its exponent is 10 = $\mathbf{3^{10}}$
b) Since 2 is multiplied 3 times itself and 4 is multiplied 5 times itself = $\mathbf{(2^3)(4^5)}$
c) Since $\frac{1}{3}$ is multiplied 5 times itself, we write = $\left(\frac{1}{3}\right)^5$
d) Since 2 is repeatedly multiplied 5 times, a 4 times , b 6 times and cc 2 times, we simplify = $\mathbf{2^5a^4b^6c^2}$

Index Laws of Multiplication

Dealing with powers and indices can be simplified further using the following Laws of Indices:

Law 1: To multiply powers with the same base, add the indices.

$$\mathbf{b^m\, b^n = b^{m+n}}$$

The notation above shows that b represents the bases while m and n represents the exponents.

Law 1 states that when multiplying powers with the same bases, simply add or get the sum of the indices.

Example 1 Simplify $5^3 5^2$

Solution:

Since both powers have the same base of 5, we write 5^{3+2} = $\mathbf{5^5}$

Therefore, $\mathbf{5^3 5^2 = 5^5}$

Example 2 Simplify $2^3 2^4 3^2 3^6$

Solution:

Since we can only add the powers of the same bases, we write $2^{3+4}\, 3^{2+6} = 2^7 3^8$

Therefore, $\mathbf{2^3 2^4 3^2 3^6 \quad = 2^7 3^8}$

Index Laws of Powers of Power

Law 2: To raise a power to a power, multiply the indices.
$\mathbf{(b^m)^n = b^{mn}}$

The second law states that if a power (b^m) is raised to another power (n), then use the same base b raised to the product of m and n.

Example 3 Simplify $(y^3)^4$

Solution:

If we will expand the given, it means that y^3 is to be multiplied 4 times itself giving us: $y^3\, y^3\, y^3\, y^3$

If we will apply Law 1 we will have: $y^{3+3+3+3} = y^{12}$

Using Law 2 we will only multiply the exponents 3 and 4 $y^{(3x4)} = y^{12}$

Therefore, $\mathbf{(y^3)^4 = y^{12}}$

Example 4: Simplify $(2^3)^2$

Solution:

$2^{3x2} = 2^6$

Index Laws of …

Law 3: To get the power of a product, distribute and multiply the indices.
$\mathbf{(ab)^m = a^m b^m}$

The third law states that if a power consisting of a product **a** times **b** (more than 1 base) is raised to another power (m), then both **a** and **b** are raised to **m**.

Example 5 Simplify $(2y)^3$

Solution:

The base consists of the product of 2 and y which is 2y. Since it is raised to the 3rd power, then we raise both 2 and y to the power of 3. $(2y)^3 = 2^3\ y^3$ = Since 2^3 is equal to 8, we further simplify 2^3y^3 as $\mathbf{8y^3}$.

Therefore, $\mathbf{(2y)^3 = 8y^3}$**.**

Example 6 Simplify $(a^2b^3c^4)^2$

Solution:

The example is a combination of Law 3 and Law 2.

Use Law 3 to distribute the power: $a^{2(2)}b^{3(2)}c^{4(2)}$

Law 2 to multiply the distributed powers: $a^4b^6c^8$

Therefore, $\mathbf{(a^2b^3c^4)^2 = a^4b^6c^8}$**.**

Index Laws of Division

Law 4 states that when dividing powers with the same base, simply subtract or get the difference of the indices.

Law 4: To divide powers with the same base, subtract the indices. $$\mathbf{\frac{b^m}{b^n} = b^{m-n}}$$ where m > n

This rule applies when the exponent of the dividend (numerator) is greater than the exponent of the divisor (denominator).

Example 1 Simplify $\frac{y^7}{y^3}$

Solution:

We apply Law 4 since the bases in the dividend and divisor are the same and 7>3 (seven is greater than 3). We simply use y as the base and raised to the difference of 7 and 3 that is $y^{7-3} = y^4$

Therefore, $\frac{y^7}{y^3} = \mathbf{y^4}$**.**

Example 2 Simplify $\frac{a^5b^9}{a^2b^4}$

Solution:

Law 4 still applies for this example; however, bear in mind that exponents of the same bases can only be subtracted. So we say: $a^{5-2}\,b^{9-4}$

Simplifying it further: a^3b^5

Therefore, $\frac{a^5b^9}{a^2b^4} = \mathbf{a^3b^5}$**.**

The 5th Law is somehow similar to Law 3 where a power consists of a difference (instead of a product) **a** divided by **b** is raised to another power (m), both **a** and **b** are raised to **m** before dividing them or getting the quotient.

Law 5: To get the power of a quotient, just find the quotient of the powers.

$$\left(\frac{a}{b}\right)^{\mathbf{m}} = \frac{a^m}{b^m}$$

where $b \neq 0$

Example 3 Simplify $\left(\frac{2}{3}\right)^2$

Solution:

Raise both numerator and denominator to 2: $\frac{2^2}{3^2}$

Simplify the numerator and denominator: $\frac{4}{9}$

Therefore, $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

The Rule for Zero as a Power and Negative Indices

Law 6: Any number or quantity raised to zero is equal to 1.

$$\mathbf{b^0 = 1}$$

This Law can actually be derived from Law 4 where m = n.

For example, we have $\frac{a^2}{a^2}$. If we apply Law 4, we will get a^{2-2} which is actually equal to a^0. But how did we make it equal to 1?

Remember the rules in fraction?

When the numerator is exactly the same with the denominator, the fraction is equal to 1. Another basic explanation of this is our simple arithmetic where $1 \div 1 = 1$, $2 \div 2 = 1$, $3 \div 3 = 1$ and so on. So, as long as any number or quantity is divided by itself, the quotient is always 1. This also applies when a number or any quantity is raised to zero.

Example 1   Simplify $(2x^3)^0$

Solution:

Apply Law 6 and we conclude that $(2x^3)^0 = 1$

> Law 7: Any number or quantity raised to a negative power is equal to its positive reciprocal.
>
> $$\mathbf{b^{-m}} = \frac{1}{b^m}$$

Example 2 Simplify the following:

a) 2^{-2}

b) $\left(\frac{1}{2}\right)^{-3}$

c) $2x^{-2}$

Solution:

a) Given 2^{-2}, applying Law 7, we get its reciprocal $\frac{1}{2^2} = \frac{1}{4}$

Therefore, $2^{-2} = \frac{1}{4}$

b) $\left(\frac{1}{2}\right)^{-3}$ = we get its reciprocal by dividing 1 by $\left(\frac{1}{2}\right)^3$ $\quad \frac{1}{\left(\frac{1}{2}\right)^3}$

Simplify the denominator: $\quad \frac{1}{\frac{1}{8}}$

Divide 1 by $\frac{1}{8}$, we get $\quad (1)\ \frac{8}{1} = 8$

Another way of simplifying it is by getting the reciprocal of the base: $\left(\frac{2}{1}\right)^3$

and raising it to a positive power 3: $\frac{2}{1}$

Apply Law 5: $\frac{2^3}{1^3}$

Simplify: $\frac{8}{1} = 8$

Therefore, $\left(\frac{1}{2}\right)^{-3} = 8$.

c) Given $2x^{-2}$, only x is raised to a negative power so we say: $(2)\ \frac{1}{x^2}$

Simplify: $\frac{2}{x^2}$

Therefore, $2x^{-2} = \frac{2}{x^2}$.

There could be more than one way of simplifying indices using the laws discussed. Sometimes, working out may vary depending on the law you choose to use first when multiple apply.

Logarithmic Laws

Consider the expression $16 = 2^4$. Remember that 2 is the base, and 4 is the power. An alternative, yet equivalent, way of writing this expression is $\log_2 16 = 4$. This is stated as '**log to base 2 of 16 equals 4'**.

Notice that the logarithm is the same as the power or index in the original expression.
It is the base in the original expression which becomes the base of the logarithm.
The two statements $16 = 2^4$ and $\log_2 16 = 4$ are equivalent statements. If we write either of them, we are automatically implying the other.

In general if we have $x = a^n$, we can write this as **$\log_a x = n$** Read as **"logarithm to base a of x equals n"**.

Example 1 Write $64 = 8^2$ using logarithms.

Solution:

Since a = 8, x = 64 and n = 2, using the logarithmic form $\log_a x = n$
We have $\log_8 64 = 2$.
Therefore, $64 = 8^2$ is written as $\log_8 64 = 2$ in logarithmic form.

Example 2 Write $\log_3 27 = 3$ in index form.

Solution:

Since a = 3, x = 27 and n = 3, using the index form $x = a^n$

We have: $27 = 3^3$ or $3^3 = 27$.

Therefore, $\log_3 27 = 3$ is written as $3^3 = 27$ in index form.

The laws of logarithm are simplified as follows:

Laws of Logarithms

Law 1:	$\log_a a = 1$
Law 2:	$\log_a xy = \log_a x + \log_a y$
Law 3:	$\log_a xm = m\log_a x$
Law 4:	$\log_a \frac{x}{y} = \log_a x - \log_a y$
Law 5:	$\log_a 1 = 0$

Example 3 Express $10 = 10^1$ in logarithmic form.

Solution:

Since $10 = 10^1$, we can write the equivalent logarithmic form $\log_{10} 10 = 1$.
This example applies Law 1. This means that the logarithm of a number to the same base is always equal to 1.

Example 4 Find $\log_2 512$.

Solution:

This is the same as being asked '**what is 512 expressed as a power of 2** ?'
Now 512 is in fact 2^9 and so $\log_2 512 = 9$.

Therefore, the logarithm of 2 to the base 512 is 9.

Example 5 Use the laws of logarithm to evaluate the following:

a) $\log_2 4$
b) log 1, 000, 000
c) $\log_7 \sqrt{7}$
d) $\log_5 \frac{1}{\sqrt[3]{5}}$
e) $\log_3 270 - \log_3 10$

Solution:

a) $\log_2 4$

$$\begin{aligned}\log_2 4 &= \log_2 2_2\\ &= 2\log_2 2\\ &= 2 \cdot 1\\ &= 2\end{aligned}$$

b) $\log 1,000,000$

$$\begin{aligned}\log 1,000,000 &= \log 10^6\\ &= 6\log 10\\ &= 6 \cdot 1\\ &= 6\end{aligned}$$

In this example **a** is not written, thus it means that it is equal to 10. Just like in the given variable x, where the exponent of x is 1, in logarithms, when **a** is not labelled, it is understood that its value is 10.

c) $\log_7 \sqrt{7}$

$$\begin{aligned}\log_7 \sqrt{7} &= \log_7 7^{\frac{1}{2}}\\ &= \frac{1}{2}\log_7 7\\ &= \frac{1}{2}(1)\\ &= \frac{1}{2}\end{aligned}$$

d) $\log_5 \frac{1}{\sqrt[3]{5}}$

$$\begin{aligned}\log_5 \frac{1}{\sqrt[3]{5}} &= \log_5 \frac{1}{5^{\frac{1}{3}}}\\ &= \log_5 5^{\frac{1}{3}}\\ &= -\frac{1}{3}\log_5 5\\ &= -\frac{1}{3}(1)\\ &= -\frac{1}{3}\end{aligned}$$

e) $\log_3 270 - \log_3 10$

$$
\begin{aligned}
\log_3 270 - \log_3 10 &= \log_3 \frac{270}{10} \\
&= \log_3 27 \\
&= \log_3 3^3 \\
&= 3 \log_3 3 \\
&= 3\ (\ 1\ ) \\
&= 3
\end{aligned}
$$

Revise the lesson and try to answer the following learning activity for your mastery.

LEARNING ACTIVITY 11.1.1.5

20 minutes

Write the correct answer on the spaces provided.

1) It tells the number of times the base is to be multiplied by itself. ________________

2) Simplify: (x)(x)(x)(x)(x)(y)(y)(y)(z)(z) ________________

3) Write $2^3x^2y^3$ in expanded form ________________

4) Simplify the following

a) $\left(\frac{1}{2}\right)^{-2}$ = ________________

b) $\frac{3^6}{3^2}$ = ________________

c) $\left(\frac{x^2y^3}{z^4}\right)^{-2}$ = ________________

d) $\frac{a^8b^{12}c^7}{a^3b^2c}$ = ________________

e) $\left(\frac{x^4}{x^4}\right)^3$ = ________________

f) $5x^0 - (2x)^0$ = ________________

g) $-y^0 - 5y^0$ = ________________

h) $\frac{2y^0}{4^2}$ = ________________

i) $\frac{(2x)^0}{2x^0}$ = ________________

j) $-3(x)^0 + (-2x)^0 + (2^3)^0$ = ________________

k) $(-3)^{-1}$ = ________________

l) $12(2x)^{-2}$ = ________________

m) $\left(\frac{x}{3}\right)^{-2}$ = ________________

n) $\left(\frac{1}{x}\right)^{-8}$ = ________________

o) $\left(\frac{2x}{5}\right)^{1}$ = ________________

5) Use the laws of logarithm to evaluate the following:

a) $\log_2 12 + \log_2 3 - \log_2 9$

b) $\log_4 8$

c) $\log_{27} \frac{1}{9}$

11.1.1.6 Standard index Form (SIF)

Standard index form or also known as the scientific notation is convenient shorthand way for writing very large or very small numbers which usually involves many zeroes.

An example of very large numbers would be the distance that light travels in a year in miles. It could also be the number of bacteria after 1 week as it doubles its number per second. On the other hand very small quantities may include the mass of a single hydrogen atom in milligrams.

If you were asked to multiply numbers in standard notation as 18, 230,000,000,000 by 0.0000000000000023 it will take time for this basic operations to perform using the conventional way of multiplying whole numbers by decimals. However, by using the standard index form make this process much simpler and doable.

First we will take a look at what standard index form or scientific notation is. The figure below is a sample of how 18, 230,000,000,000 is expressed in standard index form or scientific notation:

$$\mathbf{1.83x10^{13}}$$

The standard Index form or scientific notation shown above has two parts

1) a number in between 1 to 10, that is 1.83 (significant digits where the decimal point is in between the first and the second digits)
2) that number (1.83) is multiplied by ten to some exponent ($x10^{13}$). In the above example, the exponent is 13.

Where did the exponent 13 come from?

Given the number 18, 230,000,000,000 since it is a whole number, it means that the decimal point is located after the last zero in the numeral. From there count the number of places from right going to the left and you stop between the first and second significant digits (between 1 and 8). Did you notice that you moved 13 times? That is exactly what the exponent 13 represents.

The exponent is important when we convert between scientific notation and standard notation. The exponent tells the number of times we will multiply by 10. Multiplying by 10 will require us to move the decimal point one place. If we have 10^2 it means that we will multiply by 100 or simply say we move the decimal point two places.

Observe the tables below.

Positive Powers of 10	Equivalents
10^0	1
10^1	10
10^2	100
10^3	1000
10^4	10000
10^5	100000

Negative Powers of 10	Equivalents
10^{-1}	0.1
10^{-2}	0.01
10^{-3}	0.001
10^{-4}	0.0001
10^{-5}	0.00001

Multiplying positive powers of 10 as shown in the first table above gives us an idea that we move the decimal point to the left to express it in scientific notation and the exponent indicates the number of places we moved (we actually multiplied).

So the exponent will tell us how many times the exponent moves between scientific notation and standard notation. To decide which direction to move the decimal (left or right) we simply need to remember that positive exponents mean in standard notation we have a big number (bigger than ten) and negative exponents mean in standard notation we have a small number (less than one).

By keeping this in mind, it will be easily to convert from standard notation to scientific notation.

Example 1 Convert 26, 180 000 to standard index form. Put decimal after first nonzero number.

Solution:

First, consider the significant digits (non-zero digits) and place the decimal point in between the first and second digits, we have 2.618

Second, multiply 2.618 by 10 (2.618x10) and count the number of places by moving the decimal point to the left to determine the exponent of 10. Since you will move the point seven times to the left to place it in between the digits 2 and 6, the exponent of 10 is 7. Now we have 2.618×10^7.

Therefore, we say 26, 180 000 is written as 2.618×10^7 in standard index form

Example 2 Express 0.0027 in standard index form.

Solution:

First, consider the significant digits (2 and 7) and place the decimal point in between the first and second non-zero digits, we have 2.7

Second, multiply 2.7 by 10 (2.7x10) and count the number of places by moving the decimal point to the right to determine the exponent of 10. Since you will move the point three times/places to the right to place it in between the digits 2 and 7, the exponent of 10 is -3. But why it is negative?

Remember, when we move the decimal point to the right, we are dealing with very small numbers less than 1. Usually it is represented by decimals whose values are less than 1 and they get close to zero as the digits increases.

Now we have 2.7×10^{-3}.

Therefore, we say 0.0027 is written as 2.7×10^{-3} in standard index form.

Now we know how to convert numbers from standard form to standard index form or scientific notation. Let us try the other way around through the next examples.

Example 3 Convert 3.17×10^{5} to standard notation.

Solution:

Since 10 is raised to a positive exponent (5), it gives us an idea that the number is a whole if written in standard form. Therefore, we move the decimal point 5 places to the right and we get 317000.

Therefore, 3.17×10^{5} is written as 317000 in standard form.

Did you notice that this process is the opposite of what we did in example 1?

Example 4 Convert 2.7×10^{-9} to standard notation.

Solution:

Since 10 is raised to a negative exponent (-9), it gives us an idea that the number is a decimal and it is a smaller number if written in standard form. Therefore, we move the decimal point 9 places to the left and we get .000000027 or it can also be written as 0.000000027.

Therefore, 2.7×10−9 is written as 0.000000027. in standard form.

Knowing how to convert between standard notation and scientific notation is essential to fully understand how scientific notation works and what it does. This knowledge will help you be able to multiply and divide numbers in scientific notation using the properties of exponents or indices discussed in the previous lesson.

Example 5 Find the product of 2.13×10^{4} and 1.2×10^{7}

Solution:

Recall that we the SIF has two parts so we multiply each parts separately.
Multiplying the first parts we have (2.13)(1.2) = 2.556
Multiplying the second parts which involve exponent or indices we have $(10^{4})(10^{7}) = 10^{11}$. Recall that we applied Index Law 1 discussed previously.
Combining the two part, we now have 2.556×10^{11}.

Therefore, the product of 2.13×10^{4} and 1.2×10^{7} is 2.556×10^{11}.

Example 6 Find the product of 12 000 000 000 and 234 000 000 000 000 000 000

Solution:

This problem can possibly be solved using the conventional way, however, there might be a higher chance of committing error in handling too many digits in the solution. So a more accurate and neater working out and answer can be best achieved through the use of SIF.

First we convert the given.
12 000 000 000 = 1.2×10^{10}
234 000 000 000 000 000 000 = 2.34×10^{20}

Multiplying the first parts we have (1.2)(2.34) = 2.808
Multiplying the second parts which involve exponent or indices we have $(10^{10})(10^{20}) = 10^{30}$. Index Law 1 discussed previously also applies in multiplying the second parts.
Combining the two parts, we now have 2.808×10^{20}.
If you want to express the final answer into standard form, just move the decimal point 20 times to the right giving us 280 800 000 000 000 000 000.

Therefore, the product of the product of 12 000 000 000 and 234 000 000 000 000 000 000 is 2.808 x 10^{20} or 280 800 000 000 000 000 000.

Example 7 Find the product of 18, 230,000,000,000 and 0.0000000000000023

Solution:

First we convert the given.
18, 230,000,000,000 = 1.823×10^{13}
0.0000000000000023 = 2.3×10^{-15}

Multiplying the first parts we have (1.823)(2.3) = 4.1929

Multiplying the second parts which involve exponent or indices we have $(10^{13})(10^{-15}) = 10^{-2}$.

Index Law 1 discussed previously also applies in multiplying the second parts. In this example, the indices do not have the same sign, following the rules in adding integers [13+(-15) = -2]

Combining the two parts, we now have 4.1929×10^{-2}.
If you want to express the final answer into standard form, just move the decimal point 2 times to the left giving us .041929 or 0.041929.

Therefore, the product of the product of 18, 230,000,000,000 and 0.0000000000000023 is 4.1929×10^{-2} or 0.041929.

Example 8 Find the quotient of 1.25×10^{15} and 2.5×10^{-7}.

Solution:

Dividing the first parts we have $\frac{1.25}{2.5} = 0.5$

Dividing the second parts which involve exponent or indices we have $\frac{10^{15}}{10^{-7}} = 10^{22}$. Index Law 4 discussed previously applies in dividing the second parts. In this example, the indices do not have the same sign, following the rules in subtracting integers [15 - (-7) = 22]

Combining the two parts, we now have 0.5×10^{22}.

The quotient 0.5×10^{22} can still be simplified by moving the decimal point 1 place to the right and subtracting 1 from the exponent we get, 5×10^{21} or can be written in standard notation as 5 000 000 000 000 000 000 000.

Therefore, the product of the quotient of 1.25×10^{15} and 2.5×10^{-7} is 5×10^{21} or 5 000 000 000 000 000 000 000.

It is very interesting to know that Archimedes (287 BC - 212 BC), a Greek mathematician, developed a system for representing large numbers using a system very similar to scientific notation. He used his system to calculate the number of grains of sand it would take to fill the universe.

Revise the lesson and prepare to answer the following learning exercise to further master converting SIF and performing multiplication and division with either very large or very small quantities of number.

LEARNING ACTIVITY 11.1.1.6

20 minutes

1. Convert the following numbers in standard form to SIF.

   a) 152 300 000 000 000 000 ____________________________

   b) 14 077 000 000 000 ____________________________

   c) 0.00000000000075 ____________________________

   d) 0.000000000148 ____________________________

2. Convert the following numbers in SIF to standard form.

   a) $1.204 \text{x} 10^{-8}$ ____________________________

   b) $2.85 \text{x} 10^{7}$ ____________________________

   c) $1.874 \text{x} 10^{-10}$ ____________________________

   d) $4.002 \text{x} 10^{-4}$ ____________________________

3. Perform the indicated operation and express the final answer to SIF. Use the spaces for your working out and write the final answer on the blank provided.

   a) (23 450 000 000 000)(567 000 000)

   Answer: ______________________

   b) (240 000 000 000 000 000) (0.00000000000000018)

   Answer: ______________________

   c) $(5.23 \text{ x} 10^{-7})(2.54 \text{ x} 10^{-2})$

   Answer: ______________________

d) $6.55 \text{ x}10^{-10} \div 1.25 \text{ x}10^{5}$

Answer: ______________________

e) (1 250 000 000 000 000) (0.000000000028)

Answer: ______________________

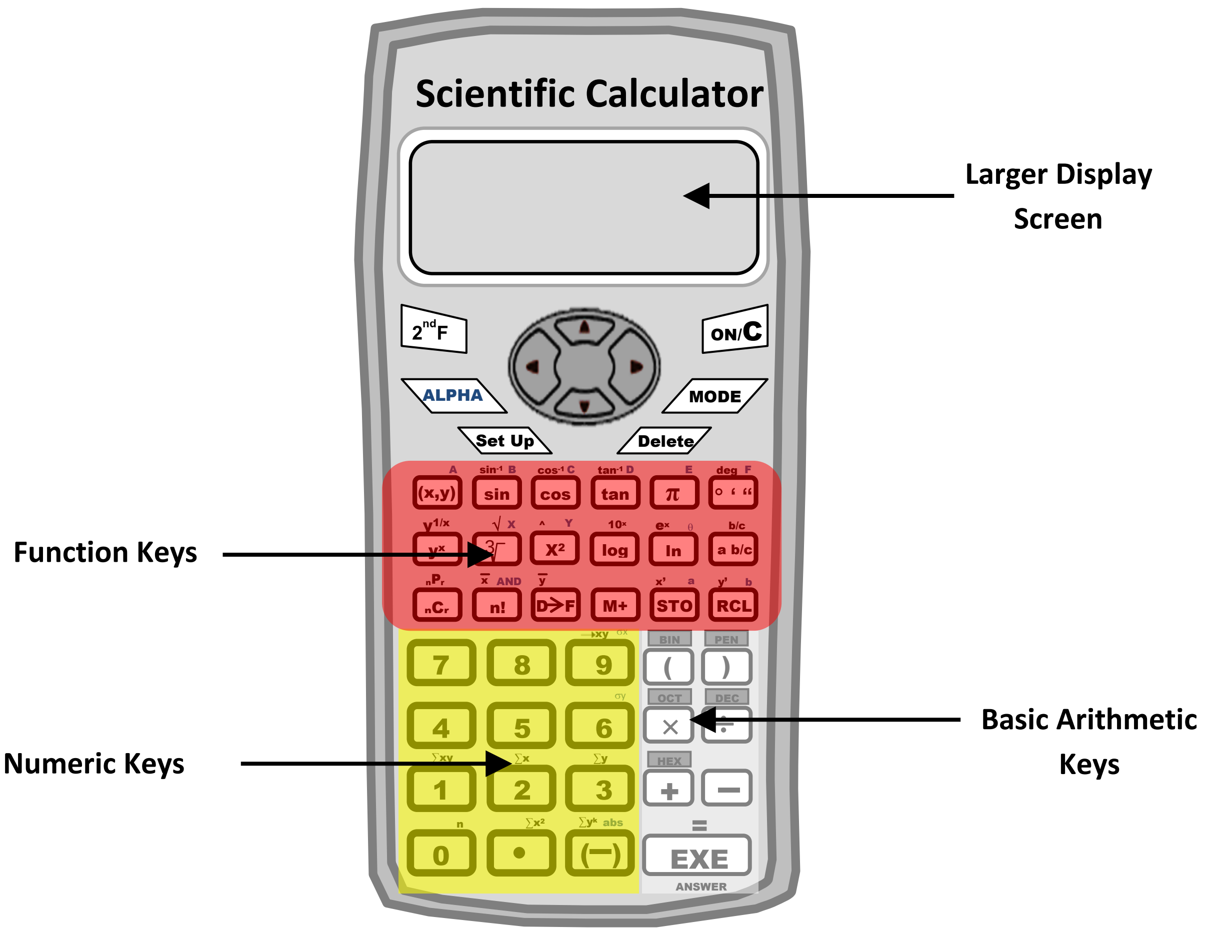
f) $(1.28 \text{ x}10^{-10})(2.5 \text{ x}10^{12})$

Answer: ______________________

11.1.1.7 The Scientific Calculator

A scientific calculator is more than the usual counting devise we know because it provides more functions used in computations as well as different modes which allows users to perform linear regression, quadratic, exponential, logarithmic, power, trigonometric and statistical computations. It basically aids the computation in higher Mathematics.

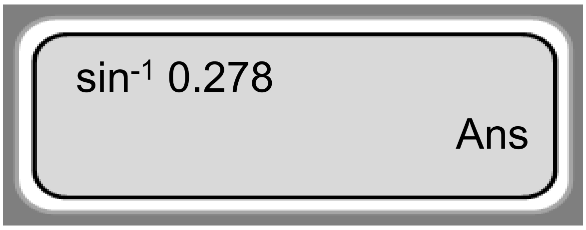
The scientific calculator can easily be identified because of the following major features as shown below.



Scientific calculators may differ in features offered but the four main features will definitely be visible and present in every scientific calculator regardless of their brands and models.

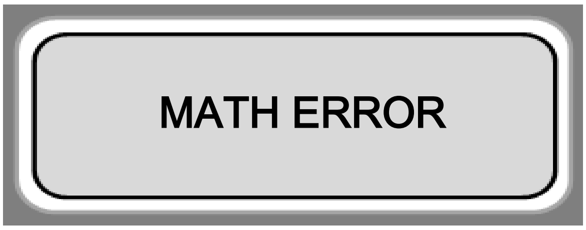
A **Larger Display Screen** as compared with the regular calculators can easily be noticed. The screen can provide displays for fractions showing the numerator and denominator, surds or radicals and other symbols. Most of the newly released scientific calculators can display the calculation the way they are written on text because they even display the function being used.

The picture below shows a sample screen display wherein the trigonometric function is displayed.



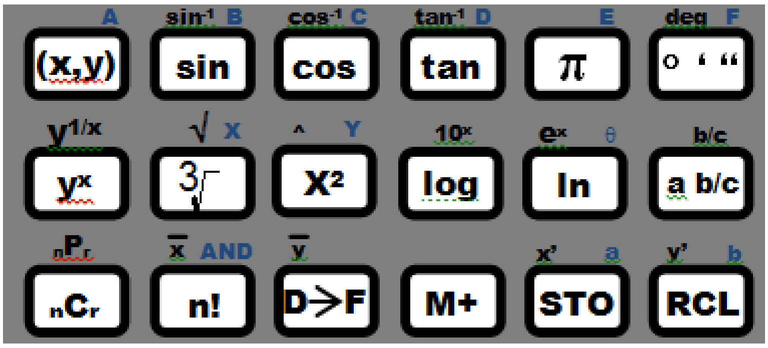
Did you notice that the screen can even display both input and answer?

You can also see the following displays on the screen.



MATH ERROR means that the calculator can no longer display the computed answer. Usually it is too large in value that the calculator can no longer handle to calculate and display while SYN ERROR or also known as the SYNTAX ERROR is displayed when there is a wrong input in either function or arithmetic keys.

Function Keys are the set of keys that cannot be found in a regular calculator or computing device. This set of keys makes the Scientific Calculator distinct.

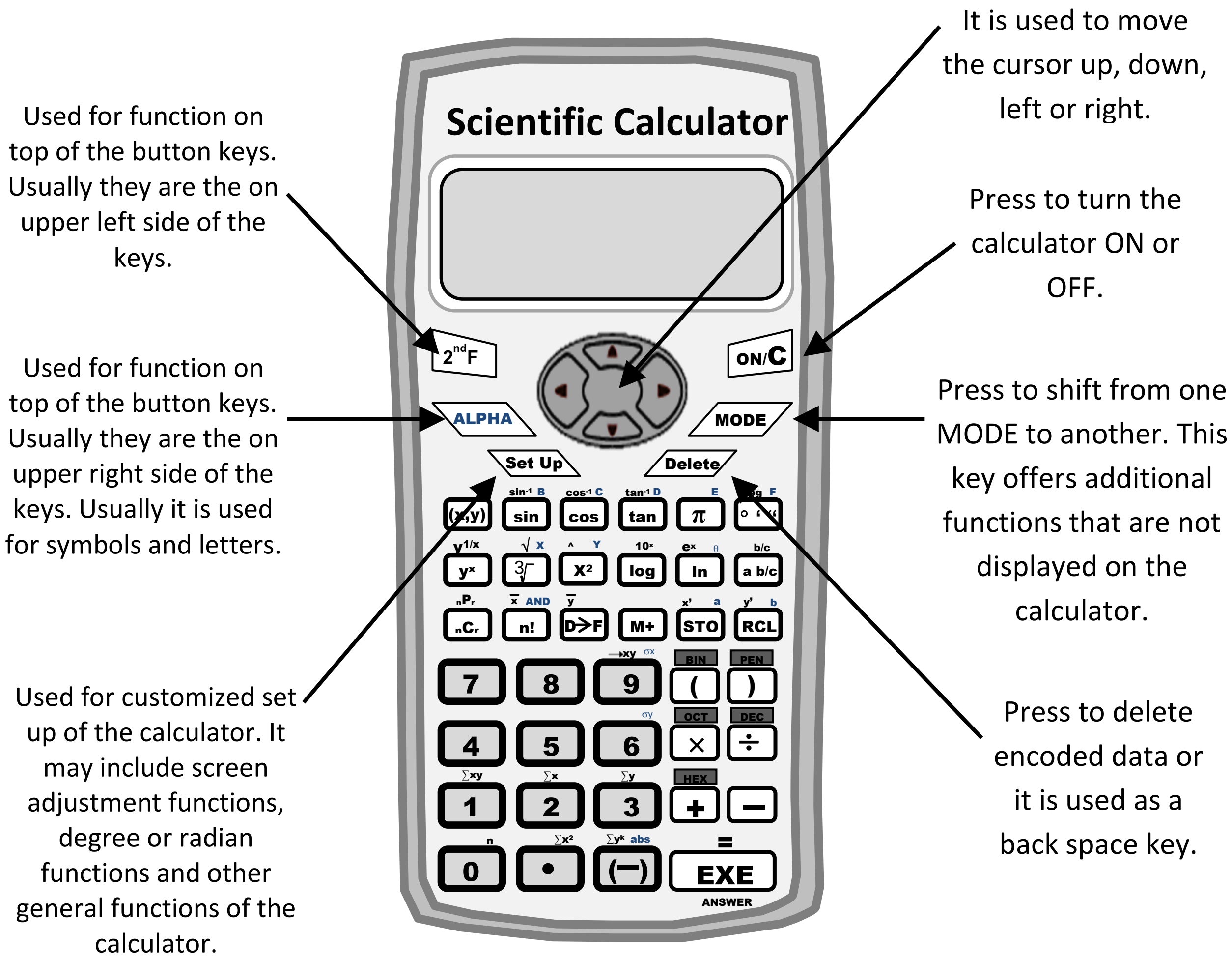


This set of keys is usually comprises of keys used for calculations involving trigonometry, logarithms, roots, powers, fractions, statistics, factorials, permutations, combinations and conversions.

Some high-end scientific calculators offer more functions which aid in statistical analysis, imaginary numbers and provides a function for the user to create his own formula to be used. A **programmable scientific calculator** allows the user to create one's own formula, programs and even store textual data. It also allows transfer of data from the calculator to the laptop and vice versa.

The **numeric keys** bear the digits from 0 to 9 and the **basic arithmetic keys** are used for adding, subtracting, multiplying and diving numbers. It also includes parentheses, equal sign, decimal point and the negative sign for integers. These two sets of keys are the usual keys found in any calculator, even the most basic of all models

Let us identify some other keys and parts of the calculator that will be used often in calculations. You also need to read the User's Manual of the calculator you are using to be familiar with it.



Remember to read your calculator's manual before using it. Some functions and positions of the keys may vary depending on your calculator's brand and model.

Function keys such as STORE STO and RECALL RCL allow the user to SAVE and RETRIEVE data. After performing a calculation, you can press STO if you intend to use or review the calculations at a latter time. Calculators have different capacity as to the number of data to store. Pressing RCL will show the previously saved data. You can modify this be inserting another value, just use the navigation keys to move the cursor where you want to insert data. You can also delete or erase data by pressing Delete after moving the cursor next to the data you want to remove.

For this lesson, we will focus our worked examples in its use in basic computation dealing with integral exponents, fractional exponents, radicals, trigonometric functions, permutations and combinations. Conversion will also be explained to make you convert from fraction to decimal and vice versa, degrees to degree-minutes-seconds, improper fractions to mixed form. Other use of the scientific calculator is embedded in the succeeding modules in Trigonometry, Probability and Statistics.

Use you scientific calculator to work on the following examples.

Example 1 Simplify 4^{12}.

Solution:

Press

[4] [^] [1] [2] [=]

Answer:
16,777,216

Example 2 Simplify $28^{\frac{3}{17}}$

Solution:

Press

[2] [8] [^] [3] [a b/c] [1] [7] [=]

Answer:
1.800449021

Example 3 Simplify the fraction $\frac{1258}{28}$. Show the answer in simplest form expressed as

a) improper fraction b) mixed number c) convert it to decimal

Solution:

To simplify fraction, press

First press

[1] [2] [5] [8] [a b/c] [2] [8]

To simplify the fraction just press [=]

Answer:
44┘13┘14

The typical scientific calculators display that kind of answer which is a mixed number read as "43 and 11 over 29". The **natural display calculators** will show $44\frac{13}{14}$. These calculators shows the natural way, fractions are written, that is why they are called natural display. But not all brands offer the same display. So check your scientific calculators.

Notice that the answer shows a mixed number.

a) to convert the simplified answer to its corresponding improper fraction, press the following 2nd F a b/c =

Answer:
629⌟14

Some calculators use the shift key shift for function on the second level.

b) If you want to view the mixed form of the fraction again, just press a b/c.

c) To convert it to decimal, press D→F =

Answer:
44.92857143

Example 4 Simplify $(5^2 + \frac{1}{8}) \div (0.12 - 4)^{\frac{3}{4}}$.

Solution:

Press the following keys. Notice that the keys are pressed in order they were written in the given $(5^2 + \frac{1}{8}) \div (4 - 0.12)^{\frac{3}{4}}$.

(5 x^2 + 1 a b/c 8) ÷

(4 - 0 . 1 2) ^ 3 a b/c 4

When you press = you will get

Answer
9.088292086

Example 5 Simplify 25!

Solution:

This example involves factorial notation that you will use in your probability module.

2 5 n! =

Answer:
$1.551121004 \times 10^{25}$

Other scientific calculators have the factorial notation on the second level. So the shift key or 2nd F key will me used. Other calculators use the following keys for factorial notation

[x!] or [!] . Notice that their similarities is the ! symbol. So when you see this symbol alone or with a letter n or x, this suggest that the key is used for factorial notation.

The answer is displayed in scientific notation as **1.551121004** $_{x10}{}^{25}$.

Example 6 Express 28.425° in degree, minutes and seconds.

Solution:

This example involves the key [o ‘ “]. It is used in dealing measures of angles in degrees and can be converted to degree, minutes, and seconds.

To convert 28.425° to degrees, minutes, seconds form, press the following keys

[2] [8] [.] [4] [2] [5] [o ‘ “] [=]

The screen will show

28.425°

Notice that the one shown in the screen is the actual given. Press the key [o ‘ “] again to get the simplified form in degrees, minutes, seconds

Answer
28°25°30

The answer displayed on the screen is 28°25’ 30” read as “**28 degrees, 25 minutes, 30 seconds**”. You will have more of these measures in your Trigonometry module.

Example 7 Find the value of tan 25.32°.

Solution:

This example makes use of the trigonometric function. To get the value of tangent 25.32° just press the following keys

[tan] [2] [5] [.] [3] [2] [o ‘ “] [=]

Answer
0.473124967

Example 8 Find the value of arc sin 0.25

Solution:

This example involves finding the measure of an angle in Trigonometry. The arc sin function appears as **sin** $^{-1}$ in the scientific calculator and usually found on the 2nd fl.

sin $^{-1}$ B

[2nd F] [sin] [0] [.] [2] [5] [=]

Answer
14.47751219

The answer is in decimal form, since this function usually deals with angles, you can easily convert this to degree-minutes-seconds form by pressing the key [o‘‘‘].

The screen will show 14°28’39.04”.

Answer
14°28°39.04

Example 9 Find the permutation of 10 taken 3 at a time.

Solution:

This example involves permutation which will be discussed in your Grade 12 Probability and Statistics module.

Permutation problems usually involve the key [$_nP_r$] in some calculator models this key is found in the upper level [$_nC_r$] ($_nP_r$).

Once you reached Probability and Statistics module you will learn that the given permutation of 10 taken 3 at a time is written as $_{10}P_3$. Using the calculator just press the following

[1] [0] [$_nP_r$] [3] [=]

Answer
720

If the function is found on the 2nd level, just press the **shift key or 2nd F** before pressing the key containing $_nP_r$ just like the sample below.

[1] [0] [2nd F] [$_nC_r$] ($_nP_r$) [3] [=]

Example 10 Find the combination of 15 taken 6 at a time.

Solution:

Dealing with combination is somehow similar with how you treat permutation. Again, you will learn about this once you reached Grade 12. For the mean time in preparation for that at least you know how to operate your scientific calculator.

Combination problems usually involve the key $\boxed{{}_nC_r}$ in some calculator models this key is found in the upper level $\overset{{}_nC_r}{\boxed{\div}}$.

The given combination of 15 taken 6 at a time is written as ${}_{15}C_6$. Using the calculator press the following

$\boxed{1}\ \boxed{5}\ \boxed{{}_nC_r}\ \boxed{6}\ \boxed{=}$

Answer
5,005

If the function is found on the 2nd level, just press the **shift key or 2nd F** before pressing the key containing ${}_nC_r$ just like the sample below

$\boxed{1}\ \boxed{5}\ \boxed{2^{nd}\ F}\ \overset{{}_nC_r}{\boxed{\div}}\ \boxed{6}\ \boxed{=}$

Knowing how to operate your scientific calculator will help you facilitate fast and more accurate computation in your higher Mathematics modules.

It is important to learn more than just the basic to keep you abreast with the fast and changing modes of learning.

Keep practicing and be ready to answer the following learning activities.

LEARNING ACTIVITY 11.1.1.7

20 minutes

Using a scientific calculator, perform the indicated operation (s).

1) $5^{\frac{3}{5}}$ ____________________________

2) $\dfrac{1}{3\left(4^2\right)^{\frac{1}{5}}}$ ____________________________

3) $(2 - \frac{3}{4})^3 + (12.5 \text{ x } 0.02)^3$ ____________________________

4) $2! + 5!$ ____________________________

5) ${}_{12}P_6$ ____________________________

6) ${}_6C_3 + 3! - 4^2$ ____________________________

7) Simplify the following fractions

	Mixed Fraction	Improper Fraction
a. $\dfrac{125}{35}$	______________	______________
b. $15\dfrac{68}{24}$	______________	______________
c. $\dfrac{6894}{52}$	______________	______________

8) Express the sum or difference of the following angle measures in degree-minute-second.
   a. $23.45° + 2.\ 002°$ ____________________________
   b. $578.234° - 234.005°$ ____________________________
   c. $12.34° + 0.99° - 2.4°$ ____________________________

9) Find the values of the following using trigonometric functions. Round off answers to two decimal places.
   a. $\sin 23.45°$ ____________________________
   b. $\tan 78.23°$ ____________________________
   c. $\cos 44.7°$ ____________________________

SUMMATIVE TASK 11.1.1

40 minutes

A. Multiple Choice. Encircle the letter of the correct answer.

1) Which of the following is an irrational number?
   A. -3 C. 2/3
   B. 3.1616… D. 0.333
2) What property of real numbers is illustrated in the statement " 2+3+5 = 5+2+3"
   A. Associative property C. Identity property
   B. Commutative Property D. Inverse property

3) It tells the number of times the base is to be multiplied by itself.
   A. Base C. exponent
   B. Radicand D. index

4) How many significant figures are there in the numeral 2.0300000000000000000?
   A. 2 C. 3
   B. 10 D. 19

5) Which function of the calculator is used to deal with fractions?
   A. Shift Key C. x^2
   B. $\wedge$ D. a b/c

6) It refers to the process also knows as approximation.
   A. Estimation C. Summation
   B. Rounding off D. Evaluating

7) 3.563×10^7 in standard index form is equal to
   A. 35 630 000 C. 356 300 000
   B. 3 563 000 000 D. 35 630 000 000 000

8) It refers to a set of keys in a scientific calculators where the digits 0-9 are found.
   A. Function keys C. Alpha keys
   B. Numeric keys D. 2^{nd} F keys

9) The fourth root of x cube is written as
   A. $\sqrt[3]{x^4}$. C. $\sqrt[4]{3x}$
   B. $\sqrt[4]{x^3}$ D. $\sqrt[4]{3x^3}$

10) Surds are also called
   A. Indices C. Radicals
   B. Conjugates D. Inverses

B. Solve the following

1) $\sqrt{80}$

2) $\sqrt{\frac{16}{25}}$

3) $18\sqrt{3} - 2\sqrt{3}$

4) $\sqrt{15} \text{ x } \sqrt{6}$

5) $2x^0 - (5x)^0$

6) $\log_2 16$

7) $\log_3 81$

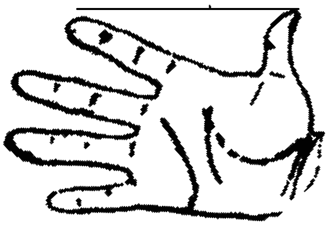
8) $2.5\text{x}10^4 + 2345\text{x}10^3$

9) 0.0000003 x 2.240000000000000 (express answer in standard index form)

10) $(2.21\text{x}10^4)(1.5\text{x}10^{-3})$

11.1.2 UNITS OF MEASUREMENTS

Weights and measures were among the earliest tools invented by man. Primitive societies needed rudimentary measures for many tasks: constructing dwellings of an appropriate size and shape, fashioning clothing and bartering food or raw materials.



Man understandably turned first to parts of his body and his natural surroundings for measuring instruments. Early Babylonian and Egyptian records and the Bible indicate that length was first measured with the forearm, hand, or finger and that time was measured by the periods of the sun, moon, and other heavenly bodies.

When it was necessary to compare the capacities of containers such as gourds or clay or metal vessels, they were filled with plant seeds that were then counted to measure the volumes. With the development of scales as a means for weighing, seeds and stones served as standards.

In the primitive years, people used their body parts to measure objects; however, this system may not be fair because people have different body dimensions.

Ancient container used to measure volume.

Plant seeds used to fill in ancient containers which were counted to measure volume.

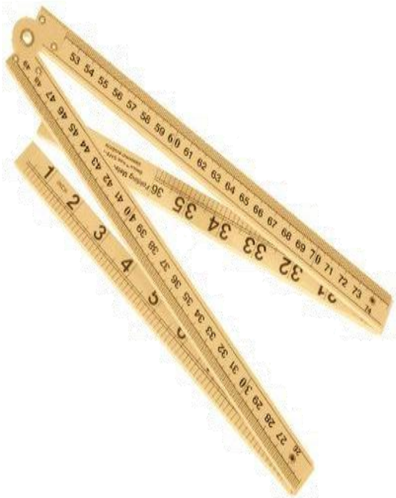
For instance, the "carat," still used as a mass unit for gems, is derived from the carob seed. As societies evolved, measurements became more complex. The invention of numbering systems and the science of mathematics made it possible to create whole systems of measurement units suited to trade and commerce, land division, taxation, and scientific research. For these more sophisticated uses, it was necessary not only to weigh and measure more complex things but also necessary to do it accurately time after time and in different places. However, with limited international exchange of goods and communication of ideas, it is not surprising that different systems for the same purpose developed and became established in different parts of the world - even in different parts of the same country.

At present, there are two commonly used and accepted standards or systems in measurement: the English System and the Metric System.

The Metric system is also known as the International System of units or SI. It is widely used all over the world because it uses bases which are multiples of 10. This gives an easier way of converting from one unit to another among the units of length, mass and volume.

The following are some of the base units of SI, with some commonly used measuring tools:

Metre (m) **Kilogram (kg)** **Litre (L)**

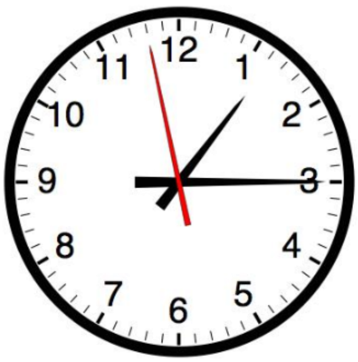


Foldable meter stick

Weighing scale

Measuring cup

Seconds(s) **Degree Celcius (°C)**



Analogue clock

Digital thermometer

11.1.2.1 Metric and Imperial Measures

Measurement is a process of comparing the dimensions of an object using a standard way of quantifying things. When we say standard, we mean something that is agreed upon and commonly used by many.

The Imperial system of units was defined in 1824 and was used by many countries in the British Empire. But before the 20th century ended, some of these countries had converted to metric units.

At present, there are two commonly used and accepted standards or systems in measurement: the English System and the Metric System or known as the International System of Units (SI).

The International System of Units (SI) defines the following seven units of measure as a basic set from which all other SI units are derived:

Units of Measure	Used to measure
Meter	length
Kilogram	mass
Second	time
Ampere	electric current
Kelvin	temperature
Candela	luminous intensity
Mole	amount of substance

Since the base unit for length is meter, the same is used in dealing with area and volume. Another unit used to measure capacity is litre. However, this is not yet formally part of SI but it is accepted for use with the SI.

In the metric system, each quantity measured has a basic unit of which other units were based using prefixes.

To easily remember these prefixes, we may use the mnemonics **K**arl **H**as **D**eveloped **M**y **D**ecimal **C**ravings for **M**etrics. This mnemonics will help you easily remember the firs letters of the prefixes used in the metric system.

Kilo (1000)	Hector (100)	Deka (10)	metric base unit	Deci (1/10)	Centi (1/100)	Milli (1/1000)
kilometre	hectometre	dekametre	**Metre**	decimetre	centimetre	millimetre
kilogram	hectogram	dekagram	**Gram**	decigram	centigram	milligram
KiloLitre	hectoLitre	dekaLitre	**Litre**	deciLitre	centiLitre	milliLitre

The common among prefixes used are: kilo, centi and milli. However, knowledge on the other prefixes will be beneficial because they are used in some specific areas such as in the fields of medicine, engineering, architecture, physics, chemistry and the like.

The table below shows some of the commonly used units in both Metric and Imperial systems.

Quantity it measures	Metric	Imperial
Length	Meter(m), kilometre (km), centimetre(cm), millimetre(mm)	inch (in) foot(ft), yard(yd), mile(mi)
Weight	gram (g), kilogram (kg) , milligram (mg)	Ounce(oz), pound(lbs), ton (t)
Capacity	liter (L), millilitre (mL)	Gallon, quarts, pint
Pressure	Kilopascals (kPa)	Pounds per square inch (psi)

LEARNING ACTIVITY 11.1.2.1

20 minutes

1) What is the importance of having and following a common standard of measurement?

____

____

____

2) List some measuring tools you use at home and identify if they are using the SI or imperial units.

____

____

____

3) List the metric unit for :

(a) 1 lb
(b) 1 oz
(c) 1 gal
(d) 1 mi
(e) 1 fl oz

4) Write imperial equivalent for :

(a) 1 m
(b) 1 kg
(c) 1 L
(d) 1 km
(e) 1 g

11.1.2.2 Conversion of Metric and Imperial

Sometimes you will need to convert or change from one system to the other especially when you travel abroad because one country may use both the imperial and SI units. In addition, the labels on the goods you usually find in the shops use either of the two units. It is also important to learn how to convert from one unit to another because your job in the near future may require you to do so. To do this, you need a chart that lists equivalent measurements.

If you need to convert units between systems when working on a job, you are usually provided with the equivalents, but you will save yourself time if you know the most common equivalents. So it will be very helpful if your try your best to learn these by heart.

Before we start converting between the two systems, we will briefly review how to convert within each of the systems.

Converting Within the Imperial System

The table below shows the equivalents used when converting within the imperial system.

IMPERIAL EQUIVALENTS

1 foot (ft)	12 inches (in)
1 yard (yd)	3 feet (ft)
1 mile (mi)	1760 yard (yd)
1 pound (lb)	16 ounces (oz)
1 quart (qt)	2 pint(pt)
1 gallon (gal)	4 quart (qt)

In this chart, the larger units are on the left. Charts might be set up with the larger units on the right, so the first line would read 1 foot = 12 inches. However, one unit, usually the larger unit, has a 1 in front of it (like 1 ft, 1 yd, 1 mi). The other unit has a number different than 1 in front of it (like 12 in, 3ft, 1760 yd).This number is the **conversion factor**. To convert an imperial amount in one unit to another unit, follow these two rules.

Rule 1: To convert **<u>from a larger unit</u>** such as feet to a smaller unit such as inches, **<u>multiply the amount of the larger unit by the conversion factor</u>**.

Example 1: Convert 5 feet to inches.

The chart tells you that 1 foot = 12 inches. The conversion factor is 12. To convert 5 feet to inches (larger unit to smaller unit), multiply by the conversion factor 12.

1 ft = 12 in

5 ft = 5 x 12

= 60 in

Rule 2: To convert **from a smaller** unit such as quarts to a larger unit such as gallons, **divide the amount of the smaller unit by the conversion factor**.

Example: Convert 10 quarts to gallons.

From the chart, 1 gallon = 4 quarts. The conversion factor is 4. Since you are going from a smaller unit to a larger one, you divide by 4.

1 gal = 4 qt

10 qt = 10 ÷ 4

= 2.5 gal

Converting within the Metric System

To convert within the metric system, you multiply or divide by 10, or a power of 10 such as 100 or 1000. To do this, you move the decimal point to the left or right the required number of places, using zeros as place holders when necessary. (You are actually multiplying or dividing by a power of ten when you move a decimal point.)

The table below lists the most commonly used prefixes with the basic units for weight (gram), length (meter), and volume (litre).

METRIC EQUIVALENTS

Prefix	Value	Basic Unit
Kilo	1000	gram metre litre
Centi	1/100 = .01	gram metre litre
Milli	1/1000 = .001	gram metre litre

To convert to or from the basic units of metre, gram or litre, look at the prefix in front of the non-basic unit to tell how many places to move the decimal point.

1. The prefix kilo means 1000 times the basic unit.
To change from a unit with kilo as its prefix to a basic unit, or to convert from a basic unit to a unit with kilo as its prefix, move the decimal point 3 places.

2. The prefix milli means .001 times the basic unit.
To change from a unit with milli as its prefix to a basic unit, or to convert from a basic unit to a unit with milli as its prefix, move the decimal point 3 places.

3. The prefix centi means .01 times the basic unit.
To change from centi to a basic unit, or to convert from a basic unit to a unit with centi as its prefix, move the decimal point 2 places.

What direction do you move the decimal point to complete the conversion?

Use the following rules:

1. To convert from a smaller unit to a larger one, move the decimal in the original amount to the left.
2. To convert from a larger unit to a smaller one, move the decimal to the right.
3. The number of places to move the decimal point in the original amount when converting to or from a basic unit depends on the prefix of the other unit.

- With the prefix kilo, move the decimal three places.
- With the prefix milli, move the decimal three places.
- With the prefix centi, move the decimal two places.

Example Convert 650 millimeters to meters.

Solution:

Converting from a smaller unit to a larger one, the decimal point moves to the left. The prefix milli indicates that the decimal point moves three places when going to or from a basic unit. Move the decimal point three places to the left. Change the unit to meters.
650 mm = .650 m

CONVERTING BETWEEN METRIC AND IMPERIAL UNITS

Sometimes you need to convert, or change, from one system to the other. To do this, you use a chart, like the table below which lists equivalent measurements.
To use the chart to convert an amount in one system to the other system, you need to determine the conversion factor.

IMPERIAL TO METRIC EQUIVALENTS

IMPERIAL Units	METRIC Units
1 inch (in)	= 2.54 centimeters (cm)
1 foot (ft)	= .305 meters (m)
1 yard (yd)	= .914 m
1 mile (mi)	= 1.6 kilometer (km)
1 ounce (oz)	= 28.38 g
1 pound (lb)	= 454 grams (g) or .454 kg
1 quart (qt) Canadian	= 1.14 liters (L)
1 gallon (gal)	= 4.56 L
1 fluid ounce (fl oz)	= 28.4 ml
degrees Fahrenheit (F)	= 9/5 degrees C +32

METRIC TO IMPERIAL EQUIVALENTS

METRIC Units	IMPERIAL Units
1 cm	= .39 in
1 m	= 39.4 in = 3.3 ft = 1.09 yd
1 km	= .62 mi
1 g	= .035 oz = .0022 lb
1 kg	= 2.2 lb
1 L	= .88 qt
1 L	= .22 gal
degrees Celsius (C)	= 5/9(degrees F - 32)

To convert an amount in centimetres to an equivalent amount in inches, look at the Metric to Imperial side of the chart. Find the cm to inch equivalent: 1 cm = .39 inches.

The number .39 in front of inches is the conversion factor. Multiply the original amount in centimetres by the conversion factor .39 and add the new unit, inches, to the answer. You have now converted an amount originally in centimeters to the equivalent amount in inches.

Suppose you are cutting a piece of block that is 4 feet long. If you are working from metric measurements on your reference chart, you might want to convert this length to meters.

Refer to the Metric side of the chart and find the conversion factor. The unit with 1 in front is the unit you are converting from. The unit with the conversion factor in front is the unit you are converting to.

Example Change 12 feet to meters.

Solution:

Look at the Imperial to Metric chart to find the line that equates feet to meters. The chart states that 1 foot = .305 meters, so .305 is the conversion factor.

12 x .305 = 3.66 multiply the original amount of 12 feet by the conversion factor .305

12 feet = 3.66 meters Place the metric unit, meters, after the answer.

To convert from an amount in an imperial unit to a metric unit, here are the steps:

1. Use the Metric to Imperial side of the chart
2. The conversion factor is in front of the metric unit.
3. Multiply the original imperial amount by the conversion factor.
4. Put the correct metric units after the multiplication answer.
5. The amount is now converted to metric.

Example Find the metric equivalent of 8 gallons.

Solution:

Since you are converting an imperial amount, 8 gallons, to a metric unit, use the Imperial to Metric chart. The chart shows 1 gal = 4.56 L. The conversion factor is 4.56. Multiply 8 times 4.56 and change the unit to litres.

1 gal = 4.56 L

8 gal = 4.56 x 8

= 36.48 L

If the chart does not show a direct equivalent, you must convert within the original system until you have a unit with a metric conversion factor. For example, if you want to convert feet to centimetres using the chart on the previous page, you must first convert the feet to inches. Then you can use the conversion factor on the chart for inches to centimetres. First convert the feet to inches, then convert inches to centimetres.

Example: Convert 5 feet to centimetres.

The tables do not show feet to centimetres, so first convert feet to inches.

Use Table for Imperial Equivalents.

1 ft = 12 inches conversion factor is 12

5 ft = 5 x 12

= 60 in

Now convert 60 inches to centimetres. Use Table for Imperial to Metric Equivalents.

1 in = 2.54 cm conversion factor is 2.54

60 in = 2.54 x 60

= 152.4 cm

Example Converting 5 feet to centimetres

Solution:

This time convert feet directly to meters and then convert the meters to centimetres.

1 ft = .305 m

5 ft = 5 x .305

= 1.525 m

1.525 m = 152.5 cm

Notice the answers to the same question vary slightly in the examples. This is because the conversion factors have been rounded off. Still remember estimation error we discussed in the previous topics? Don't worry because these estimation errors are acceptable in Mathematics.

Example If you drive 45 miles to work every day, what is the distance travelled in km.

Solution:

From the Imperial to Metric Chart:

1 mi =1.6 km conversion factor is 1.6

45 mi = 1.6 x 45

= 72 km

You might also encounter situations wherein you need to convert from metric to imperial units. Use the Metric to Imperial Equivalents. The procedure for using the chart is the same.

The unit with the 1 in front is the metric unit. The number in front of the imperial unit is the conversion factor. Because there are fewer units in the metric system, there are sometimes several conversion factors after a metric unit.

For example you will see in the table that 1 meter can equal 39.4 inches or 3.3 feet or 1.09 yards.

To convert from meters to feet, choose the conversion factor that changes meters to feet, which is 3.3. To convert from meters to inches, choose the conversion factor 39.4.

To convert from a metric unit to an imperial unit, here are the steps:

1. The conversion factor is in front of the imperial unit in the Metric to Imperial Equivalent chart.

2. Multiply the original metric amount by the conversion factor.
3. Put the correct imperial unit after the multiplication answer.
4. The amount is now converted to imperial units.

Example You are travelling to a city 450 km away. How many miles do you travel?

Solution:

Since you are converting an amount, 450 km, in a metric unit to an imperial unit, use the Metric to Imperial chart. From the chart:

1 km = .62 mi conversion factor is .62

450 km = .62 x 450

= 279 mi

Example Convert 4.5 litres to gallons.

Solution

From the Metric to Imperial chart:

1 L = .22 gal conversion factor is .22

4.5 L = 4.5 x .22

= .99 gal

Please remember!

1. When you convert, the first step is to check which system the amount is now in and which system you are converting to.
2. Next, choose the correct chart for that conversion.
3. Find the line with the required conversion factor on it.
4. Multiply the original amount by the conversion factor.
5. The multiplication answer is the amount in the new unit.
6. The last step is to write the new unit after the converted amount.

More examples.

Example A tree has a diameter of 5 in. What is its diameter in mm?

Solution:

The imperial to metric chart has no inch to millimetre conversion factor, so we will convert in two steps.

1 in = 2.54 cm First convert from inches to centimeters (**U**se Table 3).
5 in = 2.54 x 5
= 12.7 cm

Since 12.7 cm = 12.7 x 10, then convert the answer to millimetres.
= 127 mm

Example One litre of water weighs 1 kg. How many pounds does 4 L weigh?

Solution:

There are two steps to this problem. First we need to find how much 4 L weighs in kilograms and then we need to convert the kilograms to pounds.

1 L weighs 1 kg
4 L weigh 4 kg
1 kg = 2.2 lb conversion factor is 2.2
4 kg = 2.2 x 4
= 8.8 lb

Example A truck gets 30 mi/gal. How many km/L does it get?

Solution:

This conversion requires several steps.

1 mi = 1.6 km, first convert mi to km.
30 mi = 1.6 x 30
= 48 km
30 mi/gal = 48 km/gal

and

1 gal = 4.56 L Now convert gallons to liters.
48 km/gal = 48 km/4.56 L
48 ÷ 4.56 = 10.5 km/L Divide 48 by 4.56 to find km/L.
30 mi/gal = 10.5 km/L.

The truck gets 30 mi/gal or 10.5 km/L

Converting from one unit to another and from one system to another may sound difficult but it is not, right? You just have to remember the steps we have discussed in this unit.

Revise this lesson and be ready to challenge yourself in the following learning activity.

LEARNING ACTIVITY 11.1.2.2

20 minutes

1) Convert the following to the indicated unit on the right after each blank.
   a) 15 m = ______________ km
   b) 2845 g = ______________ kg
   c) 25, 000 m = ______________ km
   d) 120 ft = ______________ yd
   e) 45.6 L = ______________ gal
   f) 254 mm = ______________ in
   g) 280 km = ______________ mi

2) Solve the following problems. Use the spaces for your working out.
   a) A 120 cm piece was cut from a rope measuring 30 m. What is the length in meters of the remaining piece?

      Answer: ________________________

   b) Some selected tomatoes from the shop weigh an average of 50 g. approximately, how many pieces of tomatoes will there be in 2.5 kg?

      Answer: ________________________

   c) John wants to transfer 5 L of milk in 1 gallon container. Is the container enough to hold the milk? Justify your answer.

      Answer: ________________________

11.1.2.3 Measuring Devices and Scales

In the previous lesson, we have discussed the different units of measures commonly used, however, we have not discussed how these measures were derived. In measurement, a measuring tool is very important in order to describe the "measure" or dimensions of a specific object. Example, if you want to determine the area of your study table, it is impossible to get its area by just looking at it. You have to determine its length and width first using a measuring device.

In this lesson, we will focus on the measuring devices and scales commonly used in our everyday living.

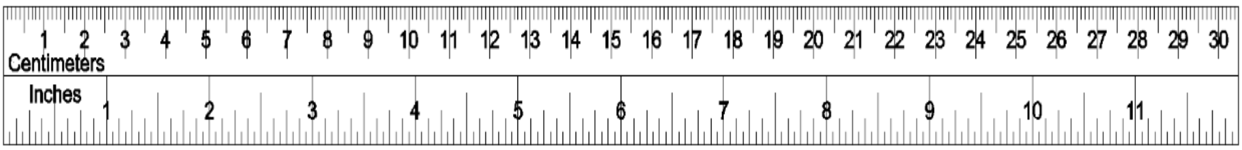
The table below shows the summary of the devices and scales that you can easily find around you.

Types of Measurement	Tools Used
Length	ruler meter sticks tape measures
Mass	balance scales, spring scales, weighing scales
Volume	graduated cylinders
Time	clocks
Temperature	thermometers

Measuring Length

The most common of all the measuring devices is the ruler. It is a straightedge commonly made from wood or light plastic which has a length of 12 inches or 30 cm in length as shown below:

Plastic Ruler



Wooden ruler

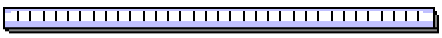
Other rulers used in school are a little shorter just the like the sample below.

Kid's ruler

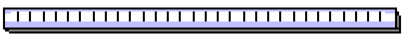
Longer rulers with a length of 18 inches (45 cm) are also available for some specific uses.

For longer measurement needs yardsticks (1 yard long), meter sticks (1 meter long) and tape measures (20 metres long) are also used.

Yard Stick vs Meter Stick

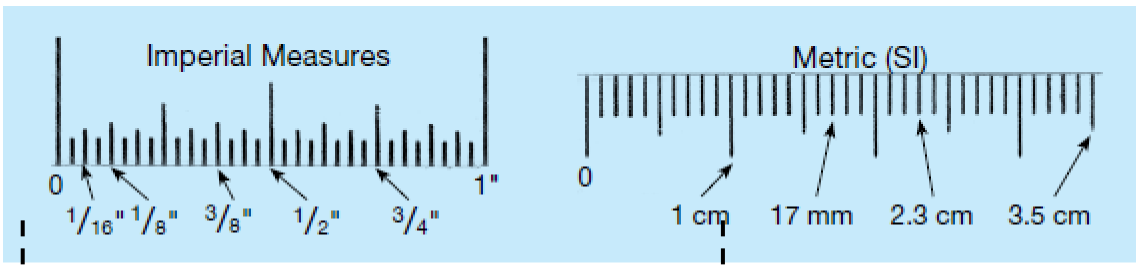


Meter Stick



Yard Stick

The following are examples of readings using both imperial and metric system found in most rulers.



Example 1 Find the measure of the digital thermometer in cm using a ruler

S

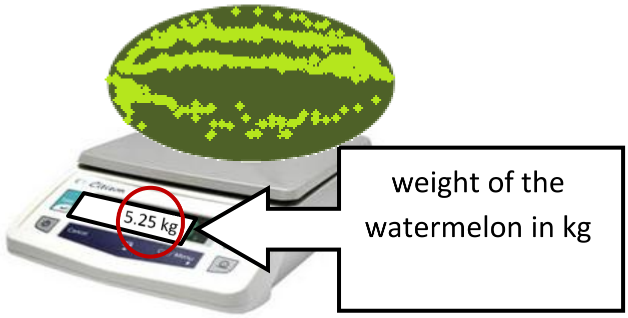
Measuring Mass

Mass is usually measured using scales. Nowadays, digital scales are already used and are common in Papua new Guinea especially in big shops like Vision city and Food World.

Reading digital scales is a lot easier as compared to the analogue or traditional ones. Just put the object being measured like vegetables, meat, sugar and other goods usually found in markets and shops and the weight usually in kilograms (kg) is displayed.

Example 2 Find the weight of watermelon in kilogram using the the digital scale.

Solution:

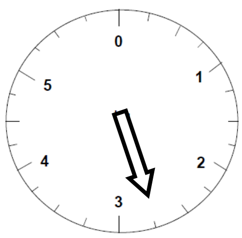


Using the digital scale, the weight of the watermelon is 5.25 kg.

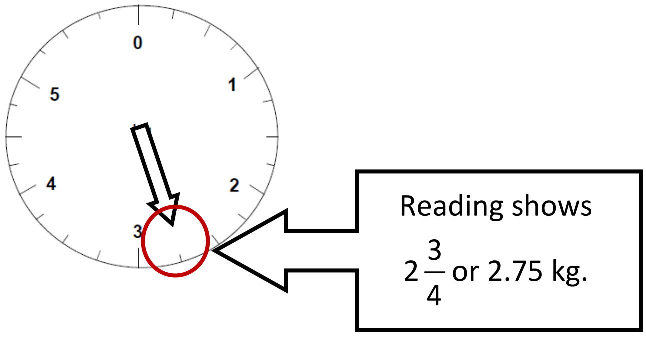
The **traditional scales** are also commonly used in PNG especially in wet markets. Others call it as clock scale because it looks like a clock and it is read in clockwise motion.

Some of these scales can measure up to a maximum of 20 kg, others 11kg and for small quantities a scale of 6 kg and 1 kg maximum are also available. The scale shown above can measure at a maximum of 11 kg.

Example 3 Read the measurement shown in the scale.



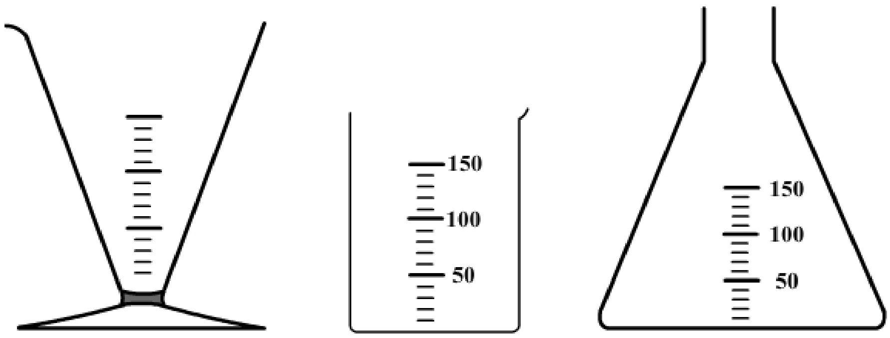
Solution:



Notice that this scale has an interval of $\frac{1}{4}$ kg or 250 grams.

Measuring Volume

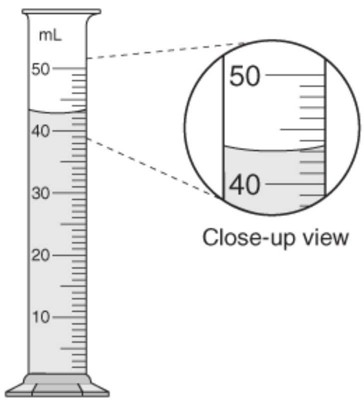
Volumes are usually measured in cubic meters for bigger amount such as the amount of water in a swimming pool or an aquarium tank. But small amount of liquid are usually measured using the following:



(Conical measure) (Beaker) (Erlenmeyer)

A **graduated cylinder** is used to more accurately measure volumes of liquids such as chemicals, medicines and other liquids used with needed accuracy in measure. They usuallu come in 10 mL, 50 mL and 100 mL.

The example below is a 50-mL graduated cylinder.



The cylinder shows a reading of 43 mL.

In cooking and baking, measuremet of liquids is also important to follow the correct recipe. Some measuring cups like the one below provides measure of both metric and imperial for flexibility in use.

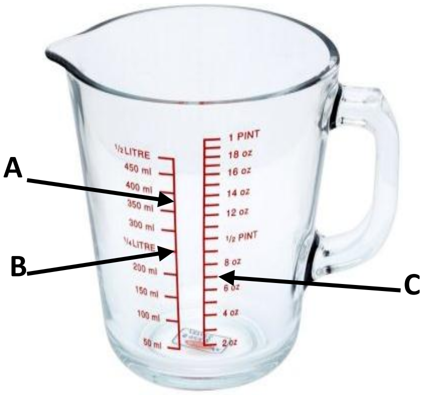
Example 4 Given the measuring cup above, identify the measure indicated by A and B in mL and C in oz.

Solution:

A is between 350 and 400 mL. Therefore, the interval is 50 mL.

Half of 50 mL is 25 mL. Adding 350mL and 25 mL gives 375 mL.

Therefore, A represents 375 mL.



B is ¼ of a Litre. Therefore, by conversion it is 250 mL.

By observation ¼ Litre is in between 200 mL and 300 mL. Knowing that the interval is by 50 mL, then we add 200 mL + 50 mL = 250 mL.

Therefore, B represents 250 mL.

C uses imperial measure, notice that there is 1 oz interval. Since C lies in etween 6 and 8 oz, then we read it as 7 oz.

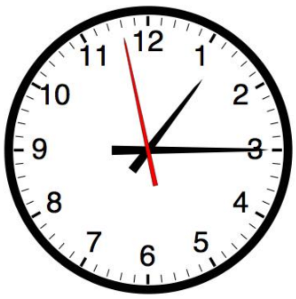
Therefore, C represents 7 oz.

Measuring Time

Measuring time is one of the most familiar as every day, we look at the clock to determine time.

Both digital and analogue clocks show the same time. Digital clocks are as easy to read just like the other digital tools. Some of them they are also designed with calendars and other informative data.

Example 5 Identify the time indicated by the clock below.



Solution:

The clock hour hand points at 1 and its minute hand points at 3. The clock has an interval of 5 minutes. Thus 3 represent 15 minutes.

Therefore, the clock shows 1:15.

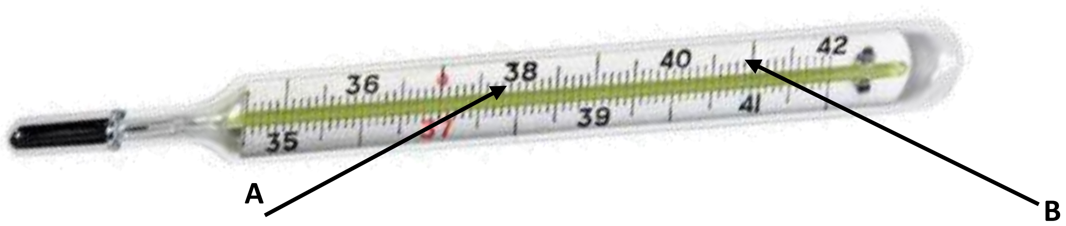
One of the limitations of the analogue clock as compared to digital clocks is that the indication whether the time is a.m. or p.m. cannot be shown.

Measuring Temperature

Temperature is usually measured by thermometers. There are different kinds of thermometers, one of the most common type is the body thermometers that are usually used in clinics and hospitals.

Nowadays, digital thermometers come in different styles and sizes and they are easy to read. While the traditional one which uses mercury remains the same over time.

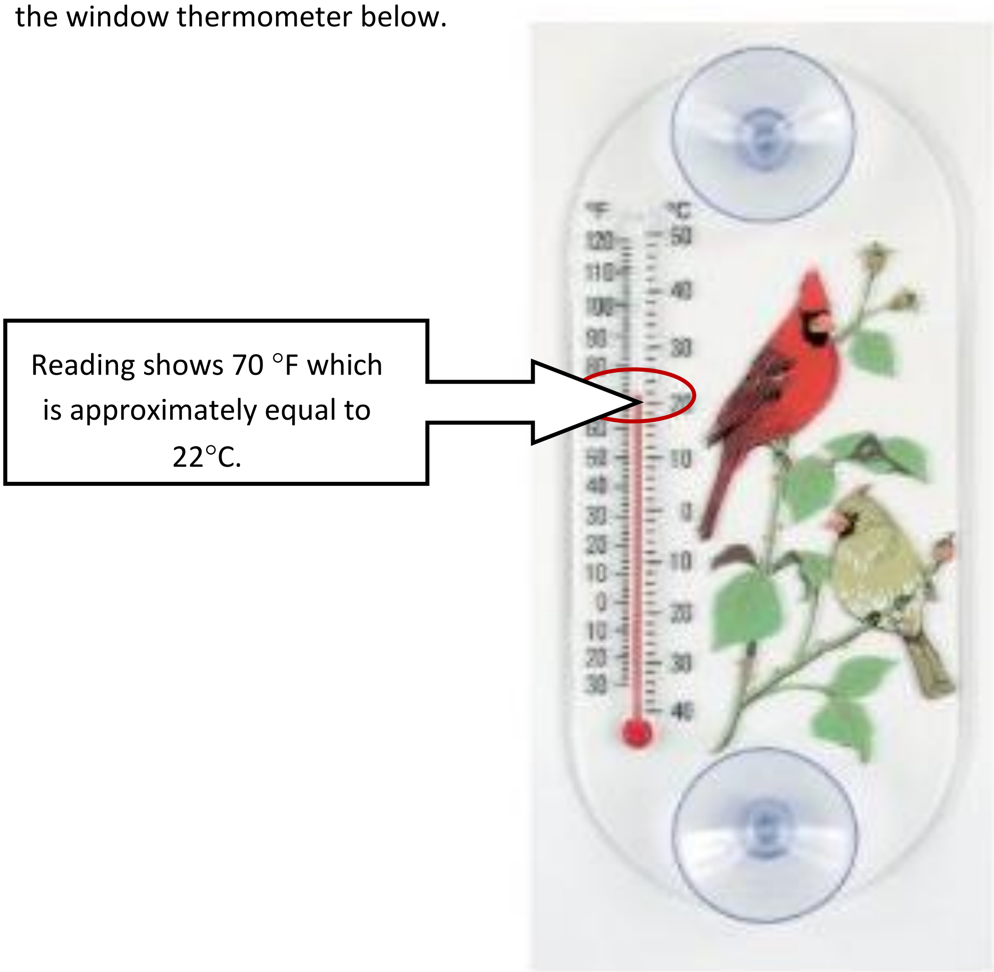
Example 6 Identify the measure indicated in the thermometer below.



Solution:

A reads 37.5 degrees while B reads 40.7 degrees.

Example 7 Identify the temperature in degree Fahrenheit and degree Celsius indicated in the window thermometer below.



Solution:

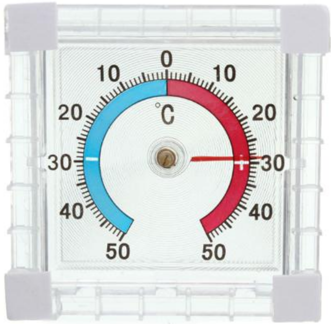
70°F or 22°C

LEARNING ACTIVITY 11.1.2.3

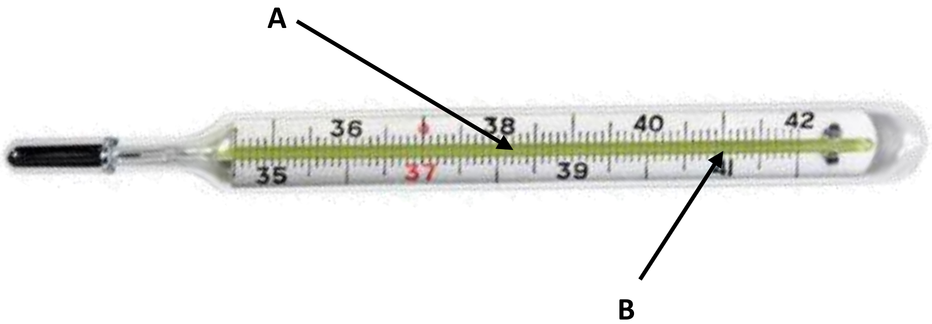
20 minutes

Identify the measurement indicated in the following measuring tools.

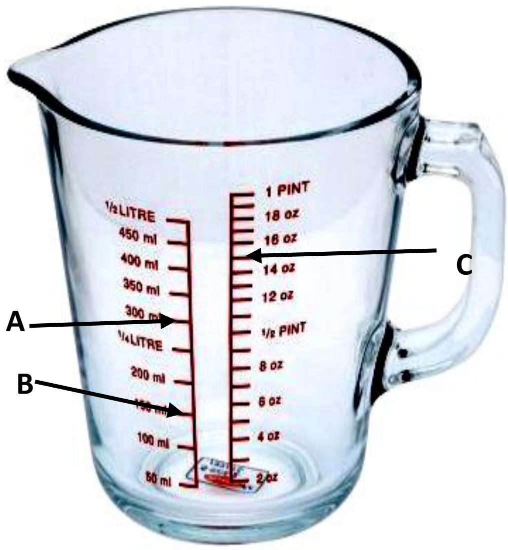
1) Temperature in degree Celsius



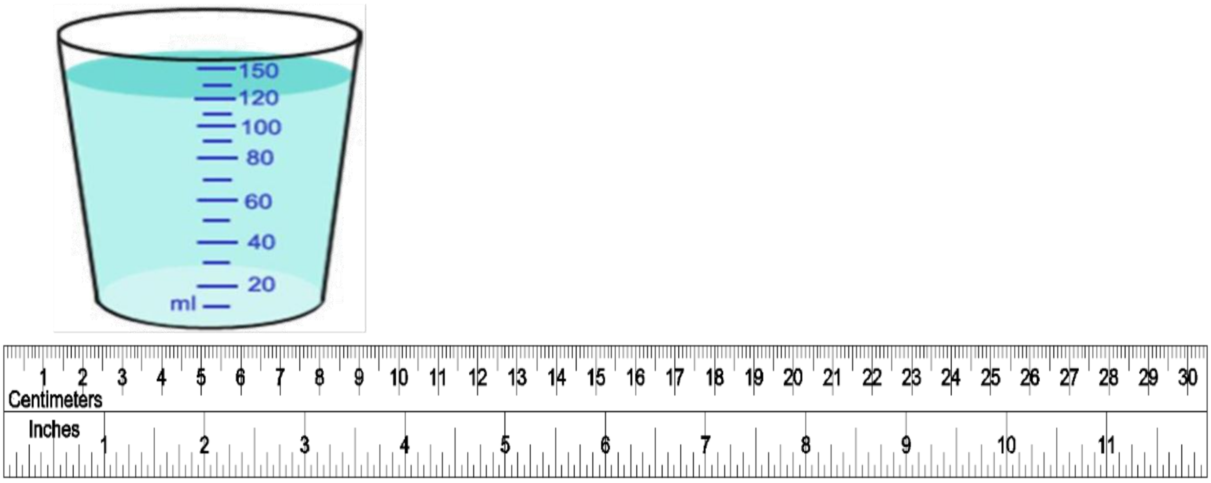
2) Temperature in degree Celsius



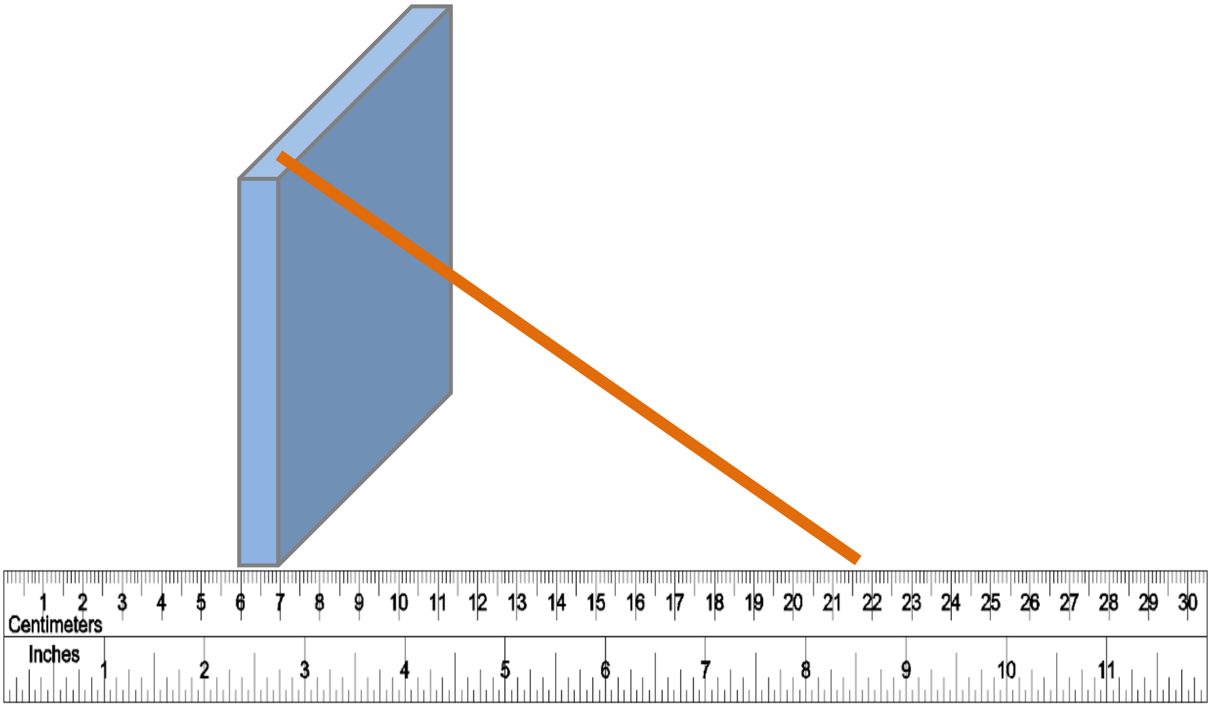
3) Volume of A and B in Ml and C in oz.



4) Volume of water inside the cup in mL and diameter of the bottom of the cup in cm.



5) What is the perpendicular distance of the wall to the end of timber on the ground?



SUMMATIVE TASK 11.1.2

40 minutes

1)Convert the following units.

a) 15 inches to centimetres

b) 260 metres to foot

c) 205 miles to kilometres

d) 270 ounces to grams

e) 120 litres to gallons

f) 85°Fahrenheit to degrees Celsius

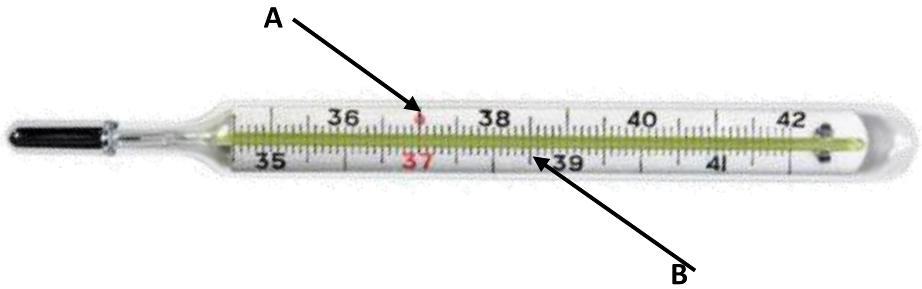
2)Solve the following problems.

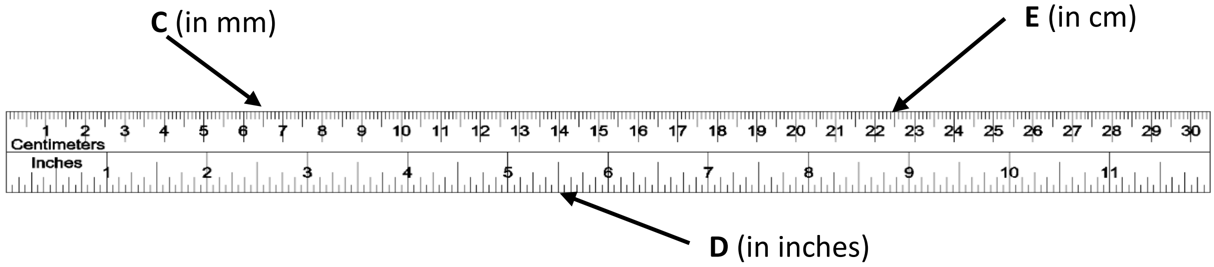
a) The amount of mangoes per kilogram is K12. How much will2 mangoes weighing 750 g costs?

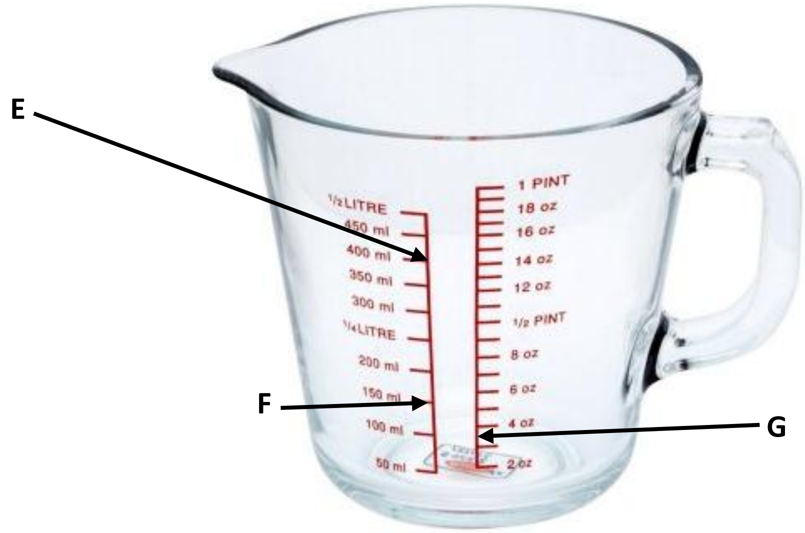
b) A tank holds 912 L of water. If John has containers which can hold a gallon of water, how many containers does he need to transfer the water from the tank?

c) If a car uses a liter of petrol per kilometres it travels. How much petrol does it need to travel 12.4 km?

3) Identify the readings in the following measuring tools.

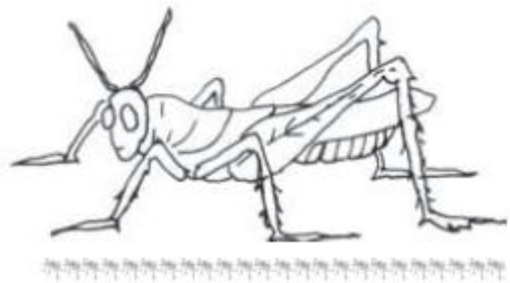






11.1.3 RATIO AND PROPORTION

In our daily life, many a times we compare two quantities of the same type. These comparison often leads to the use of Ratio and Proportion in Mathematics.



If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

11.1.3.1 Ratio

Ratio is the comparison of two quantities having the same unit of measure . It is denoted by the symbol "**:**" read as, "is to".

Consider the following:

Mikha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Mikha's weight?

Analysing the given, one can say it is three times of Mikha's weight. However, using ratio one can say that it is 75 kg : 25 kg. This ratio can also be expressed in terms of a fraction $\frac{75}{25}$ which can be simplified as $\frac{3}{1}$ or 3 in simplest form. Therefore, we can aslo say that 75:25 is the same as 3:1.

Example 1 In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.
(b) Number of boys to the total number of students.

Solution:

First we need to find the total number of students, which is, Number of girls + Number of boys = 20 + 40 = 60.

(a) Number of girls to the total number of students.

The ratio of number of girls to the total number of students is 40 : 60.

This can be expressed as a fraction $\frac{40}{60}$.

Simplify the fraction $\frac{40}{60}\frac{\div 20}{\div 20} = \frac{2}{3}$.

Therefore, the ratio of the number of girls to the number of boys is 2:3.

(b) Number of boys to the total number of students.

The ratio of number of girls to the total number of students is 20 : 60.

This can be expressed as a fraction $\frac{20}{60}$.

Simplify the fraction $\frac{20}{60}\frac{\div 20}{\div 20} = \frac{1}{3}$

Therefore, the ratio of the number of boys to the number of boys is 1:3.

Example 2 Length of a lizard is 20 cm and the length of a crocodile is 4 m. What is the ratio of the length of lizard as compared to the length of the crocodile?

Solution:

If we form a ratio right away, we will have 20: 4 that is equivalent with 5: 1. Thus this suggest that the lizard is five times the length of the crocodile.

This comparison is impossible in real life. A lizard's length cannot be 5 times of the length of a crocodile. So, what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in metres. So, we have to convert their lengths into the same unit before actually comparing their lengths.

The length of the lizard is 20 cm.

The length of the crocodile is 4 m.

4 m = 4 (100 centimetres) = 400 cm.

Forming the ratio we have 20: 400 or $\frac{20}{400}$.

Simplify the fraction $\frac{20}{400}\frac{\div 20}{\div 20} = \frac{1}{20}$

Therefore, the ratio of the lengths of the lizard and crocodile is 1:20. It is also correct to say that the crocodile is 20 times as longer as the lizard.

Note that two quantities can be compared only if they are in the same unit.

Example 3 Find the ratio of 90 cm to 1.5 m.

Solution:

The two quantities have different units. Therefore, we have to convert them into same units. 1.5 m = 1.5 (100 cm) = 150 cm.

Forming the ratio we have 90: 150 or $\frac{90}{150}$.

Simplify the fraction $\frac{90}{150}\frac{\div 30}{\div 30} = \frac{3}{5}$

Therefore, the ratio of 90 cm to 1.5 m is 3:5.

LEARNING ACTIVITY 11.1.3.1

20 minutes

Find the ratio given the following:

1) There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of :

(a) The number of females to number of males. 5:4

(b) The number of males to number of females. 4: 5

2) Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?
John lives nearer to the school (As the ratio is 2 : 1).

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km)	10		6		
Distance from John's home to school (in km)	5	4		2	1

3) Out of 30 students in a class, 6 like football,18 like cricket and remaining like tennis. Find the ratio of:

(a) Number of students who like football to number of students who like tennis.

(b) Number of students who like cricket to total number of students.

11.1.3.2 Proportion

A **Proportion** is a comparison between two ratios. It is also defined as the statement of equality between two ratios. It is denoted by the symbol "**::**" or "=" read as, "as to".

In our previous discussion, we say that Mikha's weight as compared to her father's weight is 25: 75 or simplified as 1 : 3.

With this, we can that 25:75 = 1: 3. This is read as "**25 is to 75 as to 1 is to 3**".

This is an example of a proportion wherein the product of the means is equal to the product of the extremes.

$$\underbrace{25 : \overbrace{75 = 1}^{\textbf{Means}} : 3}_{\textbf{Extremes}}$$

If the product of the means is not equal to the product of the extremes, then the two ratios are not equal and the there is no proportion formed.

Example 1 Identify which of the following is/are proportion(s):

a) 1: 2 = 2 : 6
b) 2 : 3 = 6 : 9
c) 5 : 20 = 1 : 4

Solution:

a) $$\underbrace{1 : \overbrace{2 = 2}^{(2)(2) = 4} : 6}_{(1)(6) = 6}$$

$4 \neq 6$, therefore the given is not a proportion.

b) $$\underbrace{2 : \overbrace{3 = 6}^{(3)(6) = 18} : 9}_{(2)(9) = 18}$$

18 18 , therefore the given is a proportion.

C) $$\underbrace{5 : \overbrace{20 = 1}^{\mathbf{(20)(1) = 20}} : 4}_{\mathbf{(5)(4) = 20}}$$

20 = 20 , therefore the given is a proportion.

Example 2 Cost of 105 lollies is K 35. How many lollies can be purchased for K10?

Solution:

For every 105 lollies it costs K 35, so we form the ratio 105 : 35
The number of lollies (x) is unknown for the amount K10, so we form the ratio x : 10

Form a proportion 105 : 35 = x : 10.

Get the product of the means and the extremes in the ration $$\underbrace{105 : \overbrace{35 = x}^{\mathbf{(35)(x) = 35x}} : 10}_{\mathbf{(105)(10) = 1050}}.$$

From these we form : $35x = 1050$
Divide both sides by 35: $x = 30$

Therefore, 30 lollies can be purchased for K10.

Example 3 A speedboat travels 90 km in2.5 hours

(a) How much time is required to cover 30 km with the same speed?
(b) Find the distance covered in 2 hours with the same speed.

Solution:

(a) In this case, time (t) is unknown and distance is known.

$$\underbrace{90 \text{ km} : \overbrace{2.5 = 30}^{\mathbf{(2.5)(30) = 75}} \text{ km} : t}_{\mathbf{(90)(t) = 90t}}$$

From these we form : $90t = 75$

Divide both sides by 90: $t = \frac{5}{6}$ hr (0.8333…)

To further understand this, we can convert $\frac{5}{6}$ hr to minutes.

$\frac{5}{6}$ (60 minutes) = 50 minutes.

Therefore, 30 km can be covered for 50 minutes.

(b) In this case, distance (d) is unknown and time is known

$$\overbrace{90 \text{ km} : 2.5 \text{ hrs} = d : 2 \text{ hrs}}$$

$$(2.5)(d) = 2.5d$$

$$(90)(2) = 180$$

From these we form : 2.5 d = 180

Divide both sides by 2.5: d = 72

Therefore, the distance covered in 2 hours is 72 km.

LEARNING ACTIVITY 11.1.3.2

20 minutes

1) Write P if the given is a proportion and NP if it is not a proportion.

   a. 1: 5 = 3: 15 _______________

   b. 2: 9 = 18:81 _______________

   c. 5: 12 = 10 : 36 _______________

   d. 8: 24 = 1 : 3 _______________

   e. 5 : 16 = 10 : 26 _______________

2) Solve the following:

   a. If the cost of 7 m of ply board is K294, find the cost of 5 m of ply board.

   b. Cost of 5 kg of rice is K25.
      i. What will be the cost of 8 kg of rice?

      ii. How much rice can be purchased for K85?

   c. The cost of 4 dozens of apples is K96. How many apples can be purchased for K64?

11.1.3.3 Types of Proportions

There are 3 types of proportions namely a) Direct Proportion, b) Inverse Proportion and c) Partitive Proportion.

Direct Proportion indicates that an increase in one quantity results to a proportional increase on the other quantity. On the other hand, a decrease in one quantity results a proportional decrease on the other.

This can be denoted as

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d.$$

Where: **ad = bc**

The sample problems we had in 11.1.3.2 are all examples of direct proportions.

Let us again consider example 2 in the previous lesson wherein we discussed that "If the cost of 105 lollies is K 35. Then, 30 lollies can be purchased for K10".

Note that in this example from K35, the amount of money decreased to K10. Therefore, as expected the number of lollies that can be purchased for K10 also decreases.

Example 1 A car can travel 8 km for every Litre of petrol. How far can it travel for 15 L of petrol?

Solution:

Form a proportion 8 : 1 = d : 15.

Get the product of the means and the extremes in the ration

$$\underbrace{8 : \overbrace{1 = d}^{\mathbf{(1)(d) = 1d}} : 15}_{\mathbf{(8)(15) = 120}}.$$

From these we form :

$$(1)\, d = 120$$
$$d = 120$$

Therefore, the car can travel a distance of 120 km for having 15L of petrol.

Note that as the amount of petrol increase, the distance the car can cover also increases proportionally. Thus, this is another example of direct proportion.

Inverse Proportion indicates that an increase in one quantity results to a proportional decrease on the other quantity. On the other hand, a decrease in one quantity results a proportional increase on the other.

This can be denoted as

$$\frac{a}{\frac{1}{b}} = \frac{c}{\frac{1}{d}} \quad \text{or} \quad a \times b = c \times d$$

Where: **ab = cd**

Example 2 Joey rides his bike at a speed of 10 mph.

a) How long does it take him to cycle 40 miles?
b) On the following day, he cycle the same route at a speed of 16 miles per hour. How much time did he spend on his journey?

Solution

a) To solve for the time he spent we have $t = \frac{40}{10} = 4$ hours

b) Joey took 4 hours to travel at a speed of 10 mph. Thus the portion 4: 10 can be formed.
Since the time (t) he spent cycling is unknown at 16 mph, we form the ratio t : 16.

Now we use the proportion 4:10 = t :16.

Note that this is an example of an inverse proportion. Why? Because as joey's speed increases, the time he needs to cover the same distance decreases.

Thus we use the proportion a : b = c : d, where: **ab = cd**

Given :	$4:10 = t:16$
We have:	$(4)(10) = 16t$
Simplify:	$40 = 16t$
Divide both sides by 16:	$\frac{40}{16} = t$
	$2\frac{1}{2} = t$

Therefore, it took Joey $2\frac{1}{2}$ hours to travel with a speed of 16mph.

Partitive Proportion involves identifying parts of a whole based on a given ratio of its parts. This type of proportion may involve more than 2 unknowns and the whole can be divided into more than 2 parts.

Example 3 A wire 270 m long is to be cut in the ratio 2: 3 : 4. Find the measure of each part.

Solution:

This problem involves partitive proportion. As you notice, there are 3 ratios involve.

The first step is to get the sum of the ratios 2+3+4 = 9

Then we form a ratio of each part to the total number of parts.

For the shortest part we have 2: 9 or $\frac{2}{9}$.

Then, multiply by the total length to be divided: $= \frac{2}{9}(270)$

$= 60$

For the second part we have 3: 9 or $\frac{3}{9}$.

Then, multiply by the total length to be divided: $= \frac{3}{9}(270)$

$= 90$

For the longest part we have 4: 9 or $\frac{4}{9}$.

Then, multiply by the total length to be divided: $= \frac{4}{9}(270)$

$= 120$

Therefore, the measure of each part of a 270 m wire cut in a ratio of 2:3:4 are 60m, 90m and 120m respectively.

To verify if the answers are correct simply add 60 m + 90m + 120m = 270m.

LEARNING ACTIVITY 11.1.3.3

20 minutes

Solve the following and identify the kind of proportion used to solve them.

1) Mar has to travel 280 km. How long does it take if he travels at
   a. 50 km/hr

   b. 60 km/hr

   c. How much time did he save travelling at a faster speed?

2) If K1 = Php 15.30 , convert the following:
   a. K 1250

   b. Php 2 600

3) In a small scale business, Anna invested K5000, Bea invested K3000 and Charlie invested K2000. If at the end of the year they had a profit of K20 000. How much will each of them gets?

4)Add more questions

11.1.3.4 Scales

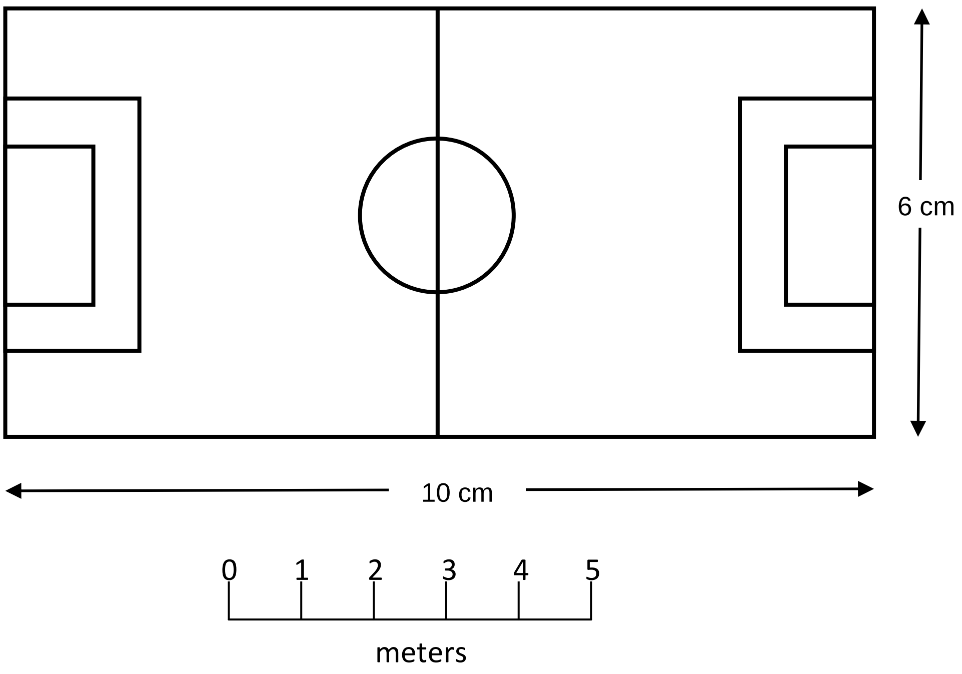
Reading maps, plans, and other blueprints representing an actual measurement on the ground in reality usually require the skill to read scales.

A **Scale** refers to the ratio of the length in a model to the actual length in reality. In plans, maps and drawings a **Bar Scale** is a way of indicating the scale used by means of a bar which shows the ratio of the measurement used is indicated.

A ratio of 1:100 on a drawing means that a scale of 1 to 100 scale is used. This mean that for every 1 centimetre (0.01 meter) measured with a ruler on the model would need to be multiplied by 100 to give the actual size of 1 metre.

Example in a 1:200 scale plan, if the distance between two objects in the figure is 1 centimetre, then the actual distance in real life is 2 meters.

The model below shows the dimensions of a field (in cm). If the measurement on the ruler is 5 cm by 10 cm and the scale used is 1:1000, then the actual dimensions of the field is 60 m (6000 cm) by 100 m (10000 cm).



How to determine the scale used?

In the scale above 1 centimetre on the map shows 10 meters on the ground, 2 centimetres shows 20 meters on the ground and so on.

So 10 m = 10 x 100 cm
= 1000 cm
Therefore the scale used is 1 : 1000

Models and drawings on a plan and maps are drawn using a scale. Standard fractions are used in relation to the actual size of the object being drawn and the dimensions used on the diagram.

Most commonly used scales are:

1)Scale 1:100
It means that 1cm on the plan or map denotes 1 meter in reality.

2)Scale1:50
It means that 1cm on the plan or map denotes 50 cm in reality.

Nowadays, reading maps is simple with the use of technology like the Google Map. You just have to point of the location and it will show the actual distance on the ground with full accuracy. However, it is still good to know how to read scales on maps and plans and not to be too dependent on technology.

Example, if you are using a map on a hiking, your gadgets' baterries may run flat and you will be in total chaos if you d not know hoe to read the maps.

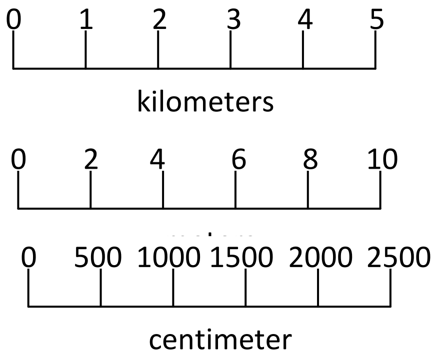
A site engineer and architect (or foreman) should not be too dependent on gadgets too because they have to be in the field most of the time.

Example 1 Write down the scales used in a map represented by the bar scales below

a)

b)

d)



Solution

a) 1 centimeter on the map shows 10 kilometers on the ground, 2 centimeters on map shows 20 km on the ground… and so on

so 10 km = 10 x 100000 cm
= 1 000 000 cm

Therefore the scale used is 1 : 1 000 000.

b) 1 centimeter on the map shows 20 meters on the ground, 2 centimeters shows 40 meters on the ground… and so on

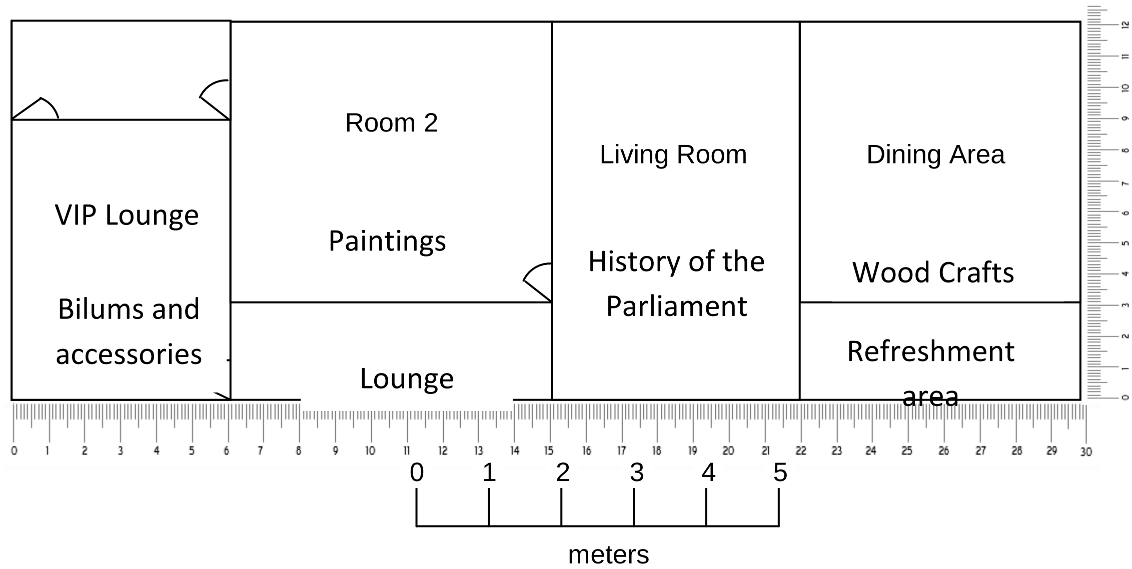
so 20 m = 20 x 100 cm
= 2000 cm

Therefore the scale used is 1 : 2000

c) 1 cm on the map shows 500 cm on the ground, 2 cm on the map shows 1000 cm on the map.

Therefore the scale used is 1:500.

Example 2 A museum which displays the different arts , crafts and history of Papua New Guinea is shown on a map below. The map shows the different rooms which contains exhibits of the same kind from different provinces.



Use the map of the museum to answer the following questions.

a) What scale was used?
b) What are the dimensions of the museum in the map?
c) What is the actual area in m^2 of the museum on the ground?
d) What is the actual distance in meter between the painting exhibits and the wood crafts exhibits?
e) What are the dimensions of the Exhibit on History of Parliament on the map?
f) Compute for the actual area in m^2 on the ground of the room where the History of the Parliament is exhibited.

Solution:

a) Since 1 m = 100 cm, we say 1 m = 1 x 100 cm = 100 cm , therefore **the scale used is 1:100**. It means that every 1 cm on the map corresponds with 100 cm (or 1m) on the ground.

b) The length of the museum on the map is 30 cm, while its width is 12 cm.

c) To solve for the actual area, we first get the actual size of the lot in m^2.

Length = 30 cm (on the plan)
= 3000 cm
= 30 m (on the ground)

Width = 12 cm (on the plan)
= 1200 cm
= 12 m (on the ground)

Area = Length x Width
= 30 m x 12 m
= $360m^2$

d) Looking at the ruler the distance between the painting exhibit and the woodcrafts exhibit are marks in between 15 and 22. To get the distance, we solve for their difference. 22 – 15 = 7. It means that on the plan the distance between the two is 7 cm. We know that 1 cm on the plan represents 1 m on the ground. Therefore we say that the actual distance between them is 7 m.

e)The dimensions of the room where the Exhibit on History of Parliament are shown are the following:
Length = (22 cm – 15cm) = **7cm**
Width = (12 cm – 0 cm) = **12 cm**.

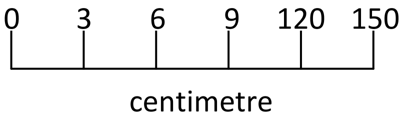
f) Since the length is 7cm and the width is 12 cm, we can say that on the ground their actual measures are 7m and 12m respectively. To solve for the area of the room where the Exhibit on History of Parliament are displayed:
Area = 7 m x 12 m
= **84 m^2**.

LEARNING ACTIVITY 11.1.3.4

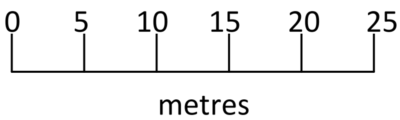
20 minutes

1. Write each bar scale in ratio form.

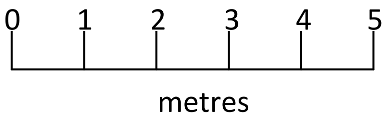
a)



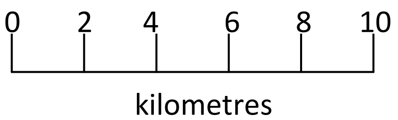
b)



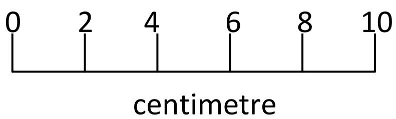
c)



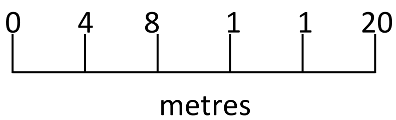
d)



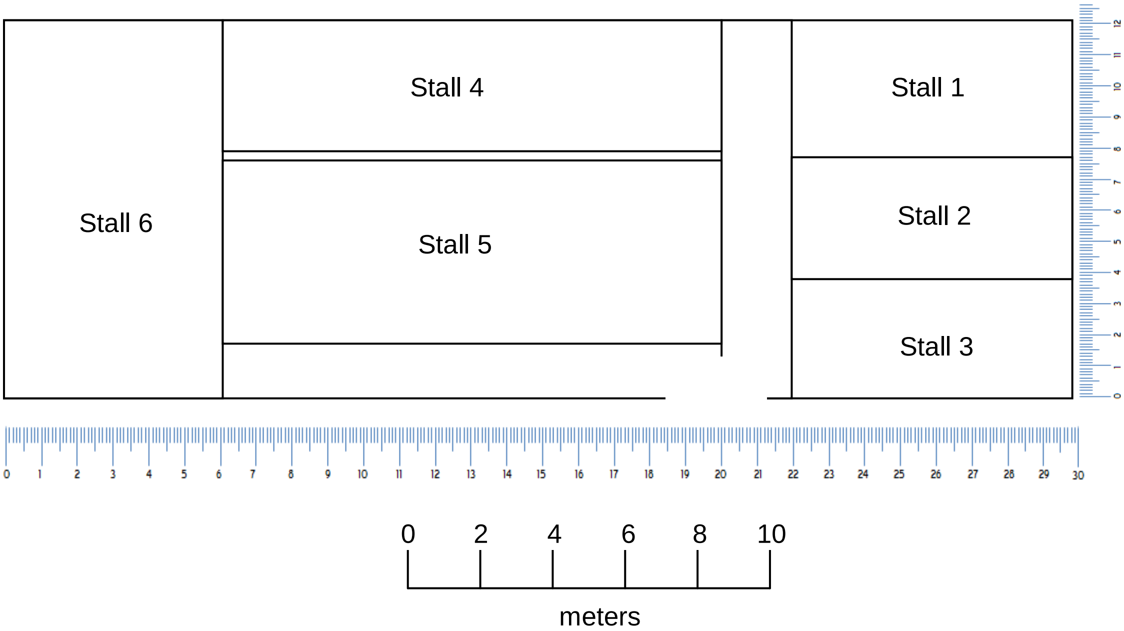
e)



f)



2. **Using the plan below, answer the questions that follow:**



a) What scale was used in the plan?

b) What are the dimensions of Stalls 1, 2 and 3 on the plan?

c) What is the actual floor area in m^2 of stall 1 on the ground?

d) What is the area in cm^2 of Stall 6 on the plan?

e) How far is stall 6 from stall 3 on the ground?

f) What is the actual perimeter of stall 5 in meters?

g) What is the actual area in m^2 of stall 4 on the ground?

11.1.3.5 Practical Applications of Ratio and Proportion

Ratio and proportion can be used in solving real life problems. Aside from the direct application we have discussed in the previous lessons, it can also be used in dealing with conversions, scaling, enlargement and reduction of objects from actual sizes to its models and vise versa.

Conversion

In our previous topic, we have discussed about conversion of one unit to another using both the imperial and the metric systems. Ratio and proportion can also be used with these conversion and other related values like currencies which make use of conversions.

Example 1

At the airport, Jasper exchanged his Philippine Peso (PhP) amounting to Php 1240 to Kina (K). If he was given K80 in exchange, what is the exchange rate from Kina to Peso?

Solution

This problem can make use of ratio, since we need to determine the value of Kina as compared to peso.

Forming a ratio of K:Php $= 80 : 1240$

This can also be written as $= \frac{80}{1240}$

Simplifying this to lowest term $= \frac{2}{31}$

We can say that it is equivalent to $= 2:31$

However, we want to know the value of every kina to peso,

We can divide both sides by 2 $= 1 : 15\frac{1}{2}$ or $1: 15.5$

Therefore, the value of K1 to Php is Php15.5.

Example 2

If 1 foot is approximately equal to 30 cm, how long is 8 ft. in cm?

Solution

Proportion can be used in this case.

We can have the proportion

$$\underbrace{1 : \overbrace{30 = 8}^{\textbf{(30)(8) = 240}} : x}_{\textbf{(1)(x) = x}}$$

We have $x = 240$

Therefore, 8 ft. is approximately equal to 240 cm.
Example 3

Found in a packaging of a burger patty, the weight is shown as 2 lbs (approx. 900 grams).

What is the weight of a packaging in grams (g) which has a label of 5 lbs?

Solution

Proportion can also be used in this case in the absence of the conversion table from imperial to metric system.

(900)(5) = 4500

We can form the proportion $2 : 900 = 5 : x$

(2)(x) = 2x

We have $2x = 4500$
Divide both sides by 2 $x = 2250$

Therefore, 5 lbs is approximately equal to 2250 g.

Example 3 An architect would like to make a sketch of a rectangular lot with the following dimensions: Length = 250 m , Width = 125 m. If he intends to draw the model wherein for every 50 m in the field, 2 cm will be used to represent it on the model.

Find, the dimensions of the field.

Solution:

To find the length of the lot in the drawing, we use a proportion

(250)(2) = 500

We can form the proportion $50 : 2 = 250 : x$

(50)(x) = x

We have $50x = 500$
Divide both sides by 50,
We have $x = 10$

To find the width of the lot in the drawing, we use a proportion

(2)(125) = 250

We can form the proportion $50 : 2 = 125 : x$

(50)(x) = x

We have $50x = 250$

Divide both sides by 50,
We have $x = 5$

Therefore, the dimensions of the field in the drawing are 10 cm (length) and 5 cm (width (respectively).

Example 4 A wooden box on the plan has a height of 3 cm. If the real box to made will use a scale factor of 1: 20, what is its height in meters?

Solution:

This problem involves the use of scale factor wherein the length on the drawing will be enlarged in actual.

We can form the proportion

$$\underbrace{3 : \overbrace{x = 1}^{(x)(1) = x} : 20}_{(3)(20) = 60}$$

We have $x = 60$

The actual length box is 60 cm. We all know that 1 m = 100 cm

Again, we can use our techniques in converting metric measures by moving the decimal point two places to the left. Or $60 \div 100 = 0.60$ m.

Therefore, the actual height of the box is 0.60m.

Ratio and proportion has many practical applications. As you progress in the modules you will find more of its application in our real life.

LEARNING ACTIVITY 11.1.3.5

20 minutes

1. Given: 1 foot = 30 cm, find :

   a) 50 cm

   b) 195 cm

2. If 1 Australian dollar (1$) is equaivalent to 1 kina and 60 toea (K1.60), find the equivalent value of the following currencies:

   a) $ 15

   b) K 8

3. In a foreign exchange, for every Australian dollar ($) , it is equivalent to the Japanese currency , Yen (Y) Y205. What is the equivalent amount of Y6560 in $? $32

4. The actual length of a wooden totem in 350 cm. If a mini model will be made using the scale 1 : 5, find its length. 70cm

SUMMATIVE TASK 11.1.3

40 minutes

I. Multiple Choice. Encircle the letter of the correct answer.

1) It refers to the comparison between two quantities.
   A. Proportion C. Ratio
   B. Scales D. Means

2) What is the symbol used for “as to”?
   A. / C. =
   B. $\neq$ D. $\approx$

3) The product of the means is equal to the product of the
   A. Inner terms B. Extremes
   B. Ratios D. Proportions

4) It refers to the comparison between two equal ratios.
   A. Proportion C. Ratio
   B. Scales D. Means

5) It is a type of proportion which results a decrease in one quantity as the other increase
   A. Inverse proportion C. Partitive proportion
   B. Direct proportion D. Equal proportions

6) In the proportion 2:4:6, how many equal parts the whole is divided?
   A. 6 C. 10
   B. 8 D. 12

7) It refers to the ratio of the length in a model to its actual size.
   A. Proportion C. Ratio
   B. Scales D. Means

8) In a direct proportion, as one quantity increases, the other
   A. Decreases C. Increases
   B. Remains the same D. Divides

9) Simplify the ratio 5:10
   A. 5: 10 C. 1:5
   B. 1:2 D. 1:10

10) What value of x completes the proportion 1:3 = x: 15
    A. 5 C. 10
    B. 15 D. 20

II. Solve the following:

1) In a map the scale used is 1:100. How long is a road in m, if the map shows a measure of 250 cm?

2)If 3 kg of meat cost K85, how much meat (in kg) will K250 buys?

3)Anna spends K80 per week. How much will she spend for 2 days?

4)If 5 sacks of grains can feed 30 chicks in a week. How long will it take the same amount of grains to feed 50 chicks?

5) A rod measuring 20 cm is to be divided in the ratio 2:3. What is the length of the shorter part?

11.1.4 BASIC ALGEBRA

11.1.4.1 Factoring

Any number or expression greater than 1 can be factored into a product of prime numbers (and variables, if any).

For example

a) 10 = (2)(5) , 2 and 5 are prime factors of 10

b)10x = (2)(5)(x), 2, 5 and x are prime factors of 10x.

Factoring (also known as Factorising) is a process of finding the prime factors of a number or an expression. It simply means writing the expression as a product of its factors.

For every given polynomial, one or more of the following may be used to factorise:

Highest Common Monomial Factor (HCMF)

In the expression ax + bx, the factors are x (a+b) simplified as

ax + bx = x(a+b)
where: a and b are any real numbers

Example 1 Factor the following:

a) 12 + 4y

b) $x^3 + x^4 + x^2$

c) $2x^3 – 6x + 10x^2$

d) $15x^2y^3 + 5x^3y$

Solution:

a) 12 + 4y

Since numerical coefficients 12 and 4 have 4 as their highest common factor (HCMF), divide both terms by 4, we get 4(3 + y).

The factored form of 12 + 4y is 4 (3+y).

b) $x^3 + x^4 +x^2$

In this example, both numerical coefficients are 1. It is understood that their common factor is 1 too. So we need to work on the variables. The highest common factor of the variables is the **variable with the least exponent**. In this case, the HCMF is x^2 .

Divide each term by x^2, we get $x + x^2 + 1$.

The factored form of $x^3 + x^4 + x^2$ is x^2, is $x(x + x^2 + 1)$.

c) $2x^3 – 6x + 10x^2$

Since numerical coefficients 2, -6 and 10 have 2 as their highest common factor and the variables have x as their highest common factor, then the HCMF of the given is 2x.

Divide each term by 2x, we get $x^2 – 3 + 5x$.

The factored form of $2x^3 – 6x + 10x^2$ is $2x(x^2 – 3 + 5x)$.

d) $15x^2y^3 + 5x^3y$

This example involves two variables, the same rule applies just like what we did on the examples a , b and c.

The numerical coefficients 15 and 5 have a common factor of 5, while the variables have a common factor of x^2y. The HCMF is 5 x^2y. Divide each term by the HCMF we have $xy^2 + x$.

The factored form of $15x^2y^3 + 5x^3y$ is $x^2y (xy^2 + x)$.

Factoring Simple Quadratic Trinomial

A simple quadratic trinomial is in the form $x^2 + bx + c$, where b and c are any real numbers.

It is a simple quadratic trinomial because the numerical coefficient of the second degree term (usually the first term denoted by x^2) is 1.

$x^2 + bx + c = (x + d) (x + e)$
where: $b = d + e$ $c = (d)(e)$

This means that , the factors of the last term gives a sum equals to the first term.

Example 2 Factorise the following:

a) $x^2 + 6x + 8$

b) $y^2 – 9y + 14$

Solution

a) Think of factor of 8 which will give a sum of 6. Trial and error method can be used. For this example, the set of possible factors are listed as follows:

Factors of 8	Sum of the factors
(8)(1)	8+ 1 = 9
(-8)(1)	-8+1 = -7
(8)(-1)	8+ -1 = 7
(-8)(-1)	-8+-1 = -9
(2)(4)	**2+4 = 6**
(-2)(4)	-2+4 = 2
(2)(-4)	2+-4 = -2
(-2)(-4)	-2+-4 = -6

From the table above, the factors of 8 which gives a sum of 6 are (2)(4). Therefore, d = 2 and e = 4.

Following the rules stated on the previous page, we have (x+ 2) (x+4).

Therefore, $x^2 + 6x + 8 = (x+ 2)(x+4)$

b) $y^2 – 9y + 14$

Since the last term is 14, possible factors are (-7)(-2). Since (-7)+(-2) = -9, then we say that d = -7 and e = -2.

Therefore, $y^2 – 9y + 14 = (x – 7) (x – 2)$.

Note: for this type of fctoring, the values of d and e can be interchanged. Since they are factors of the last term, the order of arrangement does not affect the product because multiplication is commutative.

Factoring Quadratic Expressions

> **Quadratic expressions** are any second degree polynomials. Usually in the form
> **$ax^2 + bx + c$.**
>
> Where **a, b** and **c** are any real numbers.

This method requires the use of trial and error and the use of the FOIL method to verify.

The following steps can be used to analyse.

Step 1: Find the possible factors of the first term.

Step 2: Find the possible factors of the second term.

Step 3: Verify using the FOIL method if the sum of the products of Outer terms (O) and the inner Terms (I) is equal to b in the quadratic ax2 + bx + c.

Example 3 Factorise the following:

a) $2x^2 - 5x - 3$

b) $2y^2 + 15y + 7$

Solution:

a) To factorize $2x^2 - 5x - 3$, let use the following steps:

Step 1: find the possible factors of the first term. $2x^2 = (2x)(x)$ or $(x)(2x)$

You can also write it as (2x) (x)

Step 2: find the possible factors of the second term. -3 = (-1) (3) or (1)(-3)

Step 3: Verify using the FOIL method if the sum of the products of Outer terms (O) and the inner Terms (I) is equal to b in the quadratic ax2 + bx + c.

Let us first try the factors of the second term (-1) and 3. Using the incomplete form in step 1, we write -1 and 3 as (2x - 1) (x+ 3)

Outer Terms (O)

Using the FOIL method: $(2x - 1)(x + 3)$

Inner Terms (I)

To verify: O + I = -5x (first degree term or middle term)

$$(2x)(3) + (-1)(x) = -5x$$
$$(6x) + (-x) = -5x$$
$$5x \neq -5x$$

Since O + I $\neq$ -5x, it means that this is not the correct set of factors.

Now we try 1 and (-3).

Using the incomplete form in step 1, we write 1 and -3 as (2x + 1) (x - 3)

Using the FOIL method: $(2x + 1)(x - 3)$

Inner Terms (I)

To verify: O + I = -5x (first degree term or middle term)

$$(2x)(-3) + (1)(x) = -5x$$
$$(-6x) + (x) = -5x$$

$-5x \quad = -5x$

Therefore, $2x^2 - 5x - 3 = (2x + 1)(x - 3)$.

b) $2y^2 + 15y + 7$

Step 1: find the possible factors of the first term. $2y^2 = (2y)(y)$

You can also write it as $(2y \quad)(y \quad)$

Step 2: find the possible factors of the second term. $7 = (1)(7)$

Step 3: Verify using the FOIL method if the sum of the products of Outer terms (O) and the inner Terms (I) is equal to b in the quadratic ax2 + bx + c.

Let us first try the factors of the second term (7) and 1. Using the incomplete form in step 1, we write 7 and 1 as $(2y + 1)(y + 7)$

Outer Terms (O)

Using the FOIL method: $(2y + 1)(y + 7)$

Inner Terms (I)

To verify:

$$O + I = 15y \text{ (first degree term or middle term)}$$
$$(2y)(7) + (1)(y) = 15y$$
$$(14y) + y = 15y$$
$$15y = 15y$$

Therefore, $2y^2 + 15y + 7 = (2y + 1)(y + 7)$.

Difference of Two Squares

> The quadratic expression $\mathbf{x^2 - y^2}$ are called **difference of two squares** where both terms are perfect squares
>
> $$\mathbf{x^2 - y^2 = (x+y)(x-y)}$$
>
> This means that the factors of a difference of two squares are the sum and difference of binomials having like terms.

This is very simple with the use of the following steps.

Step 1: Find the square root of the first term. $\sqrt{x^2} = (x)$

Let this be the first terms of each binomial factors: (x) (x)

Step 2: Find the square root of the second term. $\sqrt{y^2} = (y)$

Let this be the first terms of each binomial factors: (x y) (x y)

Step 3: Put + and – signs in each binomial term (x + y) (x - y)

Note: The order of the signs does not matter. It is also ok to make the first binomial a difference and the second one as sum. As long as they do not have the same signs.

Example 4 Factorise the following:

a) $x^2 - 36$

b) $9x^2 - 49$

Solution:

a) To factorize $x^2 - 36$, follow the steps.

Step 1: Find the square root of the first term. $\sqrt{x^2} = (x)$

Let this be the first terms of each binomial factors: (x) (x)

Step 2: Find the square root of the second term. $\sqrt{36} = 6$

Let this be the second terms of each binomial factors: (x 6) (x 6)

Step 3: Put + and – signs in each binomial term (x + 6) (x - 6)

Therefore, $x^2 - 36 = (x + 6)(x - 6)$.

c) To factorize $9x^2 - 49$, follow the steps.

Step 1: Find the square root of the first term. $\sqrt{9x^2} = 3x$

Let this be the first terms of each binomial factors: (3x) (3x)

Step 2: Find the square root of the second term. $\sqrt{49} = 7$

Let this be the second terms of each binomial factors: (3x 7) (3x 7)

Step 3: Put + and – signs in each binomial term (3x + 7) (3x - 7)

Therefore, $9x^2 - 49 = (3x + 7)(3x - 7)$.

Factoring Completely

A polynomial is **completely factored** if it is written as a product of a monomial and other polynomials which are already simplified. This means that every factor are no longer factorable. This process may involve a combination of two or more of the types of factoring discussed.

Example 5 Factorise the following completely:

a) $2x^2 – 8$

b) $5x^2 + 20x - 60$

Solution:

a) Factorise $2x^2 – 8$ using HCMF : $2(x^2 – 4)$
But $x^2 – 4$ can still be factorized using difference of two squares: $2(x+2)(x-2)$

Therefore, $2x^2 – 8 = 2(x+2)(x-2)$.

b) Factorise $5x^2 + 20x - 60$ using HCMF : $5(x^2+4x–12)$
But $x^2+4x–12$ can still be factorised using simple quadratic trinomial: $5(x+6)(x-2)$

Therefore, $5x^2 + 20x - 60 = 5(x+6)(x-2)$.

LEARNING ACTIVITY 11.1.4.1

20 minutes

Factorize the following and identify the type of factoring involved:

1) $x^2 - 8x + 7$

2) $4x + 10$

3) $4x^2 - 25$

4) $2x^2 + 6x - 108$

5) $5x^2 + 7x + 2$

6) $x^2 - x - 90$

7) $x^2 - 16y^2$

8) $3x^2 + 4x + 1$

9) $10x^2 + 35x$

10) $x^2 + x - 90$

11.1.4.2 Simplifying Algebraic Fractions

Much of our work with algebraic fractions will be similar to your work in dealing with fractions in arithmetic.

Algebraic Fractions are also knows as **Rational Expressions**. It is easily recognized because it is in a form of a fraction whose numerator and denominator consist of polynomials.

The fraction $\frac{10}{15}$ can be simplified by dividing both numerators and denominators by 5. Another way of doing it is by cancelling out common factors as follows:

$$\frac{10}{15} = \frac{(2)(\cancel{5})}{(3)(\cancel{5})} = \frac{2}{3}$$

Similar method can also be used to simplify algebraic fractions.

For polynomials P, Q and R

$$\frac{P}{Q} = \frac{PR}{QR}$$

When : $Q \neq 0$ and $R \neq 0$

Steps in simplifying Algebraic Fractions

Step 1 Factor the numerator and denominator (when applicable).

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). The resulting fraction will be in lowest terms or the simplified algebraic fraction.

Example 1 Simplify the following:

a) $\frac{4x^3}{6x}$

b) $\frac{10x^3y^2}{20xy^4}$

Solution:

a) Step 1 Factor the numerator and denominator (when applicable).

$$\frac{4x^3}{6x} = \frac{(2)(2)x \bullet x \bullet x}{(3)(2)x}$$

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). Or cancel out all common factors. The resulting fraction will be in lowest terms or the simplified algebraic fraction.

$$\frac{4x^3}{6x} = \frac{\cancel{(2)}(2)\cancel{x}\bullet x\bullet x}{(3)\cancel{(2)}\cancel{x}} = \frac{2x^2}{3}$$

Therefore, the simplest form of $\frac{4x^3}{6x}$ is $\frac{2x^2}{3}$.

b) Step 1 Factor the numerator and denominator (when applicable).

$$\frac{10x^3y^2}{20xy^4} = \frac{(5)(2)x\bullet x\bullet x\bullet y\bullet y}{(5)(2)(2)x\bullet y\bullet y\bullet y\bullet y}$$

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). Or cancel out all common factors. The resulting fraction will be in lowest terms or the simplified algebraic fraction.

$$\frac{10x^3y^2}{20xy^4} = \frac{\cancel{(5)}\cancel{(2)}\cancel{x}\bullet x\bullet x\bullet \cancel{y}\bullet \cancel{y}}{\cancel{(5)}\cancel{(2)}(2)\cancel{x}\bullet y\bullet \cancel{y}\bullet \cancel{y}\bullet y} = \frac{x^2}{2y^2}$$

Therefore, the simplest form of $\frac{10x^3y^2}{20xy^4}$ is $\frac{x^2}{2y^2}$.

The two examples have monomials on both numerators and denominator. In some instances, the numerator is polynomial. These polynomials can be simplified by factorising them first using the previous rules discussed.

Example 2 Simplify the following:

a) $\frac{2x\quad 4}{x^2\quad 4}$

b) $\frac{2x^2+x\quad 6}{2x^3\quad x\quad 3}$

c) $\frac{6x^2+2x}{2x^2+12x}$

Solution:

a) **Step 1** Factor the numerator and denominator.

$$\frac{2x\quad 4}{x^2\quad 4} = \frac{2(x\quad 2)}{(x+2)(x\quad 2)} \qquad \frac{\text{HCMF}}{\text{Difference of two squares}}$$

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). Or cancel out all common factors. The resulting fraction will be in lowest terms or the simplified algebraic fraction.

$$\frac{2x\quad 4}{x^2\quad 4} = \frac{2\cancel{(x\quad 2)}}{(x+2)\cancel{(x\quad 2)}} = \frac{2}{(x+2)}$$

Therefore, the simplest form of $\frac{2x\quad 4}{x^2\quad 4}$ is $\frac{2}{(x+2)}$.

b) **Step 1** Factor the numerator and denominator.

$$\frac{2x^2+x\quad 6}{2x^3\quad x\quad 3} = \frac{(x+2)(2x\quad 3)}{(x+1)(2x\quad 3)} \qquad \frac{\text{Quadratic Expression}}{\text{Quadratic Expression}}$$

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). Or cancel out all common factors. The resulting fraction will be in lowest terms or the simplified algebraic fraction.

$$\frac{2x^2+x\quad 6}{2x^3\quad x\quad 3} = \frac{(x+2)\cancel{(2x\quad 3)}}{(x+1)\cancel{(2x\quad 3)}} = \frac{(x+2)}{(x+1)}$$

Therefore, the simplest form of $\frac{2x^2+x\quad 6}{2x^3\quad x\quad 3}$ is $\frac{(x+2)}{(x+1)}$.

c) **Step 1** Factor the numerator and denominator.

$$\frac{6x^2+2x}{2x^2+12x} = \frac{2x(3x+1)}{2x(x+6)} \qquad \frac{\text{HCMF}}{\text{HCMF}}$$

Step 2 Divide the numerator and denominator by the greatest common factor (GCF). Or cancel out all common factors. The resulting fraction will be in lowest terms or the simplified algebraic fraction.

$$\frac{6x^2+2x}{2x^2+12x} = \frac{\cancel{2x}(3x+1)}{\cancel{2x}(x+6)} = \frac{(3x+1)}{(x+6)}$$

Therefore, the simplest form of $\frac{6x^2+2x}{2x^2+12x}$ is $\frac{(3x+1)}{(x+6)}$.

LEARNING ACTIVITY 11.1.4.2

20 minutes

Simplify by expressing the following Algebraic Fractions in their lowest form:

1) $\dfrac{2x-4}{x^2-4}$

2) $\dfrac{x^2 \quad 4}{x^2+6x+8}$

3) $\dfrac{9-x^2}{x^2+2x \quad 15}$

4) $\dfrac{x+1}{x^2+3x+2}$

5) $\dfrac{x^2+8x+16}{x^2-3x-8}$

6) $\dfrac{x^3-1}{x^2-1}$

11.1.4.3 Solving and Sketching Quadratic Equations

A **Quadratic Equation** is a second degree equation which is usually in the form

$$\mathbf{ax^2 + bx + c}$$

Where : $a \neq 0$.

In every quadratic equation, the numerical coefficient of the second degree term (a) cannot be equal to zero. If a = 0 then the equation becomes linear.

Solving Quadratic Equations can be dealt using the quadratic formula. This formula can be used for all cases of quadratic equations.

For any quadratic equation $ax^2+bx+c = 0$, the **Quadratic Formula** can be used.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

However, in some cases, the use of finding the roots and factoring are found to be more convenient.

Example 1 Solve for the roots (value of x) of the following:

a) $x^2 = 81$

b) $x^2 + 49 = 0$

c) $x^2 - 5x + 6 = 0$

d) $(x+3)^2 = 49$

e) $x^2 + 8x + 16 = 36$

Solution:

a) $x^2 = 81$

This is an incomplete quadratic equation because b = 0. This kind of quadratic equation is best solved by getting the square root.

Simply get the square root of both sides of the equation: $\sqrt{x^2} = \sqrt{81}$

Simplify: $x = \pm 9$

Then: $x = 9;\ x = -9$

The roots are 9 and -9.

b) $x^2 + 49 = 0$

This is also an incomplete quadratic equation. Transpose 40 on the right side of the equation: $x^2 = -49$

Get the square root of both sides of the equation : $\sqrt{x^2} = \sqrt{-49}$

Note that the square root of a negative number is **imaginary**. $x = \pm 7i$

Then: $x = 7i$; $x = -7i$

c) $x^2 - 5x + 6 = 0$

This quadratic equation can be solved using the quadratic formula or by factoring.

Let us first try factoring:

Factor the left side of the equation: $x^2 - 5x + 6 = 0$

Using simple quadratic equation: $(x - 3)(x - 2) = 0$

Equate both factors to zero:

$x - 3 = 0$ $\quad$ $x - 2 = 0$

$\mathbf{x = 3}$ $\quad$ $\mathbf{x = 2}$

Now we solve using the quadratic formula:

Given $x^2 - 5x + 6 = 0$, where a = 1, b = - 5, c = 6

Substitute in the formula:

$$x = \frac{(\ 5) \pm \sqrt{(\ 5)^2 \quad 4(1)(6)}}{2(1)}$$

$$x = \frac{(\ 5) \pm \sqrt{25 \quad 24}}{2}$$

$$x = \frac{(\ 5) \pm \sqrt{1}}{2}$$

$$x = \frac{(\ 5) \pm 1}{2}$$

$$x = \frac{(\ 5) + 1}{2} \quad ; \qquad x = \frac{(\ 5) \quad 1}{2}$$

$x = \frac{6}{2}$; $x = \frac{4}{2}$

$x = 3$; $x = 2$

Therefore the roots of $x^2 - 5x + 6 = 0$ are 3 and 2.
It doesn't matter which method you will used. What is important is that you are comfortable with the method you have chosen.

d) $(x+3)^2 = 49$

Aside from the quadratic formula, the **square root method** can also be used.

Get the square root of both sides of the equation : $\sqrt{(x+3)^2} = \sqrt{49}$

$(x+3) = \pm 7$

Then:

$(x+3) = 7$; $(x+3) = -7$

$x = 7 - 3$; $x = -7 - 3$

$x = 4$; $x = -10$

Therefore the roots of $(x+3)^2 = 49$ are 4 and -10.
You may try using the quadratic formula for your practice.

e) $x^2 + 8x + 16 = 36$

In this example, the left side of the equation can be factored:

$x^2 + 8x + 16 = 36$

$(x+4)(x+4) = 36$

$(x+4)^2 = 36$

Since both sides of the equation are perfect square: $\sqrt{(x+4)^2} = \sqrt{36}$

Then: $(x+4) = \pm 6$

To get the roots:

$x+4 = 6$; $x+4 = -6$

$x = 6 - 4$; $x = -6 - 4$

$x = 2$; $x = -10$

Therefore the roots of $x^2 + 8x + 16 = 36$ are 2 and -10.

You may try using the quadratic formula for your practice.

Graphing Quadratic Functions

The given quadratic equation in the form $ax^2+bx+c = 0$, can be transformed into a quadratic function where 0 = y.

The **Quadratic Function** is written in the form

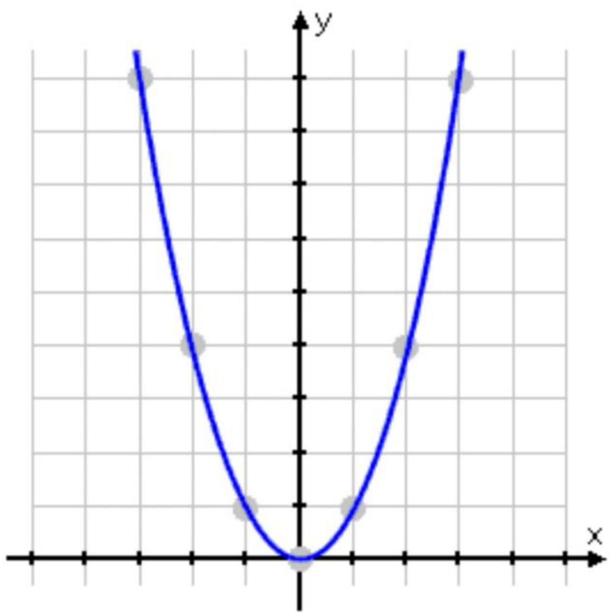
$$y = ax^2 + bx + c$$

where: a,b and c are real numbers
$a \neq 0$

The graph of a quadratic function is a **parabola**. It is a smooth curve with an axis of symmetry with its vertex (turning point).

Graphing the function $y = x^2$

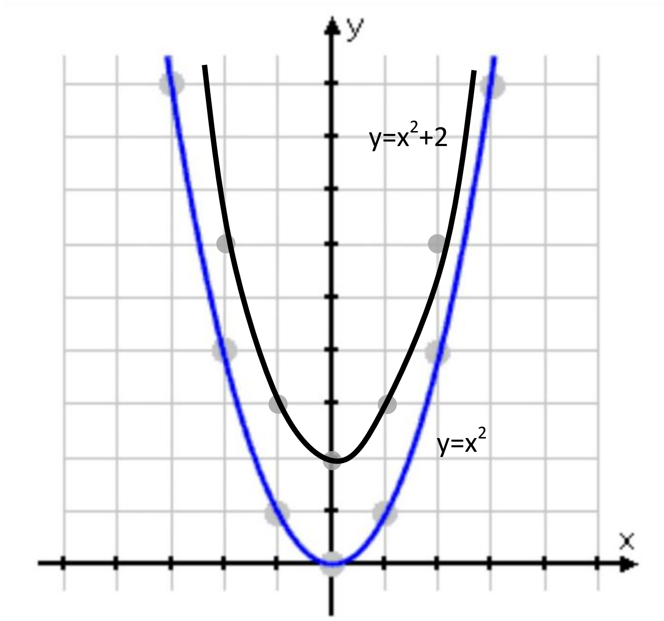
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



In this graph, the axis of symmetry is the y-axis. The vertex of the parabola is at (0,0) or the origin.

Graph of the functions $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11



Observe that when a constant 2 is added to the function $y=x^2$, the graph moves 2 units upward. Its vertex is now at (0,2). The axis of symmetry remains the same however, it is noticed that the graph is getting narrower.

Similarly, if a negative constant is added , the graph will move downward and it will become wider.

Example 2 Find the vertex and intercepts of the $y = 3x^2 + x - 2$. Show them in the graph.

Solution:

To solve for the vertex, se x=0 and substitute in the equation: $y = 3(0)^2 + (0) - 2$
$y = -2$

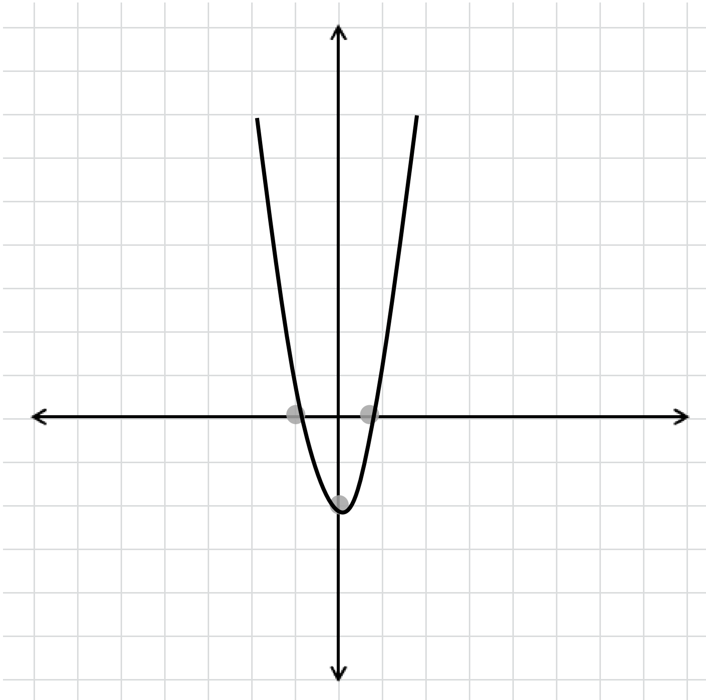
When x = 0, y = -2, therefore, the vertex is at (0,-2).

To solve for the values of x, we let y = 0. $0 = 3x^2 + x - 2$
$0 = (3x - 2)(x+1)$

Equate each factor to zero.

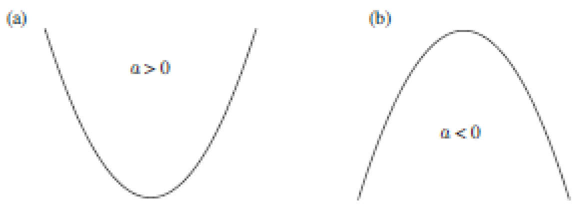
$3x - 2 = 0$; $x + 1 = 0$
$3x = 0 + 2$ $x = 0 - 1$
$3x = 2$ $x = -1$
$x = \frac{2}{3}$

The parabola passes the x-axis at points (-1,0) and ($\frac{2}{3}$, 0).

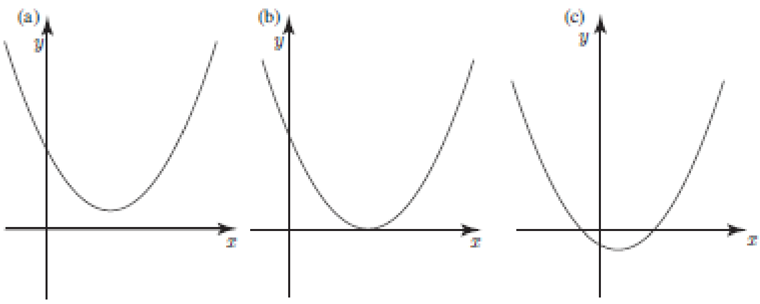


Graphing of the Function $y = ax^2 + bx + c$

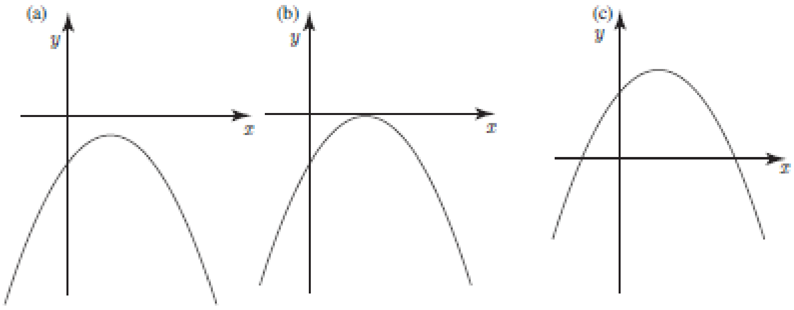
If the numerical coefficient of x^2 is positive (a>0), the parabola opens upward while if it is negative (a<0), the parabola opens downwards.



As the positive value of of increases the graph moves as in the figure below



While when the negative value of a changes, this is how the graph moves



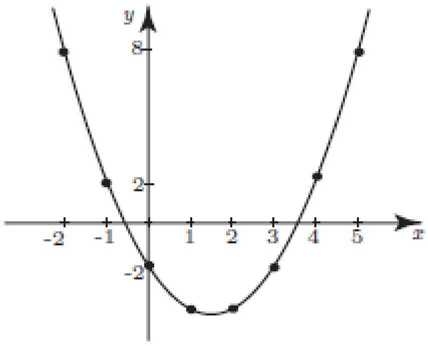
Example 3 Graph the function $y = x^2 - 3x - 2$

Solution:

First, we can use a table of values

X	-2	-1	0	1	2	3	4	5
y	8	2	-2	-4	-4	-2	2	8

From this table, the graph is plotted or sketched as follows



Student Learning Activity 11.1.4.3

20 minutes

1. Find the roots of the following using any method.

a) $X^2 = 64$

b) $3x^2 + 75 = 0$

c) $(x+3)^2 = 20$

d) X2 + 12x + 36 = 0

2. Graph the following quadratic functions.

a) $f(x) = X^2 – x + 1$

b) $Y = X^2 – 4x + 3$

(c) $f(x) = x^2 – 9$

(d) $Y = x^2 – 6x + 9$

11.1.4.4 Inequalities

An inequality involves one of the four symbols >, <, ≥, ≤.

The following statements illustrate the meaning of each symbol.

$x > 3$	:	x is greater than 3	:	4,5,6…
$x < 3$	:	x is less than 3	:	2,1,0…
$x \geq 3$	:	x is greater than or equal to 3	:	3,4,5…
$x \leq 3$	:	x is less than or equal to 3	:	3,2,1…

Inequalities can be represented on a number line.

Example 1 Represent the following inequalities on the number line.

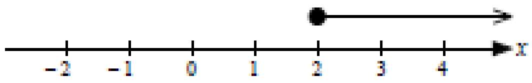
a) $X \geq 2$

b) $x < -1$

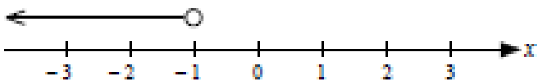
c) $-2 < x \leq 4$

Solution:

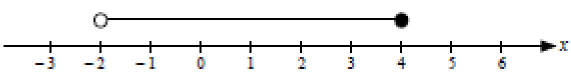
a) In the inequality $x \geq 2$, it means that the values of x includes 2,3 4 up to positive infinity. A dot is used to represent the start of this inequality.



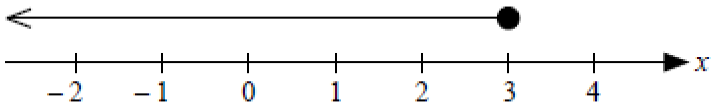
b) The inequality $x < -1$ means that the values of x does not include -1, therefore a hollow dot is usesd.

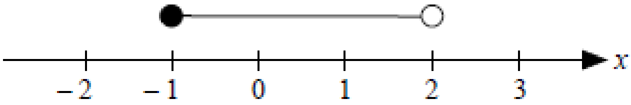


d) In the inequality $-2 < x \leq 4$, the values of x ranges from numbers more than -2 to 4. Therefore dots on both ends of the line were used.



Example 2 Write the inequality to describe the region represented by the following lines.

a) 

b) 

Solution:

a) The diagram indicates that the values of x must be less than or equal to 3 which can be written as $x \leq 3$.

b) The diagram indicates that x must be greater than or equal to -1 and less than 2. This is written as $-1 \leq x < 2$.

Solutions of Linear Inequalities

Solving for the solutions of Linear inequalities maybe compared to the way we deal with linear equations. But in this case aside from the changes in the sign, the direction of the symbols must be noted and considered.

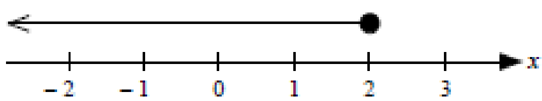
The direction of the symbol changes in multiplication and division by a negative quantity.

Example 3 Solve the inequality $6x - 7 \leq 5$

Solution:

$6x - 7 \leq 5$	Begin with the given
$6x \leq 12$	Add 7 on both sides (or transpose 7)
$x \leq 2$	Divide both sides by 6

This inequality can be represented by the number line below



Example 4 Solve the inequality $4(x - 2) > 20$

Solution:

$4(x - 2) > 20$	Begin with the given
$x - 2 > 5$	Divide both sides by 4
$\mathbf{x > 7}$	Add 2 on both sides (transpose 2)

Example 5 Solve the inequality $5 - 6x \geq -19$

Solution:

$5 - 6x \geq -19$	Begin with the given
$5 \geq -19 + 6x$	Note that the inequality contains -6x, therefore transpose the term 6x on the other side by adding 6x on both sides.
$24 \geq 6x$	Add 19 on both sides (transpose -19)
$4 \geq x$	Divide both sides by 6

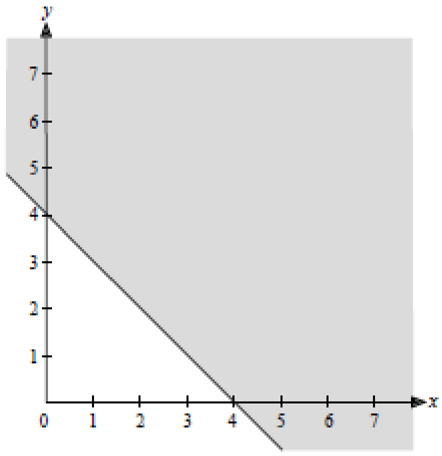
The answer can also be written as $x \leq 4$.

Unlike equalities, you can just interchange the values on both sides, in inequalities as you interchange; you have to use the opposite symbol.

Graphs of Inequalities

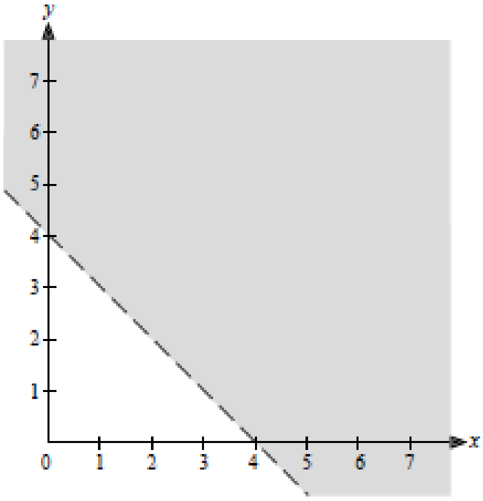
When an inequality involves two variables, it can be represented by a region (shaded part) on a graph.

The inequality $x + y \geq 4$ can be illustrated in the graph as



Notice that the line is like the graph of the equation $x + y = 4$ as it uses the coordinates of the equation. The region above the graph or the shaded area represents the inequality $x + y \geq 4$.

The inequality $x + y > 4$ can be illustrated in the graph as



Did you notice the difference?

When the symbols $\leq$ and $\geq$ are used, the line is a solid line while when the symbols are $>$ and $<$ we use a broken line.

Example 5 Show the region (by shading) that satisfies that inequality $y \geq 4x - 7$

Solution:

The region has a boundary on the line $y = 4x - 7$. Remember, as you graph the line, change the inequality symbol to = sign to determine the boundary or the line.

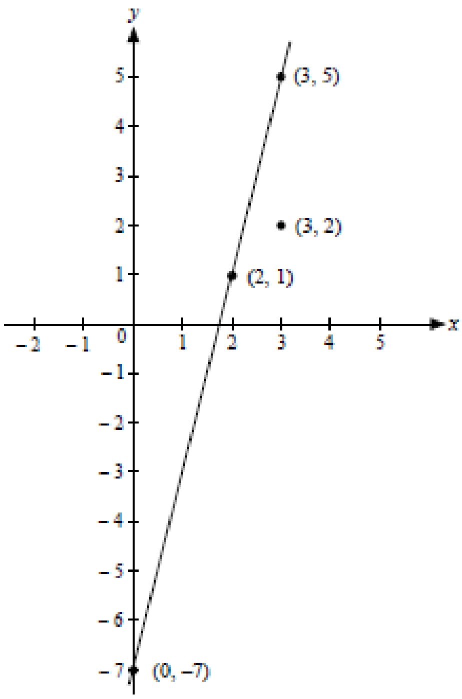
Use the table below to graph the line $y = 4x - 7$

X	3	2	1	0
y	5	1	-3	-7

Plot the points and draw the line

Note that a solid line is used because the inequality symbol in this example is $\geq$.

To determine to which region is shaded, name a point on the graph which does not belong to the line. In this example we use the point (3,2) as shown in the graph.

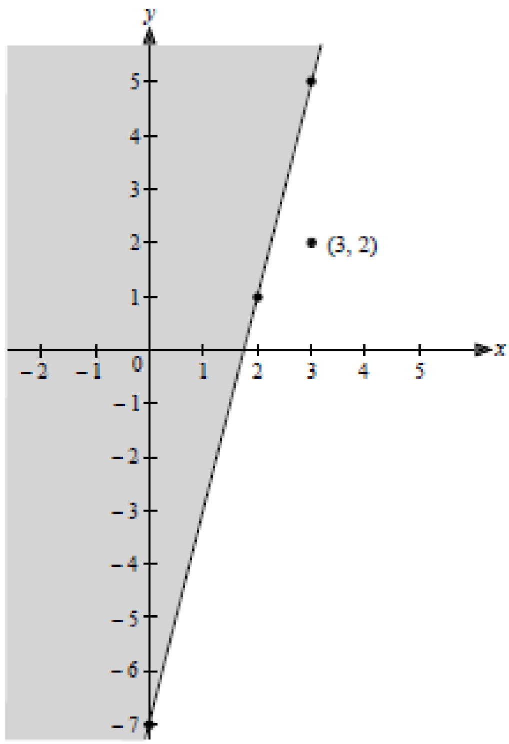


If the values of x =y and y = 2 are substituted into the inequality

$y \geq 4x - 7$

$2 \geq 4(3) - 7$

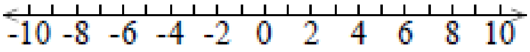
$2 \geq 5$ This is FALSE statement. Therefore, it will also be false (or not true) to any other point on the right side of the line. Thus, it suggest that the region to be shaded is the left side of the line.



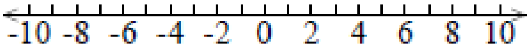
LEARNING ACTIVITY 11.1.4.4

20 minutes

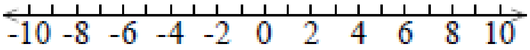
1. Plot the following inequalities on the number line.



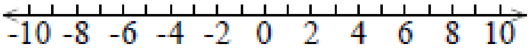
a) $x > 7$



b) $x < 4$



c) $x \geq -1$



d) $x \leq 5$

2. Determine the inequalities shown below:

a)

b)

c)

d)

3. Graph the inequality $x + 2y < 10$.

SUMMATIVE TASK 11.1.4

40 minutes

1) Factorise the following completely:

a) $x^2 + 7x = 12$

b) $x^2 + 3x - 10$

c) $x^2 - 6x + 9$

d) $8y + 12$

e) $3a + ab$

f) $m^2 + 8m + 12$

g) $y^2 - 64$

2) Simplify the following.

a) $\dfrac{5x + 8x}{x}$

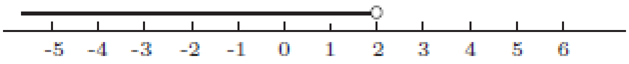
b) $\dfrac{x^2 + 5x + 6}{x + 3}$

3) Write the inequality illustrated in the diagrams below:

a). 

b) 

c) 

d) 

4) Draw the following on the number line.

a) $X > -4$

b) $4 \leq x \leq 8$

c) $-6 \leq x \leq 2$

5) Find the solutions of the following:

a) $(x - 2)(x + 3) \geq 0$

b) $x(x - 5) > 0$

c) $x^2 - 10x + 16 > 0$

d) $x^2 - 9 \leq 0$

SUMMARY

- A real number is a value that represents a quantity along a continuous line (called the number line) and is denoted by R.
- All the numbers starting from '1' to 'infinity' are natural numbers. All the numbers starting from '1' to 'infinity' are natural numbers
- The number zero was added to the set of counting numbers forming the set of whole numbers.
- An integer is a number that has no fractional part, and no digits after the decimal point. An integer can be positive, negative or zero.
- Rational numbers:
    - can be written as a ratio of two integers
    - called rational numbers because the first five letters spell the word 'ratio'.
    - can be expressed in the form Q = $\frac{a}{b}$, b ≠ 0; where, a and b are integers.
    - can be positive or negative.
    - can have a zero as numerator, e.g $\frac{0}{6}$ since this number is equal to zero.
    - cannot have a zero as denominator , e.g $\frac{6}{0}$ since this number is undefined.
- The set of rational numbers together with the set of irrational numbers make up the set of real numbers.
- Decimal numbers that have digits that repeat forever after the decimal point .are called recurring decimals.
- The absolute value of an integer tells how far the point that represents it is from zero.

- Properties of **Real Numbers**
    - Closure property of addition
    - Closure property of multiplication
    - Commutative property of addition
    - Commutative property of multiplication
    - Associative property of addition
    - Associative property of multiplication
    - Identity property of addition
    - Identity property of multiplication
    - Inverse property of addition
    - Inverse property of multiplication
    - Zero property of multiplication
    - Distributive property of multiplication over addition

- Surds are numerical expressions which involve irrational numbers. In some references, they call surds as radicals.
- Laws of Surds
    - Law 1: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

- Law 2: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- Law 3: $(\sqrt{a})^2 = a$

- Rationalizing the denominator is a way of removing or eliminating surds in the denominator.
- Rounding a number means writing the number with fewer significant figures.
- Estimation (or estimating) is the process of finding an estimate, or approximation, which is a value that is usable for some purpose even if input data may be incomplete, uncertain, or unstable.
- The base is the number to be multiplied while the exponent tells the number of times the base is to be multiplied by itself.
- Index Laws of Multiplication
  - Law 1: To multiply powers with the same base, add the indices.

    $b^m b^n = b^{m+n}$
  - Law 2: To raise a power to a power, multiply the indices.

    $(b^m)^n = b^{mn}$
  - Law 3: To get the power of a product, distribute and multiply the indices.

    $(ab)^m = a^m b^m$
  - Law 4: To divide powers with the same base, subtract the indices.

    $\frac{b^m}{b^n} = b^{m-n}$
  - Law 5: To get the power of a quotient, just find the quotient of the powers.

    $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - Law 7: Any number or quantity raised to a negative power is equal to its positive reciprocal.

    $b^{-m} = \frac{1}{b^m}$

- Laws of Logarithms
  - Law 1: $\log_a a = 1$
  - Law 2: $\log_a xy = \log_a x + \log_a y$
  - Law 3: $\log_a xm = m\log_a x$
  - Law 4: $\log_a \frac{x}{y} = \log_a x - \log_a y$
  - Law 5: $\log a\ 1 = 0$

- Standard index form or also known as the scientific notation is convenient shorthand way for writing very large or very small numbers which usually involves many zeroes.

- A scientific calculator is more than the usual counting devise we know because it provides more functions used in computations as well as different modes which allows users to perform linear regression, quadratic, exponential, logarithmic, power, trigonometric and statistical computations.

- A programmable scientific calculator allows the user to create one's own formula, programs and even store textual data. It also allows transfer of data from the calculator to the laptop and vice versa.
- The numeric keys bear the digits from 0 to 9 and the basic arithmetic keys are used for adding, subtracting, multiplying and diving numbers. It also includes parentheses, equal sign, decimal point and the negative sign for integers. These two sets of keys are the usual keys found in any calculator, even the most basic of all models.

- Measurement is a process of comparing the dimensions of an object using a standard way of quantifying things.
- There are two commonly used and accepted standards or systems in measurement: the English System and the Metric System or known as the International System of Units (SI).
- The most common of all the measuring devices is the ruler.
- Mass is usually measured using scales.
- A graduated cylinder is used to more accurately measure volumes of liquids such as chemicals, medicines and other liquids used with needed accuracy in measure.
- Ratio is the comparison of two quantities having the same unit of measure . It is denoted by the symbol ":" read as, "is to".

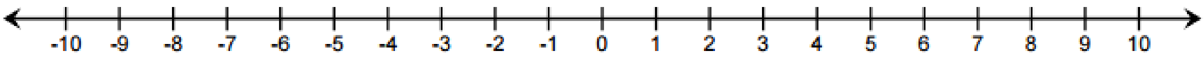
- A **Proportion** is a comparison between two ratios. It is also defined as the statement of equality between two ratios. It is denoted by the symbol "::" or "=" read as, "**is to**".
- **Direct Proportion** indicates that an increase in one quantity results to a proportional increase on the other quantity. On the other hand, a decrease in one quantity results a proportional decrease on the other.
- **Inverse Proportion** indicates that an increase in one quantity results to a proportional decrease on the other quantity. On the other hand, a decrease in one quantity results a proportional increase on the other.
- **Partitive Proportion** involves identifying parts of a whole based on a given ratio of its parts. This type of proportion may involve more than 2 unknowns and the whole can be divided into more than 2 parts.
- A **Scale** refers to the ratio of the length in a model to the actual length in reality.

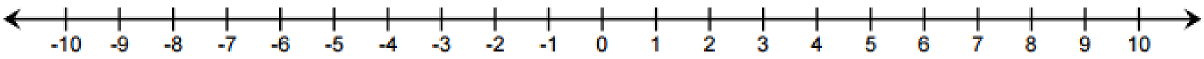
- Factoring (also known as Factorising) is a process of finding the prime factors of a number or an expression. It simply means writing the expression as a product of its factors.
- A simple quadratic trinomial is in the form $\mathbf{x^2 + bx + c}$, where b and c are any real numbers.

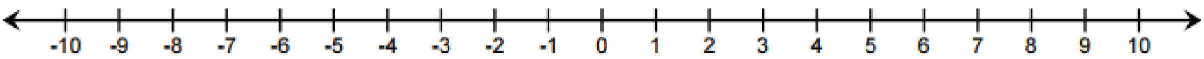
- Quadratic expressions are any second degree polynomials. Usually in the form **$ax^2 + bx + c$**. Where a,b and c are any real numbers.
- The quadratic expression $x^2 – y^2$ are called difference of two squares where both terms are perfect square terms. **$x^2 – y^2 = (x+y) (x-y)$**
- A polynomial is completely factored if it is written as a product of a monomial and other polynomials which are already simplified.
- Algebraic Fractions are also knows as Rational Expressions. It is easily recognized because it is in a form of a fraction whose numerator and denominator consist of polynomials.
- A Quadratic Equation is a second degree equation which is usually in the form **$ax^2 + bx + c$**. Where : $a \neq 0$.

ANSWERS

LEARNING ACTIVITY 11.1.1.1

1. + 100
2. - 500
3. + 9
4. + 5
5. - 7
6. + 3
7. - 7
8. 

9. 

10. 

11. - 3; 3
12. + 5; 5
13. - 23; 23
14. $-\frac{6}{5}$; $\frac{6}{5}$
15. $-\frac{10}{3}$; $\frac{10}{3}$
16. 32 + 21
17. (2 + 3) + 5
18. 1
19. Commutative
20. Zero
21. Associative

LEARNING ACTIVITY 11.1.1.2

1. $7\sqrt{5}$
2. $8\sqrt{5}$
3. 5
4. $\frac{2\sqrt{3}}{3}$
5. $9\sqrt{3}$
6. $-11\sqrt{5}$
7. $8\sqrt{5}$
8. $162 + 13\sqrt{3}$
9. $\frac{\sqrt{2}}{2}$
10. $-4\sqrt{6}$
11. 62
12. $\frac{\sqrt{6}}{3}$
13. $\frac{2\sqrt{10}}{5}$
14. - 5

15.

Conjugate	Product
$-7 - \sqrt{5}$	44
$\sqrt{2} - \sqrt{3}$	-1
$\sqrt{2} + 5$	-23
$-\sqrt{6} - 4$	-10
$-2y^2 - \sqrt{y^4}$	$3y^4$

16. $\frac{8\sqrt{2}+4\sqrt{6}}{13}$ **17.** **2-** $\sqrt{3}$ **18.** $5\sqrt{5}+3$

LEARNING ACTIVITY 11.1.1.3

1.SF

	No of SF	1 SF	2 SF
a	3	0.000 03	0.000 025
b	3	2×10^{5}	2.2×10^{5}
c	2	100 000 000	120 000 000
d	7	10	12
e	4	20	15
f	3	2×10^{8}	2.1×10^{8}
g	2	3×10^{-3}	3.4×10^{-3}
h	7	2	2.0
i	1	300 000 000	300 000 000
j	2	3×10^{-7}	2.5×10^{-7}

2.

	Given	2dp	3dp
a	12.3428	12.34	12.343
b	0.89923	0.90	0.899
c	17.00297	17.00	17.003

LEARNING ACTIVITY 11.1.1.4

1. (a) 780 (b) 2 040 (c) 21 980 (d) 98 700 (e) 0.002 9
2. (a) 7.77 (b) 20.4 (c) 21.984 (d) 9.875 (e) 9.00
3.

	Whole	1 SF
12.65	13	10
9.42	9	9
15.12	15	20
18.52	19	20
12.86	13	10

4. (a)58 + 29 + 11 = 98 (b)74 – 29 = 45 (c) (13 - 6) + 16 = 23
5. (a) 900 (b) 1 (c) 1 200 (d) 20

LEARNING ACTIVITY 11.1.1.5

1. power, index or exponent
2. $x^5y^3z^2$
3. 2.2.2.x.x.y.y.y

4. (a) 1
   (b) 81
   (c) $\frac{z^8}{x^4y^6}$
   (d) $a^5b^{10}c^6$
   (e) 1
   (f) 4
   (g) -6
   (h) $\frac{1}{8}$
   (i) $\frac{1}{2}$
   (j) - 1
   (k) $-\frac{1}{3}$
   (l) $\frac{3}{x^2}$
   (m) $\frac{9}{x^2}$
   (n) x^8
   (o) $\frac{5}{2x}$

5. (a) 2 (b) 3/2 (c) -2/3

LEARNING ACTIVITY 11.1.1.6

1. (a) 1.52×10^{17} (b) 1.4077×10^{13} (c) 7.5×10^{-13} (d) 1.48×10^{-7}

2. (a) 0.000 000 012 04 (b) 285 000 000 (c) 0.000 000 000 187 4
   (d) 0.000 400 2

3. (a) $1.329\ 615 \times 10^{22}$ (b) 4.32×10^1 (c) $1.328\ 42 \times 10^{-10}$ (d) 2.7×10^6
   (e) 3.5×10^4 (f) 3.1×10^2

LEARNING ACTIVITY 11.1.1.7

1. 2.626 5 (4 dp)
2. 0.1914
3. 1.265 625
4. 122
5. 665 280
6. 710
7. (a) $3\frac{4}{7}$; $\frac{25}{7}$
   (b) $17\frac{5}{6}$; $\frac{107}{6}$
   (c) $132\frac{15}{26}$; $\frac{3447}{26}$
8. (a) 25°27′7.2″
   (b) 344°13′44.4″
   (c) 10°55′48″
9. (a) 0.40
   (b) 4.80
   (c) 0.71

SUMMATIVE TASK 11.1.1

1. B
6. B

2. B
3. D
4. C
5. D
7. A
8. B
9. B
10. C

Part B

1. $4\sqrt{5}$

LEARNING ACTIVITY 11.1.2.1

1. Establish common understanding among different people and countries; unify the world.
2. 30 cm Ruler, Bathroom Scale etc.

3) (a) 454 g or 0.454 kg
(b) 454 g
(c) 4.56 L
(d) 1.6 km
(e) 28.4 mL

4) (a) 1.09 yd
(b) 2.2 lb
(c) 0.22 gal
(d) 0.62 mi
(e) 0.035 oz

LEARNING ACTIVITY 11.1.2.2

1. (a) 0.015 km
(b) 2.845 kg
(c) 25 km
(d) 40 yd
(e) 10 gal
(f) 10 in
(g) 175 mi
2. (a) 28.8 m
(b) 50

(c) NO. Extra 440 mL.

LEARNING ACTIVITY 11.1.2.3

1. -30°C
2. A = 38.2°C, B = 41°C
3. A = 200 mL, B = 1.2 L and C = 15 fl oz
4. 120 mL
5. 14.5 cm

SUMMATIVE TASK 11.1.2

1. (a) 28.1 cm (b) 852 ft (c) 328 km
   (d) 7662.6 g (e) 26.4 gal (f) 29.4°C
2. (a) K9.00 (b) 2 000 containers (c) 12.4 L of Petrol
3. (a) 37 C (b) 38.5 C (c) 65 mm (d) 5.5 in
   (e) 22.4 cm (f) 150 mL (g) 400 mL (h) 3 ½ fl oz

LEARNING ACTIVITY 11.1.3.1

1. (a) 5:4 (b) 4:5
2. (a) John
   (b)

Distance from Mary's home to school (in km)	10	**8**	6	**4**	**2**
Distance from John's home to school (in km)	5	4	**3**	2	1

3. (a) 1:1 (b) 3:5

LEARNING ACTIVITY 11.1.3.2

1. (a) P (b) P (c) NP (d) P (e) NP
2. K210.00
3. K40.00
4. 17 kg
5. $2\frac{2}{3}$ dozens

LEARNING ACTIVITY 11.1.3.3

1. (a) 5 hrs 36 min (b) 4 hrs 40 min (c) 56 min saved

2. (a) Php19 125 (b) K169.93
3. Anna K10 000.00, Bea K6 000.00 and Charlie K4 000.00
4. (a) X = 15 (b) X = 2 (c) X = 3 (d) X = 12
5. (a) 7 fl oz (b) 44.05 lb (c) 15.24 cm (d) 17.62 oz (e) 4.57 m

LEARNING ACTIVITY 11.1.3.4

1. (a) 1:3 (b) 1: 500 (c) 1: 100
   (d) 1: 200 000 (e) 1: 2 (f) 1: 400

2. (a) 1: 200 (b) 4.4 cm X 7.8 cm (c) 1 372 800 m^2 (d) 73.2 cm^2 (e) 3 200 m (f) 7 800 m (g) 2 539 200m^2

LEARNING ACTIVITY 11.1.3.5

1. (a) $1\frac{2}{3}$ ft (b) 7.5 ft
2. (a) K240.00 (b) $5.00
3. $32.00
4. 70 cm

SUMMATIVE TASK 11.1.3

Part A	Part B
1. A	1. 250 m
2. D	2. 8.8 kg
3. B	3. K22.00
4. A	4. 8⅓ shacks
5. A	5. 8 cm
6. D	
7. B	
8. C	
9. B	
10. A	

LEARNING ACTIVITY 11.1.4.1

1. (x - 7)(x - 1)
2. 2(2x + 5)
3. (2x - 5)(2x + 5)
4. 2(x - 6)(x + 9)
5. (5x + 2)(x + 1)
6. (x - 10)(x + 9)
7. (x – 4y)(x + 4y)
8. (3x + 1)(x + 1)
9. 5x (2x + 7)
10. (x + 10)(x - 9)

LEARNING ACTIVITY 11.1.4.2

1. $\frac{2}{x+2}$

2. $\frac{x-2}{x+4}$

3. $\frac{3+x}{x+5}$

4. $\frac{1}{x+2}$

5. $\frac{x+4}{x-7}$

6. $\frac{x^2+x+1}{x+1}$

LEARNING ACTIVITY 11.1.4.3

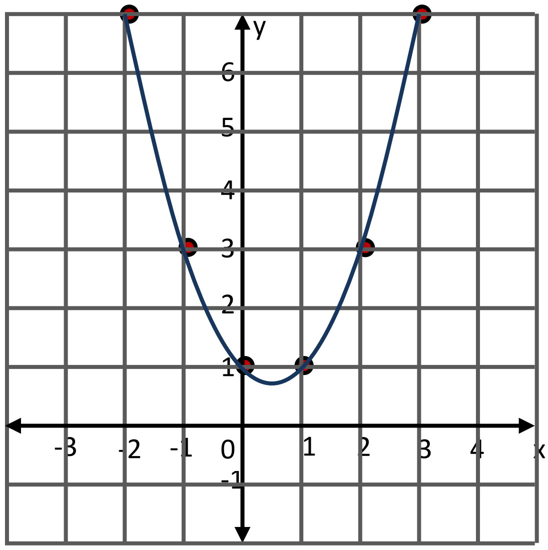
1. (a) ±8

(b) ±5i

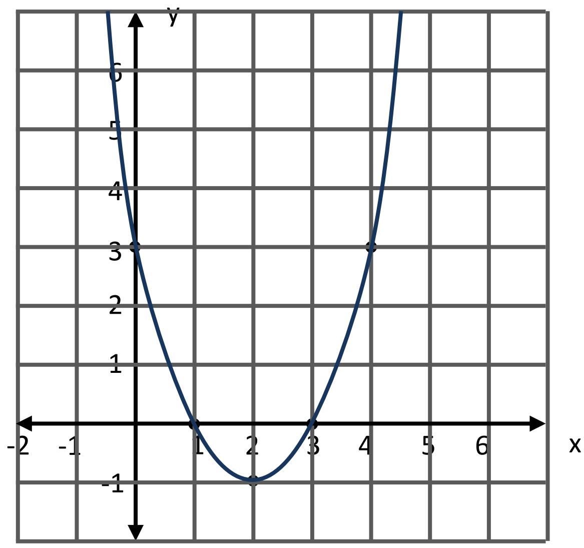
(c) -3 -2√5 or -3+√5

(d) -6 or -6

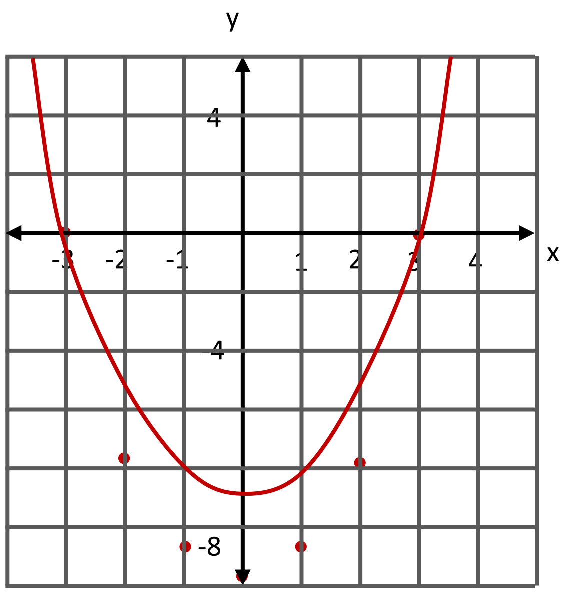
2. (a) $y = x^2 - x + 1$



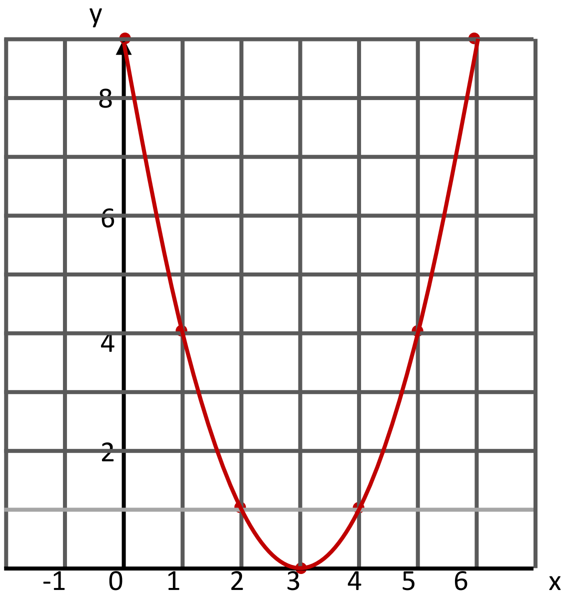
(b) $y = x^2 - 4x + 3$



(c) $Y = x^2 - 9$

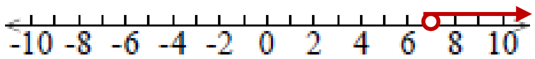


(d) $Y = x^2 - 6x + 9$

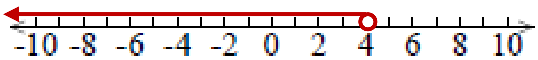


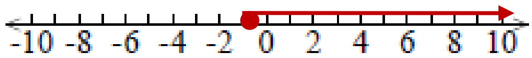
LEARNING ACTIVITY 11.1.4.4

1. a) $x > 7$

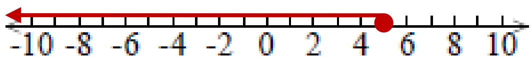


e) $x < 4$





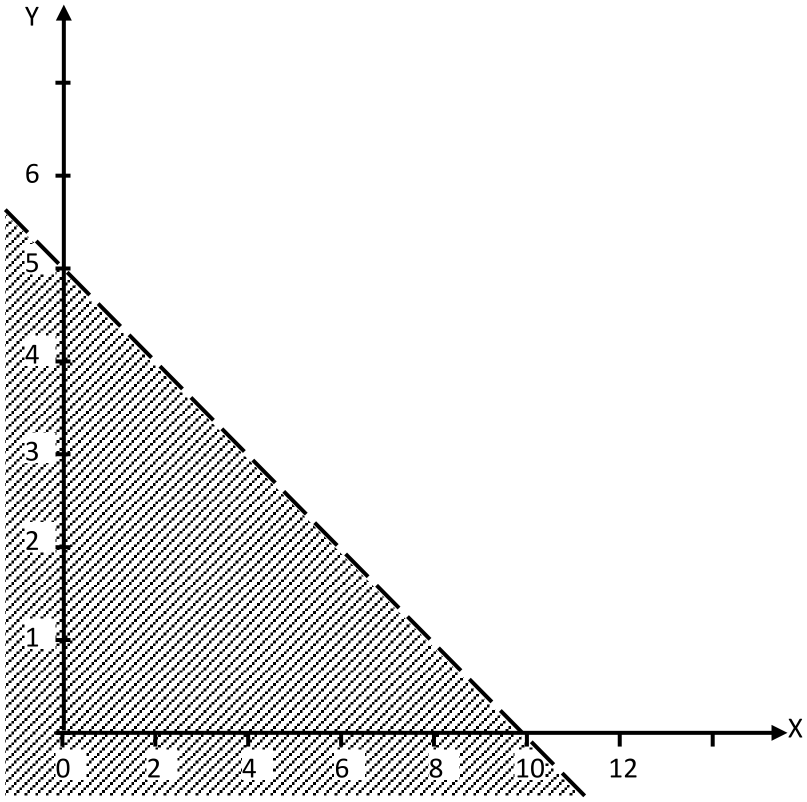
c) $x \geq -1$



d) $x \leq 5$

2. a) $x < 7$
   b) $x \geq -2$
   c) $x > -8$
   d) $x < -1$

3. Graph of x + 2y < 10.



SUMMATIVE TASK 11.1.4

1. (a) (x - 3)(x - 4)

   (b) (x + 5)(x - 2)

   (c) $(x - 3)^2$

   (d) 4(2y + 3)

   (e) a(3 + b)

   (f) (m + 2)(m + 6)

   (g) (y + 8)(y - 8)

2. (a) 13 (b) x + 2

3. (a) x ≤ -5 (b) x>2 (c) -5 ≤ x < 2 (d) x < 2

4. (a)

(b)

(c)

5. (a) $x \geq 2$ or $x \leq -3$ (b) $x < 0$ or $x > 5$ (c) $x < 2$ or $x > 8$ (d) $x \leq -3$ or $x \geq 3$

REFERENCES

http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8_7.pdf
http://www.hawkeslearning.com/Documents/FoundationsOfMath/FOM_Chapter%204.pdf

http://www.ncert.nic.in/ncerts/l/femh112.pdf

http://www2.lhric.org/pocantico/math/course_2/chap08-e.pdf

http://www.math.utah.edu/~wortman/1050-text-fp.pdf

https://www.mcckc.edu/common/services/BR_Tutoring/files/math/factoring/Factoring_Practice.pdf

http://www.ams.org/journals/mcom/1975-29-129/S0025-5718-1975-0371800-5/S0025-5718-1975-0371800-5.pdf

https://www.kutasoftware.com/FreeWorksheets/Alg1Worksheets/Factoring%201.pdf

http://www.mhhe.com/math/devmath/streeter/ba/graphics/streeter5ba/ch05/others/stre_5.1.pdf

http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-simpfrac-2009-1.pdf

https://maths.mq.edu.au/numeracy/web_mums/module2/Worksheet23/module2.pdf

http://www.cse.salford.ac.uk/physics/gsmcdonald/M1A_02_Algebra_Main_07_Algebraic%20Fractions%20(simplification,%20simple%20partial%20fractions).pdf

http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-quadeqns-2009-1.pdf

http://people.ucsc.edu/~miglior/chapter%20pdf/Ch08_SE.pdf

http://www.shoreline.edu/faculty/leyden/archives-from-sharepoint/ArchiveDocuments/Math141_Fall2012/pamphlet-on-quadratics_july2010.pdf

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf

http://www.cse.salford.ac.uk/physics/gsmcdonald/M1A_02_Algebra_Main_02_Simultaneous%20equations.pdf

http://www.bilingualmaths.com/2nd%20level/2ndequationsII/Unit%20IV%20Equations%20II%20pdf/Simultaneous%20equations.pdf

http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-inequalities-2009-1.pdf

https://www.nsa.gov/academia/_files/collected_learning/middle_school/pre-algebra/intro_solving_graph_inequal.pdf

http://people.ucsc.edu/~miglior/chapter%20pdf/Ch09_SE.pdf

http://www.cimt.plymouth.ac.uk/projects/mepres/allgcse/bkc16.pdf

http://www.emedinfo.com/Gallery/PhysicianScale.jpg

http://www.globalspec.com/ImageRepository/LearnMore/20134/ohaus-bw-standard-compact-washdown-scale6a4fd63c49d0440282878b6c45fd4b7b.png http://dialnsearch.com/productimage/High%20Precision%20Balances-%20Master%20Series-2012_12_13_13.jpg

http://www.11plusforparents.co.uk/Maths/images/Measurement/meas20.gif

http://www.chemistryland.com/CHM130FieldLab/Lab1/Digital-Thermometer512.jpg

http://www.globalspec.com/ImageRepository/LearnMore/20122/flexible-digital-thermometer--waterproof-93a79560729248a497ab68097b382876.png

http://m.c.lnkd.licdn.com/mpr/mpr/AAEAAQAAAAAAAAJXAAAAJDA2YWVlZWJiLWRkNDgtNDc0Yi05ZTVmLTZjNzcwYjdhMzVhYg.jpg

http://www.thepricecut.com/wp-content/uploads/2015/03/what-temperature-is-considered-a-fever-3.jpg

http://ecx.images-amazon.com/images/I/418sVoHmWbL._SY300_.jpg

http://g02.a.alicdn.com/kf/HTB1yVOrIFXXXXc5XpXXq6xXFXXXr/-font-b-Window-b-font-Indoor-font-b-Outdoor-b-font-Wall-Greenhouse-Garden-Home.jpg

http://ecx.images-amazon.com/images/I/415SCDUkphL._SY300_.jpg

http://blog.gcawood.com/wp-content/uploads/2011/12/452122.jpg

http://www.technologyuk.net/physics/measurement_and_units/images/measurement_0019.jpg

http://www.dimensionsinfo.com/wp-content/uploads/2010/10/measuring-cup.jpg

http://www.physics-chemistry-class.com/graduated-container.jpg

http://chemwiki.ucdavis.edu/@api/deki/files/426/20graphicaa.gif?revision=1