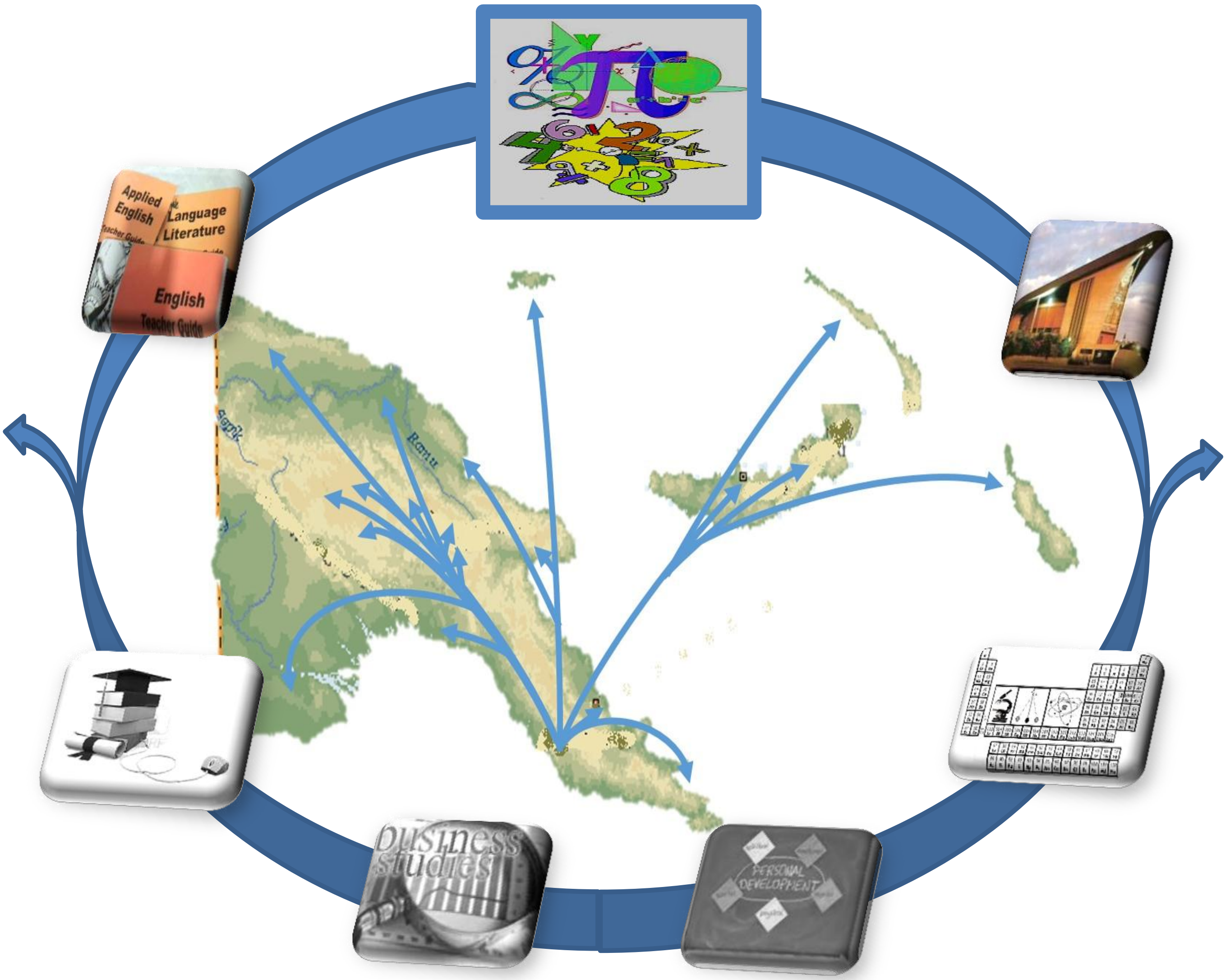
DEPARTMENT OF EDUCATION

GRADE 11 ADVANCE MATHEMATICS

11.2: GRAPHS AND FUNCTION



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GRADE 11

MATHEMATICS A

UNIT MODULE 2

GRAPHS AND FUNCTIONS

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SECRETARY'S MESSAGE

Achieving a better future by individual students, their families, communities or the nation as a whole depends on the curriculum and the way it is delivered.

This course is part and parcel of the NDOE new reformed curriculum. Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in its constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable pathways for students and adults to complete their education, through one system, many pathways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers who have contributed so much in developing this course.

Secretary for Education

LEARNING OUTCOMES

On successful completion of this module, you will be able to:

- form algebraic expressions from word problems
- simplify algebraic expressions
- apply the factor and remainder theorem
- simplify exponential and logarithmic expressions
- express exponential expressions to logarithmic expressions
- solve and sketch linear and quadratic equations
- write the domain an range of polynomial functions
- sketch and write quadratic, and absolute value functions
- solve linear inequalities and representing solutions on a plane
- discuss linear functions

TIME FRAME

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again.

If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not pass any question without solving it first.

UNIT 2: GRAPHS AND FUNCTIONS

Introduction

Algebra is one of the most important foundations in Mathematics as it deals with representations and axioms of logical Mathematics. The focus of this unit is on Algebra and graphs of functions. Solutions and sketching of graphs are promoted to illustrate the acquisition of the concept developed. The unit develops from the basic concept of algebra, expressions, equations to graphs of functions

Algebraic Expressions:

This topic includes translating word to algebraic expressions. It provides good depth of key words that define operators to be used to bind terms. It then expands on to polynomials and factorization by zeros of P [P(0)] , factor theorem and remainder theorem. It also includes simplifying and evaluating indicial and logarithmic expressions.

Linear Equations and Inequalities :

This topic covers linear equation of the form $y = mx + c$. Then it expands on to describe relations between two of the linear pairs in parallel lines, perpendicular lines and intersecting lines and deriving equations based on the given relationship. Distance between points is also discussed. Inequality of linear systems is also defined and illustrated followed with practice activity.

Quadratic Equations:

The topic covers factorizing; and solving by factorizing, identity, square-root method, completing the square and quadratic formula. It also expands your knowledge on plotting and sketching of the quadratic curves and identifying turning points.

Graphs of Functions:

This topic includes plotting and sketching of graphs of absolute value functions, hyperbolas or rational functions of the form $xy = k$, cubic graphs, and logarithmic curves. The graphs restricted to are that of the functions.

11.2.1 ALGEBRAIC EXPRESSIONS

Algebra is one of the fundamentals in Mathematics. Mastery of its basics will enable you to facilitate a more organize analysis in your higher Mathematics courses.

In this Topic we will focus on how to simplify expressions which will assist in dealing with equations and graphing functions in the succeeding topics.

11.2.1.1 Basic Concepts on Polynomials

A **constant** is a symbol that can take only one value.

A **variable** is a symbol that can take one or more values from a given set of permissible values.

For example, if we write the sum $4x^2 + 2x + 3$, x is symbol for the variable while 4, 2 and 3 the constants.

An **algebraic expression** is a term used to mean a constant, a variable or a combination of variables and constants involving a finite number of indicated operations (addition, subtraction, multiplication, division, raise to powers and extraction of roots) on them.

Examples: a+b, 3a $3a + \frac{2b}{a+b}$, $\frac{3ab - 5c2 + d}{a}$

Some more examples of Algebraic Expressions.

Algebraic Expression	Constant(s)	Variable(s)
3x	3	x
5y – 6	5 and 6	y
$\frac{(3s+1)}{4}$	3, 1 and 4	s
7(4b+2c)	7, 4 and 2	b and c
- 4x – 3y	- 4 and 3	x and y

In each of these algebraic expressions, we see that the constants and the variables are all attached by arithmetic operations. So, we need to find out which phrases are used to stand for different operations. Then, we can represent a verbal phrase as an algebraic expression.

A **polynomial** is an algebraic expression involving only non-negative powers of one or more variables and containing no variable in the denominator.

A polynomial of the degree n in x is in the form

$$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$$

A **term** of an expression or a polynomial is any symbol between two plus signs, two minus signs or a plus and a minus sign. A polynomial be considered as having a finite number of terms . In a polynomial $3x^2-2x-5$

The **coefficient** of the remaining factor is any factor of a term. For example , in 4xyz, the explicit number 4 is the numerical or constant coefficient, while the coefficient of 4y is xz, the coefficient of x is 4yz and so on.

Two monomials are considered similar, or like terms, if they differ only in their constant or numerical coefficients.

Examples

$-2xy$ and $6xy$, $-3x^2y^5$ and $7x^2y^5$, $18x^3y^2z$ and $\frac{3}{4}x^3y^2z$

On the other hand, the following are NOT similar terms

$-2x^3y$ and $6xy$, $-3x^2y^5$ and $7x^2y^6$, $18x^2y^2z^4$ and $\frac{3}{4}x^3y^2$

Did you observe the difference?

If the exponents of the variables, the variables and the whole of the literal coefficients have differences, then the two terms are not similar.

Degree of Polynomial

The **degree of a polynomial** in one variable is the greatest or highest exponent of its variable. The term with the highest degree is called **leading term.** The leading coefficient is the (numerical) coefficient of the leading term.

For polynomials with more than one variable, the degree is the highest sum of the exponents of its variables. But there is no leading term or leading coefficient because it is possible that two or more terms may have the same degree.

Observe the table below.

Polynomial with 1 variable	Degree	Leading coefficient
12	0	none
$6x + 3$	1	6
$4x^2 + 3x - 1$	2	4
$2x^3 - x + 9$	3	2
$2 - y^7$	7	-1
$4y^3 + 8y^2 - 5y^{12}$	12	-5

Polynomial with more than 1 variable	Degree
$x + 3x^2y$	3
$x^4y^3 + 3x^2y^5$	7
$xy^2 + 3xy – 8$	3
$x^3y^2z – xyz + 9x^2y^5z^3$	10

These terms are important because they are used as we move along with our discussion.

Now do the learning activity.

LEARNING ACTIVITY 11.2.1.1

20 minutes

1) Identify the number of terms in the following algebraic expressions. Write S if they are similar terms and NS if they are not similar terms.

Given	Number of terms	S or NS
a) $xy + 3xy – 4$	________________	________________
b) $-7x + x$	________________	________________
c) $xy + 3xy – 4xy + 8xy$	________________	________________
d) $x^3y^2z + 2\, x^3y^2z$	________________	________________
e) $x^3y^2z + 4$	________________	________________

2) Complete the table below. Write the degree and leading coefficient of the following polynomials. If the polynomial has more than 1 variable write NA (not applicable) for its leading coefficient.

Polynomial	Degree	Leading coefficient
$x + 12$		
$6xy + 4$		
$3x^3 + 7x^2 + 3x – 2$		
$X^4y^2z – x^2yz + 4x^2y^5z^5$		
$2 – y^8$		
$-5y^9 + 8y^2 – 5y^8$		

11.2.1.2 Simplifying Algebraic Expressions

Simplifying Algebraic Expressions involves performing operations, factoring and expanding expressions.

In Module 1 we have discussed factoring and simplifying rational expressions. This section provides more examples of simplifying algebraic expressions that may involve one or more processes.

Example 1

Simplify the following:

a) $(a + b)(c + d + e)$
b) $(2x + 3)(x^2 + 4x + 5)$
c) $(x + 2)^3$
d) $x(x + 1)(x + 2)$

Solution:

a) $(a + b)(c + d + e)$

$= (a + b)c + (a + b)d + (a + b)e$ Use distributive law and multiply (a+b) with each term c , d and e.

$= ac + bc + ad + bd + ae + be$ Use distributive law to remove parenthesis.

b) $(2x + 3)(x^2 + 4x + 5)$

$= 2x^3 + 8x^2 + 10x$ multiply 2x with $x^2 + 4x + 5$

$+ \; 3x^2 + 12x + 15$ multiply 3 with $x^2 + 4x + 5$

$= 2x^3 + 11x^2 + 22x + 15$ add similar terms

c) $(x + 2)^3$

$= (x + 2)\;(x + 2)^2$

$= (x + 2)(x^2 + 4x + 4)$ expand $(x + 2)^2$

$= x^3 + 4x^2 + 4x$ multiply x with $x^2 + 4x + 4$

$+ 2x^2 + 8x + 8$ multiply 2 with $x^2 + 4x + 4$

$= x^3 + 6x^2 + 12x + 8$ add similar terms

d) $x(x + 1)(x + 2)$

$= (x^2 + x)(x + 2)$ multiply x with x + 1

$= x^3 + 2x^2 + x^2 + 2x$ Multiply $(x^2 + x)(x + 2)$ using FOIL method

$= x^3 + 3x^2 + 2x$

The above examples made use of combinations of the distributive law with multiplication rules of polynomials discussed in your previous grades.

Example 2

Simplify:

a) $\frac{4x^3}{6x}$

b) $\frac{10x^3y^2}{20xy^4}$

c) $\frac{6x^2+2x}{2x^2+12x}$

Solution:

a) $\frac{4x^3}{6x} = \frac{(2)(2)x \bullet x \bullet x}{(3)(2)x}$ Factor the numerator and denominator

$= \frac{(2)(2)x \bullet x \bullet x}{(3)(2)x}$ Cancel out all common factors

$= \frac{2x^2}{3}$

b) $\frac{10x^3y^2}{20xy^4} = \frac{(5)(2)x \bullet x \bullet x \bullet y \bullet y}{(5)(2)(2)x \bullet y \bullet y \bullet y \bullet y}$ Factor the numerator and denominator

$= \frac{(5)(2)x \bullet x \bullet x \bullet y \bullet y}{(5)(2)(2)x \bullet y \bullet y \bullet y \bullet y}$ Cancel out all common factors

$= \frac{x^2}{2y^2}$

c) $\frac{2x \quad 4}{x^2 \quad 4} = \frac{2(x \quad 2)}{(x+2)(x \quad 2)}$ Factor out using $\frac{\text{HCMF}}{\text{Difference of two squares}}$

$= \frac{2(x \quad 2)}{(x+2)(x \quad 2)}$ Cancel out all common factors

$= \frac{2}{(x+2)}$

The key in simplifying algebraic expressions is to master the basics of factorising and multiplying algebraic expressions.

Now do the learning activity.

LEARNING ACTIVITY 11.2.1.2

20 minutes

Simplify the following algebraic expressions.

1) $-9 - 6(-v + 5)$

2) $1 + 4(2 - 3k)$

3) $7(1 + 9v) - 8(-5v - 6)$

4) $(2x + 1)^3$

5) $(x + 1)(x - 2)(x + 2)$

6) $\dfrac{6x^2 + 2x}{2x^2 + 12x}$

7) $7x^2 - 3y + 2x^2 - x + 2y$

8) $\dfrac{2x^2 + x}{-6x^2 - 3x}$

9) $(x - 3)(x^2 + 3x + 9)$

10) $\dfrac{x^2 + 12x + 36}{x^2 - 36}$

11.2.1.3 Translating Word Problems To Algebraic Expressions

In the last two lessons, we discussed variables and how to solve equations for a given variable. These skills will be used in dealing with word problems.

First and foremost, it is important to remember that a variable is used to represent an unknown value. In some cases, a phrase or sentence will tell us which variable we should use to represent the unknown value. However, it is more common for the reader to create the variable using a **let statement**.

A **let statement** is used to help solve a word problem by creating a variable to represent the unknown value in the word problem.

For example, "Let x = the unknown number."

Now that we have created a variable for our word problem, it is time to figure out which phrases mean the different operations.

The following will help one translate mathematical word expressions into algebraic expressions:

Operation	Word Expression	Algebraic Expression
Addition	Add, Added to, the sum of, more than, increased by, the total of,plus	+
	Add x to y	x + y
	y added to 7	7+ y
	The sum of a and b	a + b
	m more than n	n + m
	p increased by 10	p + 10
	The total of q and 10	q + 10
	9 plus m	9 + m
	Subtract, subtract from, difference, between, less, less than, decreased by, diminished by, take away, reduced by, exceeds, minus	–
Subtraction	Subtract x from y	y - x
	From x, subtract y	x - y
	The difference between x and 7	x -7
	10 less m	10 - m
	10 less than m	m - 10

	p decreased by 11	p - 11
	8 diminished by w	8 - w
	y take away z	y - z
	p reduced by 6	p - 6
	x exceeds y	x - y
	r minus s	r - s
Multiplication	Multiply, times, the product of,multiplied by, times as much, of	x
	7 times y	7y
	The product of x and y	xy
	5 multiplied by y	5y
	one-fifth of p	(1/5)p
Division	Divide, divides, divided by, the quotient of, the ratio of, equal amounts of, per	÷
	Divide x by 6	x÷6 or x/6
	7 divides x	x÷7or x/7
	7 divided by x	7÷xor 7/x
	The quotient of y and 5	y/5
	The ratio of u to v	u/v
	u separated into 4 equal parts	u/4
	5 parts per 100 parts	5/100
Equals	Is equal to, the same as, is, are, the result of, will be, are, yields	=
	x is equal to y	x = y
	p is the same as q	p = q

Example 1

Translate “the number of cents in a given number of quarters.”

Solution:

One quarter is 25 cents. There are (25 × 2) cents in two quarters.
There are (25 × 3) cents in three quarters.

If we let q equal the unknown number of quarters, then the number of cents in q quarters is 25q.

When we translate a word expression into an algebraic expression, it is very important to preserve the order of operations. Algebraic expressions must be written and interpreted carefully, so that everyone understands the same meaning.

The algebraic expressions on the right represent the word expressions on the left.

The variable n will be used to hold the place of the unknown number.

Word Expression	Algebraic Expression
Three times a number	3n
Five more than three times a number	3n + 5
Three times, the sum of a number and five	3(n+5)

In the second expression, 3n + 5 , the number is first multiplied by three; then, that product is added to five. In the third expression, 3(n + 5) , the sum is found first; then, the sum is multiplied by three. The word sum is underlined, because it often means we group the addition in parentheses. The word difference is also important, because it often means we group the subtraction in parentheses.

Example 2

“Five years ago, I was half the age I will be in eight years. How old am I now?”

Solution:

Let a = my age now

As we can see, the word “is” is not in the word problem, but the word “was” is the past tense of the word “is”. So, we will double underline the word “was”, because that is where the equals sign goes.

“Five years ago, I was half the age I will be in eight years. How old am I now?”

Now we must find the key words that represent operations and numbers. There are no key words in this problem that match the ones given to us already.

" Five years ago I was half the age I will be in eight years. How old am I now?"

$$a - 5 = \frac{1}{2}(a + 8)$$

Now we can start to solve the equation for **a**.

$$a - 5 = \frac{1}{2}(a + 8)$$

$2a - 10 = a + 8$	[multiply each term by 2]
$2a - 10 + 10 = a + 8 + 10$	[add 10 to both sides]
$2a = a + 18$	[simplify]
$2a - a = a - a + 18$	[subtract **a**]
$a = 18$	

Example 3

Five more than twice a number is three times the difference of that number and two. What is the number?

Solution:

First, we need to create a let statement for our unknown value.

Let n = the number

Next, we will underline all the key words in the sentence.

Make sure you double underline the word "is", because that is where the equal sign goes.

"More than" means addition. More specifically, "five more than" means we add 5 to something. "Twice a number" is 2 times the variable. So, five more than twice a number is the same as 2n + 5 .

"Three times" means we multiply something by 3. "Difference" means subtraction. More specifically, "the difference of that number and two" means we group the subtraction of the variable and 2. So, three times the difference of that number and two is the same as 3(n - 2) .

Since those two phrases were separated by the word "is", we set them equal to each other.

2n + 5 = 3(n - 2)

Solve the equation:

2n + 5 = 3(n - 2)	
2n + 5 = 3n – 6	[clear the grouping symbol]
2n + 5 – 5 = 3n – 6 - 5	[subtract 5 to from both sides]
2n = 3n – 11	[simplify]
2n – 3n = 3n – 3n – 11	[subtract 3n from either sides]
-n = - 11	[simplify]
n = 11	[divide either side by -1]

So, the number is 11.

Let's check our answer.

Twice the number is 22, and five more than that is 27. The difference of the number and two is 9, and three times that is 27. Since 27 = 27, we know the number 11 is the correct answer.

Notice that when we check our answer, we check it using the written statement instead of the algebraic equation we translated. If we translated our algebraic equation wrong, the answer we get is wrong, but when we check it, it seems right.

Therefore, we must check our answer by using the original written statement.

Example 4

Three consecutive integers have a sum of 99. What are the three numbers?

Solution:

Before we can start to solve this, we need to understand what consecutive integers are.

Consecutive integers are integers that come right after another on the number line. The following are examples of consecutive integers. {1, 2, 3, 4, 5} {13, 14, 15, 16} {-11, -10, -9}.

As you can see, consecutive integers increase by one each time. But for our problem, the three consecutive integers are unknown. So, we will use a variable to represent the first integer.

If we use x as the first integer, then the second integer should be 1 more than that. Also, the third integer should be 2 more than the first one. So, the following let statements can be used to represent our three numbers.

Let x = the first integer

$x + 1$ = the second integer

$x + 2$ = the third integer

We add our three integers and set that equal to 99: **$x + (x + 1) + (x + 2) = 99$**

We can get rid of the parentheses because there is no number that needs to be distributed through them.

$x + x + 1 + x + 2 = 99$

$3x + 3 = 99$

$3x = 99 - 3$

$3x = 96$

$x = 32$

If x = 32 is the first integer, then the second integer is 1 more than that, x + 1 = 33. The third integer is 2 more than the first integer x + 2 = 34.

Proof: $32 + 33 + 34 = 99$

LEARNING ACTIVITY 11.2.1.3

20 minutes

1. Form an algebraic expression using the following statements. Use the variable n to represent unknown number.

a. A number subtracted from 15

b. 17 more than a number

c. A number increased by 12

d. A number decreased by 12

e. The sum of a number and 8

f. 15 less than a number

g. four less than a number b

h. six more than a number r

i. the quotient of eleven and a number t

j. three-fifths of a number y

k. a number z times 11

l. six less than a number x

m. the difference of 9 and twice a number n

n. twice the difference of 9 and a number n

o. twice the difference of a number n and 9

p. Rosa’s age in x years if she is now 15

2. Solve the following word problems.

a. Twice the sum of four and a number is six less than that same number. What is the number?

b. Three years from now, Alejandra will be triple her age from seven years ago. How old is Alejandra?

c. Three consecutive integers sum to 123. What are the three integers?

d. The sum of three consecutive odd numbers is 33. What are the three numbers?

e. The product of two consecutive even numbers is 528. Find the two numbers.

f. If 10% of an amount of money is K300, what is the amount?

g. A rectangle has the width being 7 cm less than the length. If the area is 198 cm^2, find the perimeter of the rectangle.

11.2.1.4 Factor and Remainder Theorem

When you divide a polynomial P(x) by a divisor d(x), you get a quotient polynomial q(x) and a remainder polynomial r(x). We write this as:

$$\frac{P(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor.

In your lower Mathematics, the long division was introduced to get the quotient and remainder.

In this section, you will learn theorem which will help you determine the remainder without actually performing the long division. We will also use a theorem to determine if a given is a factor of a polynomial without performing the long division method or techniques in factoring.

The Remainder theorem
If a polynomial P(x) is divided by **x – k**, then the remainder is **r = P(x)**

This theorem will help you determine the remainder when the polynomial **P(x)** is divided by the binomial **x – k**.

We will use synthetic substitution or sometimes called synthetic division. It can be used to divide a binomial by an expression of the form **x - k**.

Example 1

Divide $x^3 + 2x^2 - 6x - 9$ by

(a) x - 2

(b) x + 3.

Solution:

(a) To divide $x^3 + 2x^2 - 6x - 9$ by x – 2 , first we identify k. Since the binomial is x – 2 , we equate it with zero getting x – 2 = 0, transpose 2 on the other side, we have x = 2. Therefore k = 2. A technic to get k is simply get the additive inverse of the numerical term. Example x – 2 , the additive inverse of -2 is 2.

Arrange the polynomial P(x) in descending order. $x^3 + 2x^2 - 6x – 9$

Copy all numerical coefficients of each term

2	1	2	-6	-9
	1			

Drop down the numerical coefficient of the first term

2	1	2	-6	-9
	1			

Multiply the numerical coefficient of the first term (1) by k (2) . Write the product below the second term.

2	1	2	-6	-9
		2		
	1			

Add the numerical coefficients in column two (2 + 4 = 4)

2	1	2	-6	-9
		2		
	1	4		

Multiply 4 and k (2)

2	1	2	-6	-9
		2	8	
	1	4	2	

Find the sum of -6 and 8

2	1	2	-6	-9
		2	8	
	1	4	2	

Multiply 2 and 2

2	1	2	-6	-9
		2	8	4
	1	4	2	

Find the sum of -9 and 4

2	1	2	-6	-9
		2	8	4
	1	4	2	-5

Since the last sum is -5, it means that the remainder is -5.

To get the quotient, use the numerical coefficients and use the variable whose exponent is 1 degree lover than the given.

So, the quotient is $x^2 + 4x + 2 + \frac{-5}{x - 2}$.

(b) To divide $x^3 + 2x^2 - 6x - 9$ by $x + 3$, first we identify k. Since the binomial is $x + 3$, we equate it with zero getting $x + 3 = 0$, transpose 3 on the other side, we have $x = -3$. Therefore $k = -3$.

Arrange the polynomial f(x) in descending order: $x^3 + 2x^2 - 6x - 9$

Copy all numerical coefficients of each term

$$
\begin{array}{r|rrrr}
-3 & 1 & 2 & -6 & -9 \\
 & & & & \\
\hline
 & 1 & & &
\end{array}
$$

We follow the steps in example a and we will have the following.

$$
\begin{array}{r|rrrr}
-3 & 1 & 2 & -6 & -9 \\
 & & -3 & 3 & 9 \\
\hline
 & 1 & -1 & -3 & 0
\end{array}
$$

Since the last sum is zero (0), this means that the polynomial $x^3 + 2x^2 - 6x - 9$ has NO REMAINDER when divided by $x + 3$.

The quotient is $x^2 - x - 3$.

The Factor Theorem
A polynomial P(x) has a factor $x - k$, if and only if $P(k) = 0$.

In example 1 b , since $P(k) = 0$, using the theorem, we can say that $x^2 - x - 3$ and $x + 3$ are factors of $x^3 + 2x^2 - 6x - 9$.

Example 2

Determine if the following are factors of $P(x) = 2x^3 + 11x^2 + 18x + 9$.

a) $x + 3$
b) $x - 1$
c) $x - 2$

Solution:

a) Since the binomial is x + 3, then x = -3

Substitute x = -3 in the polynomial

$$P(x) = 2x^3 + 11x^2 + 18x + 9$$
$$= 2(-3)^3 + 11(-3)^2 + 18(-3) + 9$$
$$= 2(-27) + 11(9) + 18(-3) + 9$$
$$= -54 + 99 - 54 + 9$$
$$= 0$$

Since P(x) = 0, then x + 3 is a factor of $2x^3 + 11x^2 + 18x + 9$.

b) Since the binomial is x - 1, then x = 1

Substitute x = 1 in the polynomial

$$P(x) = 2x^3 + 11x^2 + 18x + 9$$
$$= 2(1)^3 + 11(1)^2 + 18(1) + 9$$
$$= 2(1) + 11(1) + 18(1) + 9$$
$$= 2 \quad + 22 \quad + 18 \quad + 9$$
$$= 51$$

Since P(x) = 51, then x + 1 is NOT a factor of $2x^3 + 11x^2 + 18x + 9$.

c) Since the binomial is x - 2, then x = 2

Substitute x = 1 in the polynomial

$$P(x) = 2x^3 + 11x^2 + 18x + 9$$
$$P(2) = 2(2)^3 + 11(2)^2 + 18(2) + 9$$
$$= 2(8) + 11(4) + 18(2) + 9$$
$$= 16 \quad + 44 \quad + 36 \quad + 9$$
$$= 105$$

Since P(x) = 105, then x - 2 is NOT a factor of $2x^3 + 11x^2 + 18x + 9$.

Now do the activity.

LEARNING ACTIVITY 11.2.1.4

20 minutes

1) Use synthetic division to find the quotient of the following:

a) $(3x^3 – 2x^2 – 7x + 6) \div (x + 1)$

b) $(2x^3 + 7x^2 – 6x - 8) \div (x + 4)$

2) Using the remainder theorem, find the remainder when f(x) is divided by (x – k).

a) $(3x^3 + 4x^2 – 5x + 3) \div (x + 2)$

b) $(3x^3 + 4x^2 – 5x – 2) \div (x – 1)$

c) $(2x^3 - 5x^2 + x – 3) \div (x – 1)$

d) $(2x^3 – 6x – 5) \div (x + 3)$

3) Factorize the following using the Factor Theorem.

a) $P(x) = x^3 + 2x^2 - 5x - 6$

b) $P(x) = 2x^3 + x^2 - 2x - 1$

c) $P(x) = 2x^4 - 3x^3 - 12x^2 + 7x + 6$

d) $P(x) = 9x^3 - 9x^2 - 4x + 4$

e) $P(x) = x^4 - 10x^2 + 9$

11.2.1.5 Exponents and Logarithms

Exponential functions and logarithm functions are important in both theory and practice. When asked to solve an exponential equation such as $\mathbf{2^{x+6} = 32}$ or $\mathbf{5^{2x-3} = 18}$, the first thing we need to do is to decide which way is the "best" way to solve the problem. Some exponential equations can be solved by rewriting each side of the equation using the same base. Other exponential equations can only be solved by using logarithms.

11.2.1.5.1 Exponents

Let us revise the rules for exponents, indices or powers.

Definition	Power
Zero Power	$a^0 = 1$
Power of 1	$a^1 = a$
Negative Power	$a^{-1} = \frac{1}{a}$
Product Rule	$a^m \times a^n = a^{m+n}$
Quotient Rule	$a^m \div a^n = a^{m-n}$
Power of Power	$(a^m)^n = a^{mn}$

Example

Simplify the following:

(a) 17^0

(b) 349^1

(c) 3^{-2}

(d) $4^2 \times 4^{½}$

(e) $7 \div 7^{-2}$

(f) $(5^6)^{½}$

Solution:

(a) $17^0 = 1$ [Power of 0]

(b) $349^1 = 349$ [Power of 1]

(c) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ [Negative Power]

(d) $4^2 \times 4^{½} = 4^{2 ½}$ [Product Rule]

$= 4^{5/2}$

$= (\sqrt{4})^5$ [Fractional Power]

$= 2^5$

$= 32$

(e) $7 \div 7^{-2} = 7^3$ [Quotient Rule]

$= 343$

(f) $(5^6)^{½} = 5^3$ [Powers of a Power]

$= 125$

11.2.1.5.2 Solving Exponential Equations with the Same Base

After deciding the best way to solve an exponential equation is by rewriting each side of the equation using the same base.

Solving Exponential Equations with the Same Base:
If $B^M = B^N$, then M = N.

This statement simply says that if the bases are the same then the exponents must be the same.

To solve an exponential equation with the same base, first we need to rewrite the problem using the same base and after getting the bases the same we can drop the bases and set the exponents equal to each other.

Steps for Solving Exponential Equations with the Same Base

Step 1: Determine if the numbers can be written using the same base.

Step 2: Rewrite the problem using the same base.

Step 3: Use the properties of exponents to simplify the problem.

Step 4: Once the bases are the same, drop the bases and set the exponents equal to each other.

Step 5: Finish solving the problem by isolating the variable.

Example 1

Solve $9^{2x-5} = 27$.

Solution:

$9^{2x-5} = 27$	Determine if 9 and 27 can be written using the same base. In this case both 9 and 27 can be written using the base 3.
$(3^2)^{2x-5} = 3^3$	Rewrite the problem using the same base.
$3^{4x-5} = 3^3$	Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.
$4x - 5 = 3$	Since the bases are the same, we can drop the bases and set the exponents to be equal to each other.
$x = 2$	Add 5 to each side and then dividing each side by 4.

Therefore, the solution to the problem $9^{2x-5} = 27$ is x = 2.

11.2.1.5.3 Logarithms

Given $\mathbf{\log_x y = a}$ means base **x** raised to the power **a** equals **y**, that is $\mathbf{x^a = y}$. Or $\mathbf{\log_x y}$ means base **x** raised to an unknown power (**?**) is equal to **y**, that is $\mathbf{x^? = y}$.

Suppose $\mathbf{\log_x y}$ then it is an expression and $\mathbf{\log_x y = a}$ is an equation. The expression can be rewritten as $\mathbf{\log_x y} = \frac{1}{\log_y x}$. That is saying, how can **y** be expressed as a power of **x**?

What is logarithm?

Logarithm enables us to linearize the exponential equations. When an exponent is the unknown and the base is large, logarithm enables us to solve for the exponent more faster.

Say if given is $\mathbf{1\,000\,000 = 50\,000\,(1 + 8\%)^n}$, which simplifies to $1\,000\,000 = 1.08^n$. The equation is not solvable by basic algorithm. Linearizing by applying logarithmic power property as $\log 1\,000\,000 = \log 1.08^n$, thus yields:

$$n = \frac{\log 1\,000\,000}{\log \cdot\cdot 1.08}$$

We can then use the calculator to find value of n from the given equation above. You need to be familiar with the calculator buttons and functions in-order to use the calculator at this stage. Some calculators may function by the sequence **log** 1 000 000 while others go by the sequence 1 000 000 **log**.

We must note that we are restricted to write **positive** log expression for a given exponential expression or equation. Also, we **cannot** have **0** and **1** as base of a log expression.

Let us relate the exponential properties to logarithmic properties.

Property	**Exponents**	**Logarithms**
Zero Power	$a^0 = 1$	$\log_a 1 = 0$
Power of 1	$a^1 = a$	$\log_a a = 1$
Negative Power	$a^{-1} = \frac{1}{a}$	$\frac{1}{\log_a b} = -1$
Product Rule	$a^m \times a^n = a^{m+n}$	$\log_a MN = \log_a M + \log_a N$
Quotient Rule	$a^m \div a^n = a^{m-n}$	$\log_a M/N = \log_a M - \log_a N$
Power of Power	$(a^m)^n = a^{mn}$	$\log_a M^N = N\log_a M$

Say given log 6, means that $\log_{10} 6$. So when a base is not indicated, it means the base of the log expression is 10.

We will restrict our discussions to Logarithms and not touch on Niperian or Natural logarithm **$\log_e x = \ln x$**. Where **e** is an **exponential constant,** with an approximate value of 2.718 (4 significant figures).

Example 2

Evaluate the following expressions and equations:

(a) $\log_7 1$

(b) $\log_6 6$

(c) $\log_{25} 5$

(d) $\log_2 16 = x$

(e) $\log_3 x = 5$

(f) $\log_x 8 = 3$

(g) log10

Solution:

(a) $\log_7 1 = 0$ [Zero Power]

(b) $\log_6 6 = 1$ [Power of 1]

(c) $\log_{25} 5 = \frac{1}{\log_5 25} \rightarrow l\frac{1}{\log_5 25} = \frac{1}{\log_5 5 + \log_5 5} = \frac{1}{1+1} = \frac{1}{2}$ [$25^x = 5$]

(d) $2^x = 16 \rightarrow 2^x = 2^4$, $x = 4$

(e) $3^5 = x$, $x = 243$

(f) $x^3 = 8 \rightarrow x^3 = 2^3$, powers are the same, therefore $x = 2$

(g) $\log 10 = \log_{10} 10 = 1$

Example 3

Expand and simplify the following where possible:

(a) $\text{Log}_3 15$

(b) $\text{Log}_2 \frac{9}{2}$

(c) $\text{Log}_5 3^7$

Solution:

(a) $\text{Log}_3 15 = \log_3 (3 \text{ x } 5)$

$= \log_3 3 - \log_3 5$ [Product Property]

$= 1 - \log_3 5$ [$\log_a a = 1$]

Log₃ 15 is the condensed form of 1 – log₃ 5.

(b) $\text{Log}_3 \frac{9}{2} = \log_3 (9 - 2)$

$= \log_3 9 - \log_3 2$ [Quotient Property]

$= \log_3 3 + \log_3 3 - \log_3 2$ [Product Property on $\log_3 9$]

$= 1 + 1 - \log_3 2$ [$\log_a a = 1$]

$= 2 - \log_3 2$

$\text{Log}_3 (9/2)$ is the condensed form of $2 - \log_3 2$.

(c) $\text{Log}_5 10^7 = 7\log_5 10$ [Power of Power]

$= 7\log_5 2 + 7\log_5 5$ [Product Property]

$= 7\log_5 2 + 7 \text{ x } 1$ [$\log_a a = 1$]

$= 7 + 7\log_5 2$

$\text{Log}_5 10^7$ is the condensed form of $7 + 7\log_5 2$.

11.2.1.5.4 Solving Exponential Equations with the Different Bases

After deciding the only way to solve an exponential equation is to use logarithms, what is the next step? The next step is to take the common logarithm or natural logarithm of each side.

By taking the logarithm of each side, we can use the properties of logarithms, specifically property 5 from our list of properties, to rewrite the exponential problem as a multiplication problem. After changing the problem from an exponential problem to a multiplication problem using the properties of logarithms we will be able to solve the problem.

Example 3

Let's solve the problem $5^{2x-3} = 18$.

Solution:

In this problem we have already seen that it is impossible to rewrite the numbers 5 and 18 using the same base, so we must use logarithms. Continuing on here is what we get:

$\log(5^{2x-3}) = \log(18)$	Take the common logarithm or natural logarithm of each side.
$(2x - 3)(\log 5) = \log 18$	Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.
$2x - 3 = \dfrac{\log 18}{\log 5}$	Divide each side by log 5.
$2x - 3 \approx 1.795889$	Use a calculator to find log 18 divided by log 5. Round the answer as appropriate, these answers will use 6 decimal places.
$x \approx 2.397944$	Finish solving the problem by adding 3 to each side and then dividing each side by 2.

Therefore, the solution to the problem $5^{2x-3} = 18$ is $x \approx 2.397944$.

From this example we can derive the following steps.

Solving Exponential Equations with Different Bases

Step 1: Determine if the numbers can be written using the same base. If so, stop and use Steps for solving an Exponential Equation with the Same Base. If not, go to Step 2.

Step 2: Take the common logarithm or natural logarithm of each side.

Step 3: Use the properties of logarithms to rewrite the problem. Specifically, use Property 5 which says $\mathbf{\log_a x^y = y\log_a x}$.

Step 4: Divide each side by the logarithm.

Step 5: Use a calculator to find the decimal approximation of the logarithms.

Step 6: Finish solving the problem by isolating the variable.

Example 4

Solve $8^{4x+1} = 205$.

Solution:

Solve $8^{4x+1} = 205$	Determine if 8 and 205 can be written using the same base. In this case 8 and 205 cannot be written using the same base, so we must use logarithms.
$\log(8^{4x+1}) = \log(205)$	Take the common logarithm or natural logarithm of each side.
$(4x+1)(\log 8) = \log 205$	Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.
$4X + 1 = \frac{\log 205}{\log 8}$	Divide each side by log 8.
$4x + 1 \approx 2.559827$	Use a calculator to find log 205 divided by log 8. Round the answer as appropriate, these answers will use 6 decimal places.
$x » 0.389957$	Finish solving the problem by subtracting 1 from each side and then dividing each side by 4.

Therefore, the solution to the problem $8^{4x+1} = 205$ is $x \approx 0.389957$.

11.2.1.5.5 Change of Base Formula.

Calculators operate in base 10 or base e, thus there is a need to change say, loga b to base 10 or base e.

Formula

$$\log_a b = \frac{\log_{10} b}{\log_{10} a} \text{ or } \frac{\ln b}{\ln a}$$

Example 1

Write $\log_2 3$ to base 5.

Solution:

$$\log_2 3 = \frac{\log_5 3}{\log_5 2}$$

Example 2

Write $\log_2 3$ to base 10.

Solution:

$$\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2}$$

The reverse can be written for the formula.

$$\boxed{\frac{\log_b a}{\log_b c} = \log_c a}$$

Example 1

Express as a single log: $\frac{\log_9 12}{\log_9 4}$.

Solution:

$$\frac{\log_9 12}{\log_9 4} = \log_4 12$$

Example 2

Express as a single log, then simplify where possible: $\frac{\log_7 32}{\log_7 4}$.

Solution:

$$\frac{\log_7 32}{\log_7 4} = \frac{\log_7 2^5}{\log_7 2^2}$$

$$= \frac{5\log_7 2}{2\log_7 2}$$

$$= \frac{5}{2} \text{ or } 2\tfrac{1}{2}$$

Or we can solve by expressing it as a single log expression as:

$$\frac{\log_7 32}{\log_7 4} = \log_4 32$$

$= \log_4 (2 \times 4 \times 4)$
$= \log_4 2 + \log_4 4 + \log_4 4$
$= \log_4 2 + 1 + 1$
$= \frac{1}{2} + 2$
$= 2\frac{1}{2}$

We can also transform $\log_4 32$ to an indicial equation and solve.

We can also transform $\log_4 32$ to an indicial equation and solve.

$\log_4 32 \rightarrow 4^x = 32$, thus

$2^{2x} = 2^5$ [base is the same]

$2x = 5$ [equate powers]

$X = 2.5$ or $2\frac{1}{2}$

So $\log_7 32 \div \log_7 4 = 2\frac{1}{2}$.

Now do the learning activity.

STUDENTLEARNING ACTIVITY 11.2.1.5

20 minutes

1. Simplify the following indicial expressions:

   (a) $2^0 + 2^{-1}$

   (b) $3^{-2} \times 3^4$

   (c) $4^6 \div 4^4$

   (d) $(5^{1/3})^9$

   (e) $3^{-1} \times 6^2$

2. Solve for x in the following:

   (a) $4^{2x-3} = 8^{x-3}$

   (b) $3^{2x-5} = 243$

   (c) $3^{3x} = 729$

   (d) $7^x = 49^{x-2}$

   (e) $3125 = 25^{(x+2)}$

3. Write as an indicial equation:

   (a) $\text{Log}_5 125$

   (b) $\text{Log}_{125} 5$

   (c) $\text{Log}_2 16 = x$

   (d) $\text{Log}_3 x = 5$

   (e) $\text{Log}_x 8 = 3$

4. Evaluate the log expressions and equations given below.

(a) $\log_{100} 1$

(b) $\log_{3x} 3x$

(c) $\log_{81} 3$

(d) $\log_4 64 = x$

(e) $\log_2 x = 5$

5. Expand the log expressions:

(a) $\log_2 12 =$

(b) $\log_3 \dfrac{21}{5} =$

(c) $\log_2 6^3 =$

(d) $\log_3 5 =$

(e) $\log_8 \dfrac{3}{2} =$

6. Expand using quotient and product properties and simplify where possible:

(a) $\text{Log}_3 \dfrac{12}{7} =$

(b) $\text{Log}_4 20 =$

(c) $\text{Log}_6 \dfrac{27}{4} =$

(d) $\text{Log}_5 30 =$

(e) $\text{Log}_2 210 =$

7. Condense the log expressions:

(a) Log 6 + log 3 =

(b) Log8 – log 5 =

(c) $\text{Log}_2 7 + \log_2 3 + \log_2 5 =$

(d) $\text{Log}_3 5 - \log_3 7 - \log_3 2 =$

(e) $\dfrac{\log 7}{\log 2} =$

SUMMATIVE TASK 11.2.1

20 minutes

1. Write the expression for "5 less than the quotient of 13 and a number n".

2. The difference of five – eighth of a number y and 3 is 7. What is the value of the number y?

3. The sum of the first and third consecutive even numbers is equal to twice the second consecutive even number. What are the three consecutive even numbers?

4. Simplify $\frac{-8x^2 - 8x}{2x^2 - 2}$.

5. Simplify by collecting like terms: $6x^2 - 2xy + 2x^2y - 4x^2 + 2xy + 4x^2y$

6. What is the remainder when $(x^3 + 2x^2 – 3x + 4) \div (x - 2)$

7. If P(-6) = 0 given $P(x) = (x^4 + 3x^3 - 34x^2 – 108x – 72)$,find the other three P(x) = 0.

i. [(x- 6)(x + 6)(x + 2)(x + 1)]

8. $3^{½ x – 1} = 729$

9. Simplify $\log_3 80 – \log_3 24$.

10. Express as an indicial equation for $\log_4 84$.

11.2.2 Linear Equations and Inequalities

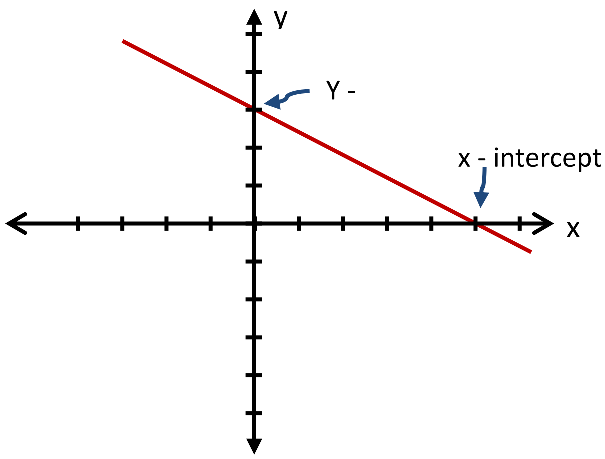
This topic focuses on the linear and quadratic equations which serve as the foundations of equations. Other graphs of functions depend on the basic properties of the two. Any equation is linear if it is univariable equation with a degree of 1.

11.2.2.1 Linear Equations and Functions

As previously described in your lower Mathematics, a **linear equation** can be defined as an equation in which the highest exponent of the equation variable is one. A linear function is a function of the form f(x) = ax + b or y = mx + c.

The graph of a linear equation is a graphical view of the set of all points that make the equation true. The graph of any linear function is **a straight line**.

Form	Equation	Note
General	Ax + Bx + C = 0	A and B are not 0.
Standard or Slope-intercept	Y = mx + c	M is the slope of the line and c is the y - intercept



To find the **x-intercept**:

1. Set y = 0 in the equation.
2. Solve for x. The value obtained is the x-coordinate of the x-intercept.
3. The x-intercept is the point (x, 0), with x the value found in step 2.

To find the **y-intercept**:

1. Set x = 0 in the equation.
2. Solve for y. The value obtained is the y-coordinate of the y-intercept.
3. The y-intercept is the point (0, y), with y the value found in step 2.

Example 1

Find the intercepts of the straight-line y = $\frac{1}{5}$x - 3.

Solution:

y = $\frac{1}{5}$x – 3, when y = 0, 0 = $\frac{1}{5}$x – 3 [substitute for y]

0 = x – 15 [multiply each term by 5]

3 = x [add 15 to each side]

X = 15

When x = 0, y = $\frac{1}{5}$(0) – 3 [substitute for x]

Y = 0 – 3 [zero times any number is 0]

Y = - 3 [simplify RHS]

The intercepts are x = 15 and y = 3. In point form (15, 0) and (0, -3)

Example 2

Find the intercepts of the straight-line 4y + x – 28 = 0.

Solution:

4y + x – 28 = 0 in general form is the same as y = $-\frac{1}{4}$x + 7 expressed in slope-intercept form.(You can use either form to solve for intercepts).

When x = 0, 4y + 0 – 28 = 0 [substitute for x]

4y – 28 = 0 [simplify LHS]

4y = 28 [add 28 to either side]

Y = 7 [divide by either side by 4]

When y = 0, 4(0) + x – 28 = 0 [substitute for y]

0 + x – 28 = 0 [multiply 4 by 0]

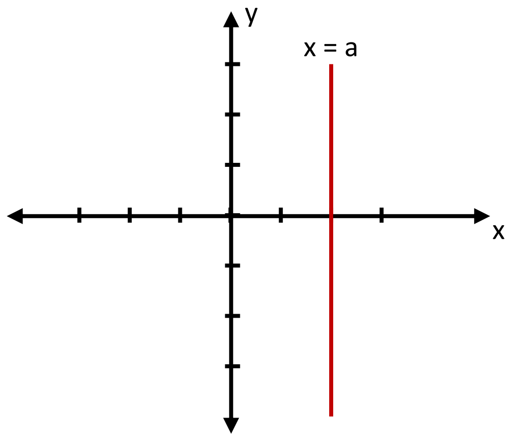
X – 28 = 0 [simplify LHS]

X = 28 [add 28 to both sides]

The intercepts are y = 7 and x = 28. In point form (0, 7) and (28, 0)

Vertical Lines

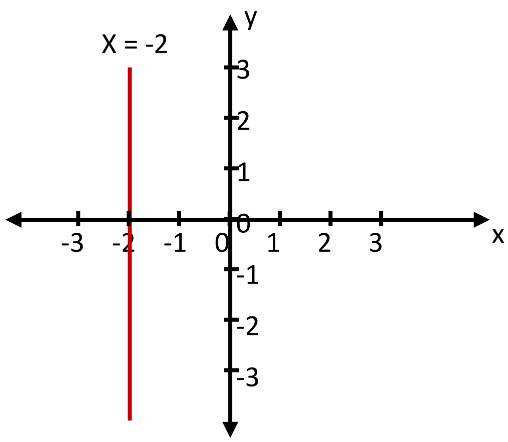
Equations of the form x = a are vertical lines. The x-coordinate of every point on the vertical line x = a has the value "a," always, for any given value.



Example 3

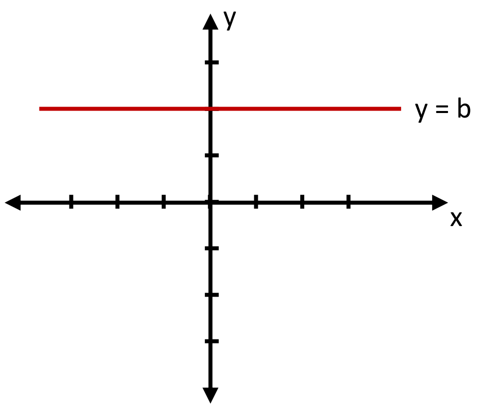
Draw a graph of x = -2.

Solution:



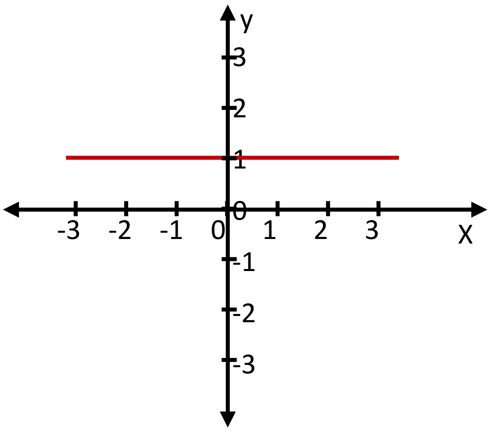
Horizontal Lines

Equations of the form y = a are horizontal lines. The y-coordinate of every point on the horizontal line y = b has the value "b," always, for any given x value.



Example 4

Write equation of the straight-line graph below.

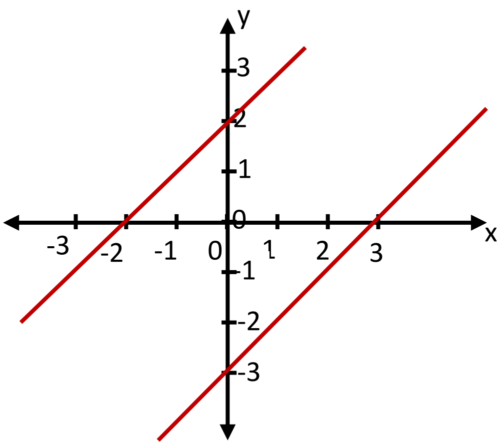


Solution:

Equation: y = 1

Parallel Lines

In the slope -intercept form equation y = mx + c, m is the slope and c is the y-intercept. Two lines are parallel if their slopes are the same ($m_1 = m_2$) and their y-intercepts are different, ($c_1 \neq c_2$).



Example 5

Find the equation of the straight-line **P** parallel to y = $\frac{1}{2}$x + 4 and has a y – intercept of 0.

Solution:

P: $m_2 = \frac{1}{2}$ since $m_1 = m_2$ if two straight-lines are parallel lines

C = 0, Given

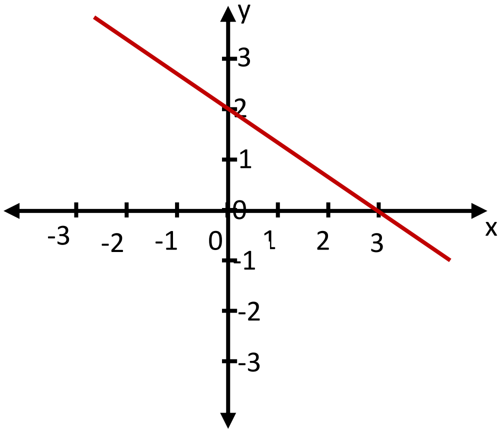
Equation: $y = \frac{1}{2}x + 0$

$Y = \frac{1}{2}x$ [simplify]

Thus $y = \frac{1}{2}x + 4$ is parallel to $y = \frac{1}{2}x$.

Example 6

Below is a graph of a straight-line 3y + 2x – 6 = 0. Write equation of another straight-line **P** parallel to it and has the point (- 6,2).



Solution:

Expressing 3y + 2x – 6 = 0 in slope-intercept form is $y = -\frac{2}{3}x + 2$.

Thus $m_1 = -\frac{2}{3}$, therefore m of **P**: $m_2 = -\frac{2}{3}$. P has the point (-6,2).

To find y- intercept, substitute for x and y in **P**.

Thus, in P: $y = -\frac{2}{3}x + c$ and contains (-6,2).

$2 = -\frac{2}{3}(6) + c$ [multiply $-\frac{2}{3}$ by 6]

$2 = -4 + c$

$2 + 4 = c$ [add 4 to both sides]

$C = 6$

Now substitute for c into P, given $m_2 = -\frac{2}{3}$.

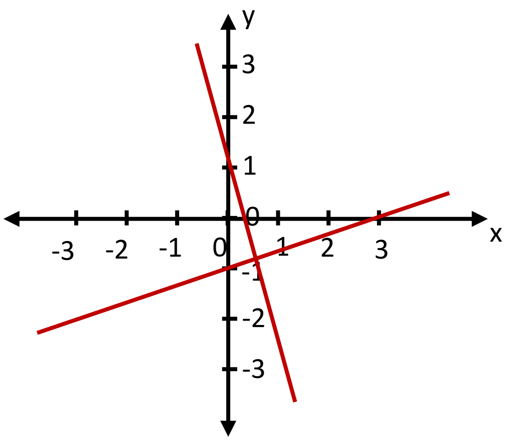
Equation of P: $y = -\frac{2}{3}x + 6$

Perpendicular Lines

Two lines are perpendicular if the slopes are the negative reciprocal of each other: $\mathbf{m_1 = -\frac{1}{m_2}}$ **or** $\mathbf{m_1 \times m_2 = -1}$

Example	1)	$y = 4x + 7$	$m_1 = 4$	$c_1 = 7$
	2)	$y = -\frac{1}{4}x + 2$	$m_2 = -\frac{1}{4}$	$c_2 = 2$

The slope of equation 2 is the negative reciprocal of the slope of equation 1; therefore, the lines are perpendicular.



Example 7

Prove that the two straight - lines **L**: $5y – 2x = 5$ and **M**: $2y + 5x - 4 = 0$ are perpendicular.

Solution:

Express **L** and **M** in gradient – intercept forms (slope - intercept).

L: $y = \frac{2}{5}x - 1$, hence $m_1 = \frac{2}{5}$

M: $y = -\frac{5}{2}x + 2$, hence $m_2 = -\frac{5}{2}$

$m_1 \times m_2 = \frac{2}{5} \times -\frac{5}{2} = -\frac{10}{10} = -1$

Since product of the slopes is – 1, L is perpendicular to M.

Line **L** $\perp$ Line **M**

Example 8

A straight-line **L**: y = $\frac{3}{2}$x – 10 is perpendicular to straight-line **M** at (10,5). Write equation of line **M**.

Solution:

L: y = $\frac{3}{2}$x – 10, therefore $m_1 = \frac{3}{2}$

Since L ⊥ M (given), therefore $m_1 \times m_2 = -1$ (property)

$\frac{3}{2} \times m_2 = -1$

$3m_2 = -2$

$m_2 = -\frac{2}{3}$

L ⊥ **M** at (10,5) and m of **M** = $-\frac{2}{3}$

Thus M: $5 = -\frac{2}{3}(10) + c$

$15 = -20 + 3c$

$35 = 3c$

$\frac{35}{3} = c$

$c = 11\frac{2}{3}$

Therefore M: y = $-\frac{2}{3}$x + $\frac{35}{3}$ or 3y + 2x – 35 = 0.

Mid-Point Formula

Given two points on a Cartesian Plane, we can calculate the centre point of the two given points by the formula:

$$M.P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + Y_2}{2}\right).$$

Example 8

Find the mid-point of the points (-3,8) and (3,- 4).

Solution:

$$M.P = \left(\frac{-3+3}{2}, \frac{8+-4}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0,2)$$

Two-Points Formula

Given two points we can derive the formula of a SL that contains the two points by the formula :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This formula can be re-written as:

$y - y_1 = m(x - x_1)$ which is called **Point-Slope Formula**.

This is because the ratio of the differences of y and x yields the slope. That is m = $\frac{y_2 - y_1}{x_2 - x_1}$.
So if you are given two points, you can apply the formula to derive its slope or gradient. That formula is called **Gradient Formula**.

Example 9

Use two-point formula to derive equation of a SL that runs through (2, ½) and (4, -3 ½).

Solution:

Points (2, ½) and (4, -3 ½).

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - \frac{1}{2} = \frac{-3\frac{1}{2} - \frac{1}{2}}{4-2}(x - 2)$$
$$y - \frac{1}{2} = \frac{-4}{2}(x - 2)$$
$$y - \frac{1}{2} = -2(x - 2)$$
$$y - \frac{1}{2} = -2x + 4$$
$$y = -2x + 3\tfrac{1}{2} \text{ or } y = -2x + 3.5$$

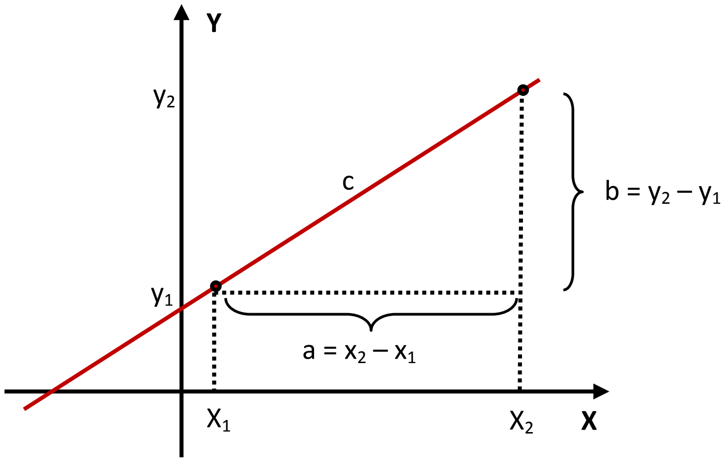
The SL y = -2x + 3.5 runs through (2, ½) and (4, -3 ½).

Distance Formula

Distance between two points can be calculated using the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad .$$

The formula is derived from Pythagoras theorem $c^2 = a^2 + b^2$. This is a theorem derived from a right triangle.
Study the illustration below and relate it to the example 10 that follows.



Example 10

Calculate the distance between the two points given in example 9.

Solution:

Points **(2, ½) and (4, -3 ½)**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4-2)^2 + (-3\tfrac{1}{2} - \tfrac{1}{2})^2}$$
$$= \sqrt{2^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= \sqrt{4x5}$$
$$= 2\sqrt{5}$$

The distance between the points is 2√5 units or 4.47 units.
(2√5 units is exact, 4.47 units is the approximate distance)

Now do the learning activity.

LEARNING ACTIVITY 11.2.2.1

30 minutes

1. Write TRUE if the given is correct, write FALSE if it is not correct.

   a) The standard form of a linear equation is y = mx+b

   b) The point (4, 0) is on the graph of 3x – 4y = 12

   c) The graph of 3x – 4y = 12 passes the vertical line test for functions.

   d) The y-intercept of of 3x – 4y = 12 is (0, –3)

   e) To find the x-intercept of a graph, set y equal to zero and solve the equation for x.

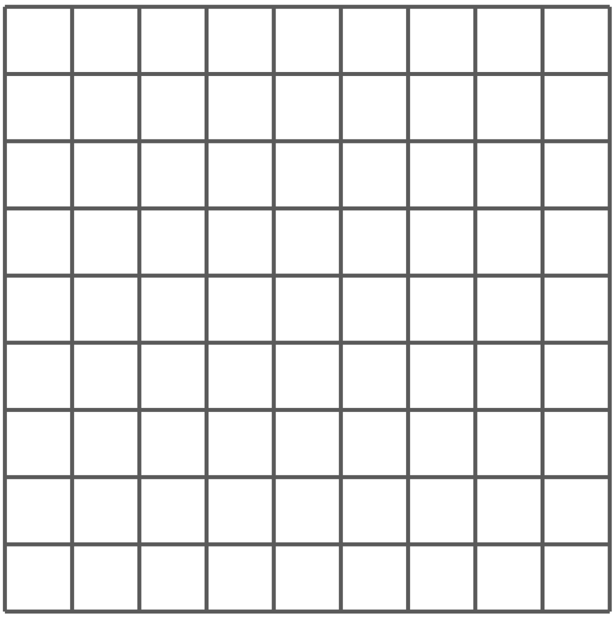
   f) The graph of x = 3 is a horizontal line three units above the x-axis.

   g) Generally speaking, it is a good idea to plot three points when constructing the graph of a linear function.

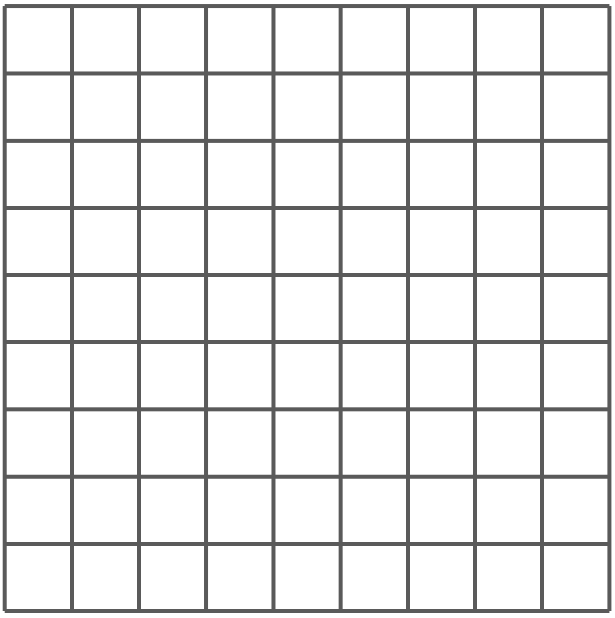
   h) To find the y-intercept of a graph, set x equal to zero and solve the equation for y

2. Graph the following:

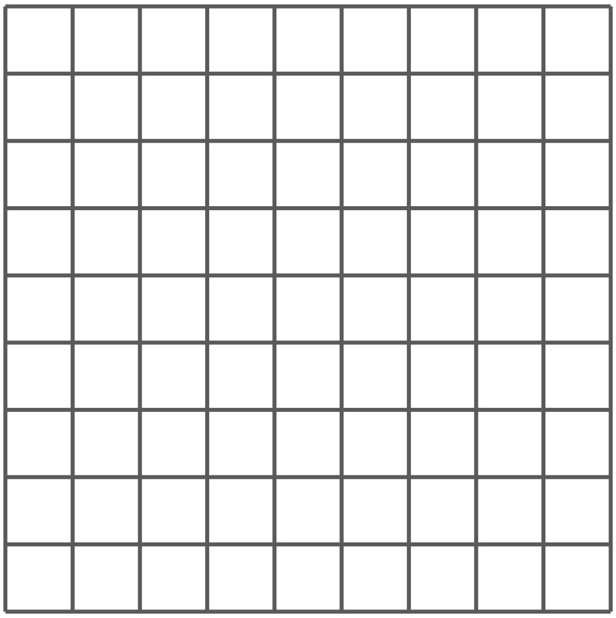
   a) 2x + 1 = 0



b)x = -3



c) y = 4



d) A straight line y = 2x – 8 is parallel to another straight-line (SL) **P** such that **P** contains the point (0, 2). Write equation of the SL **P**.

e) A straight line y = -3x + 4 is perpendicular to another straight-line **Q** such that **Q** contains the point (-3, 10). Write equation of the SL **Q**.

f) A SL **M** is perpendicular to SL **N** at the point (6,1). If **M**: 4y = 2x – 12, derive equation of SL **N**.

g) Calculate slope of the SL that runs through the points (-3, -4) and (1,6).

h) Calculate the mid-point of **LM** when **L** is the point (-5, 2) and **M** is (7, -4).

i) Use two-points formula to derive equation of a SL passing through (-1,5) and (3, 1).

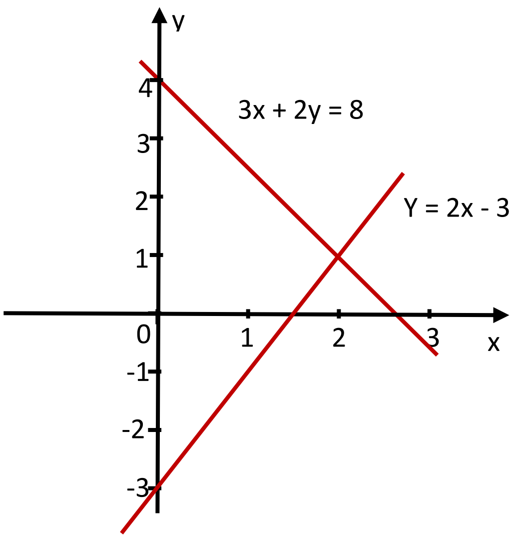
j) Calculate the distance between the points (2, 6) and (-1,2).

11.2.2.2 Simultaneous Linear Equations

Let's just have a look at a relatively straightforward single equation. The equation we are going to look at is $2x - y = 3$. We can plot these as points on a graph. We can plot the first as the point (1,−1). We can plot the second one as the point (2, 1), and the third one as the point (0,−3) and so on. Plotting the points on a graph, as shown in Figure 1, we see that these three points lie on a straight line.

This is the line with equation $y = 2x - 3$. It is a straight line and this is another reason for calling the equation a linear equation.

Suppose we take a second linear equation $3x + 2y = 8$ and plot its graph on the same plane.



When we solve a pair of simultaneous equations what we are actually looking for is the intersection of two straight lines because it is this point that satisfies both equations at the same time. From the figure above we see that this occurs at the point where $x = 2$ and $y = 1$.

If we have two **parallel** lines; they would never meet, and hence the simultaneous equations would not have a solution. We shall observe this behaviour in one of the examples which follows.

When solving a pair of simultaneous linear equations we are, in fact, finding a common point- the point of intersection of the two lines.

Solving Simultaneous Equations - Method of Substitution

How can we handle the two equations algebraically so that we do not have to draw graphs? We are going to look at two methods of solution. In this Section we will look at the first method -the method of substitution.

Let us use the following equations:

$$2x - y = 3 \qquad [1]$$

$$3x + 2y = 8 \qquad [2]$$

By rearranging Equation [1] we find

$$y = 2x - 3 \qquad [3]$$

We can now substitute this expression for y into Equation [2].

$$3x + 2(2x - 3) = 8$$

$$3x + 4x - 6 = 8$$

$$7x - 6 = 8$$

$$7x = 14$$

$$x = 2$$

Solving Simultaneous Equations - Method of Elimination

We illustrate the second method by solving the simultaneous linear equations:

$$7x + 2y = 47 \qquad [1]$$

$$5x - 4y = 1 \qquad [2]$$

We are going to multiply Equation (1) by 2 because this will make the magnitude of the coefficients of y the same in both equations. Equation [1] becomes

$$14x + 4y = 94 \qquad [3]$$

If we now add Equation (2) and Equation (3) we will find that the terms involving y disappear:

$$\begin{array}{l} 5r \quad - 4y = 1 \\ \underline{14r + 4y = 94} \\ 19x \qquad = 95 \end{array}$$

And so,

$$x = \frac{95}{19} = 5$$

Now that we have a value for x we can substitute this into Equation (2) in order to find y.

Substituting:

$$\begin{aligned} 5x - 4y &= 1 \\ 5 \times 5 - 4y &= 1 \\ 25 &= 4y + 1 \\ 24 &= 4y \\ y &= 6 \end{aligned}$$

The solution is x = 5, y = 6.

Now do the learning activity.

Learning Activity 11.2.2.2

30 minutes

Solve the simultaneous equations using any method.

1) $3x + 7y = 27$ and $5x + 2y = 16$

2) $5x + 3y = 9$ and $2x - 3y = 12$

3) $5x + y = 10$ and $7x - 3y = 14$

4) $Y = 3x + 11$ and $2y - x = 13$

5) $Y - 2x = 2$ and $y + 2x = 4$

6) $5y + x = 1$ and $5y = 20x - 37$

7) $2y + x = -3$ and $3y + 12 = x$

8) $3y + 2 = x$ and $5y + 6 = x$

9) $Y + 2x - 7 = 0$ and $8y - 2x - 65 = 0$

10) $30y - 45x = 28$ and $3y - 5x + 2 = 0$

11.2.2.3 Linear Inequalities

In real life situations we are often faced with inequalities, and not equations. The situation may give rise to more than one inequality (systems of linear inequality). And as such, we may arrive at more than one possible solution.

We will use the symbols: **<,>,≤** and **≥**.

a > b	**a** is **greater** than **b**
a < b	**a** is **less** than **b**
a ≥ b	**a** is **greater than or equal** to **b**
a ≤ b	**a** is **less than or equal** to **b**

Linear Inequality with one variable such as x -5 < 2, has one variable, 'x'. And Linear Inequality with two variables such as 2y ≥ 3x + 8 has two variables, 'x' and 'y'.

Linear inequality with one variable can be illustrated or represented on a number line. The process of solving linear inequality is similar to solving linear equality or linear equations. The slight difference in inequality occurs when the inequality sign changes direction when multiplied or divided by a negative integer.

Example 1

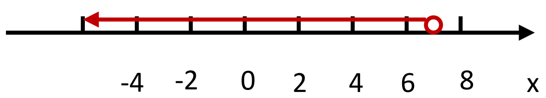
Show on the number line the inequality x – 5 < 2.

Solution:

x – 5 < 2

x -5 + 5 < 2 + 5

x < 7



Note: We have an open circle on 7, indicating that all possible solutions are less than 7.

Example 2

Show on the number line the inequality $5 - 3x \geq 2$.

Solution:

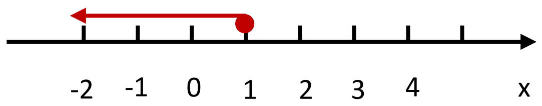
$$5 - 3x \geq 2$$

$$5 - 5 - 3x \geq 2 - 5$$

$$-3x \geq -3$$

$$\frac{-3x}{-3} \leq \frac{-3}{-3}$$

$$X \leq 1$$



Note: We have a closed circle on 1, indicating that all possible solutions are equal to or less than 1.

Example 3

Solve the inequality $-2(x + 3) < 10$

Solution:

$$-2(x + 3) < 10$$

$$-2x - 6 < 10$$

$$-2x - 6 + 6 < 10 + 6$$

$$-2x < 16$$

$$\frac{-2x}{-2} > \frac{16}{-2}$$

$$X > -8$$

Now do the learning activity.

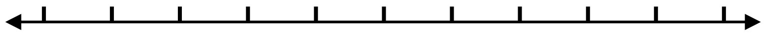
LEARNING ACTIVITY 11.2.2.3

30 minutes

Show on the number line the following:

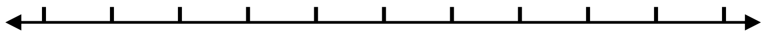
1. X > -2

2. X ≥ 1

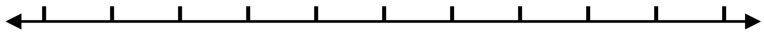


3. X < 5

4. X ≤ 0

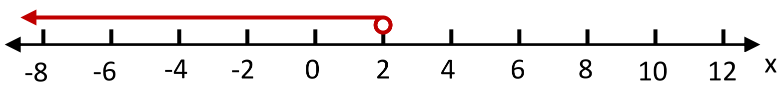


5. -3 < x ≤ 4

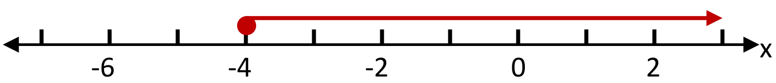


Write inequality of the following:

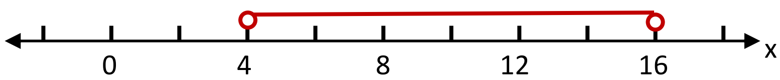
6.



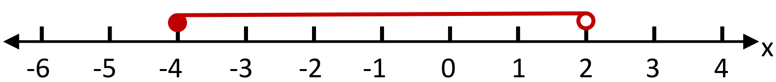
7.



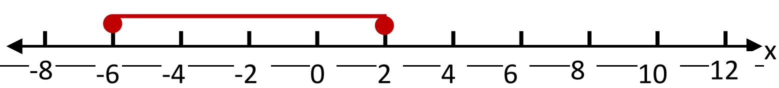
8.



9.



10.



11.2.2.4 Systems of Linear Inequalities

Linear inequality of systems of two unknowns or two variables can be treated like systems of two equations in two unknowns.

We can solve them simultaneously to solve for their points of intersection, or we can plot or sketch them and show feasible region and feasible solutions.

Example 1

Solve the systems of inequalities $y + 2x \leq 8$ and $3y + 4x < 6$.

Solution:

$y + 2x \leq 8$ $\quad\quad$ $y + 2x = 8$ [1]

$3y + 4x < 18$ $\quad\quad$ $3y + 4x = 18$ [2]

From [1] $y = 8 – 2x$, substitute into [2], yields

$$3(8 – 2x) + 4x = 18,$$
$$24 – 6x + 4x = 18$$
$$24 – 2x = 18$$
$$- 2x = - 6$$
$$X = 3$$

Now, when $x = 3$, in [1] $y + 2.3 = 8$, $y + 6 = 8$, $y = 2$ $\quad$ (3, 2)

A straight line in a Cartesian plane divides the plane in two equal regions. Two straight lines that are not parallel divide the Cartesian plane into four regions.

The purpose of sketching and plotting systems of inequalities is to provide us illustrations so we are able to visualize and solve the problem. Many problems in real life do not give us equations, but are inequalities.

Example 2

Sketch the systems of inequalities $y + 2x \leq 8$ and $3y + 4x < 6$ and show the feasible region.

Solution:

$y + 2x \leq 8 \Rightarrow y = -2x + 8$

$3y + 4x < 6 \Rightarrow y = -4/3\,x + 2$

For $y = -2x + 8$, when $x = 0$, $y = 8$, and when $y = 0$, $x = 4$ Intercepts (0, 8) and (4, 0)

For $y = -4/3x + 2$, when $x = 0$, $y = 2$, and when $y = 0$, $x = 1.5$ Intercepts (0, 2) and (1.5, 0)

Test Point (3,1)

When $y + 2x \leq 8$, $1+2.3 \leq 8$

$1 + 6 \leq 8$

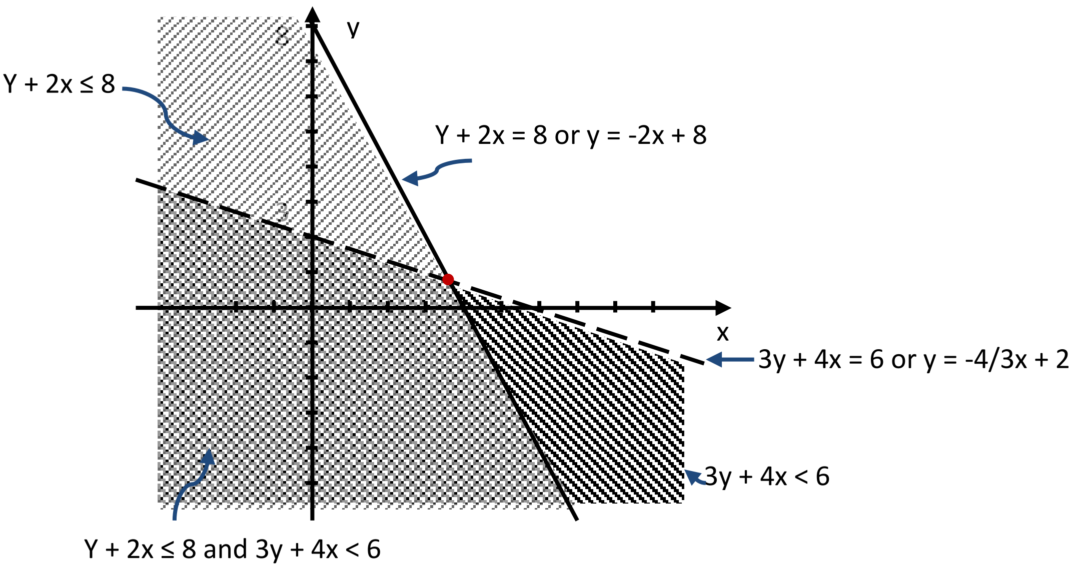
$7 \leq 8$ **True**

When $2y + x < 6$, $2.1 + 1.3 < 6$

$2 + 3 < 6$

$5 < 6$ **True**

Shade region containing the **test-point**. Shade region containing the **test-point**.



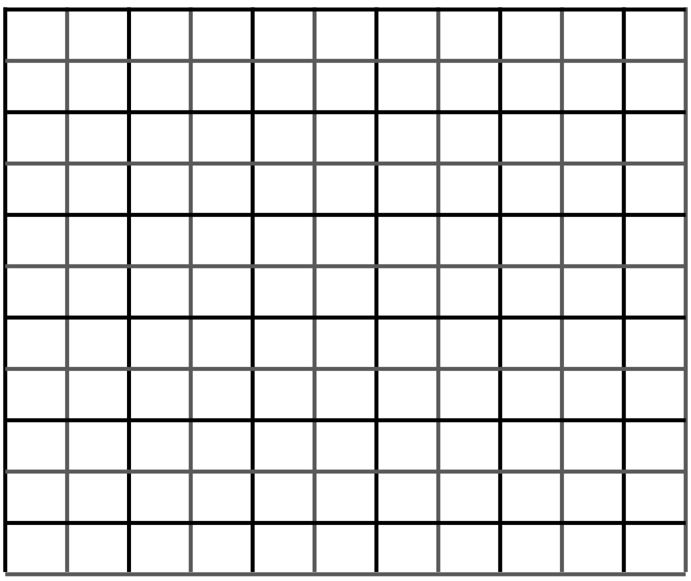
LEARNING ACTIVITY 11.2.2.4

30 minutes

1. Sketch the region $y > 2x + 3$.

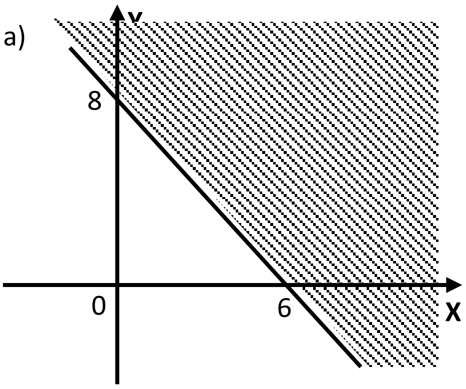
2. Show that $x \geq 3$ on a graph paper.

3. On a graph paper, rule a straight line through the points (0,4) and (4,0). Then rule another dash line (or cut-line) through (2,3) and (-1,2). Check that the straight lines have the equations $y = -x + 4$ and $3y = x + 7$. If so, then shade the region specified by $y + x \leq 4$ and $3y - x < 7$.

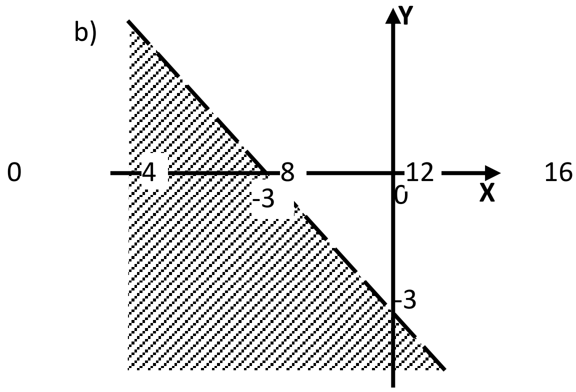


4. Derive linear equations of the sketches then, write inequalities that specify the shaded regions:

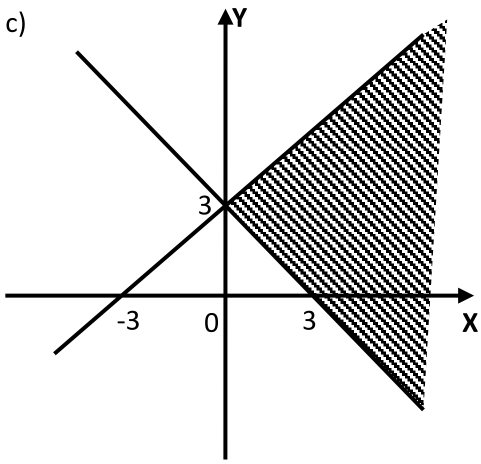
a)



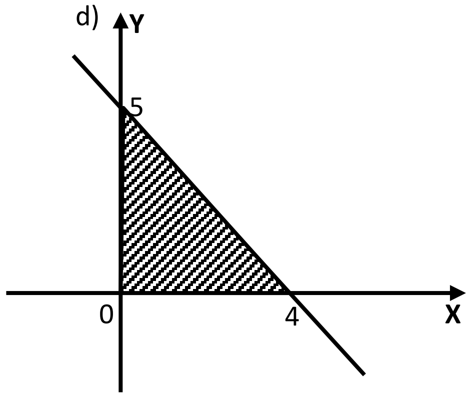
b)



c)



d)



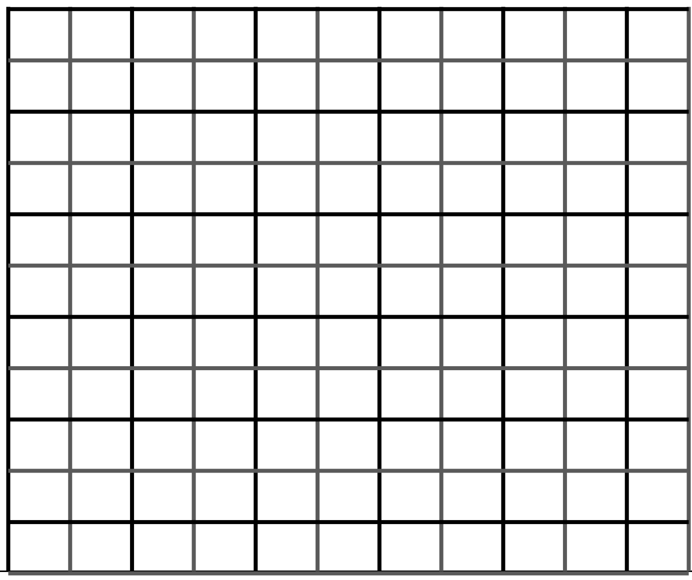
5. Show the region specified by:

$Y > 2x - 2$

$Y \leq -x + 5$

$Y > 0$

$X > 0$



SUMMATIVE TASK 11.2.2

30 minutes

1) Solve simultaneously by elimination method the equations $3y - x = 5$ and $2y + x = -15$.

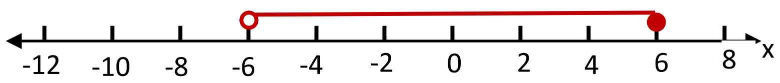
2) Solve by substitution method the equations $3y - 2x = 11$ and $y = -2x - 7$.

3) A straight line $y = -2x - 3$ is parallel to another straight-line (SL) **P** such that **P** contains the point (2, 2). Write equation of the SL **P**.

4) A straight line $y = -\frac{4}{5}x + 3$ is perpendicular to another straight-line **Q** such that **Q** contains the point (3, 5). Write equation of the SL **Q**.

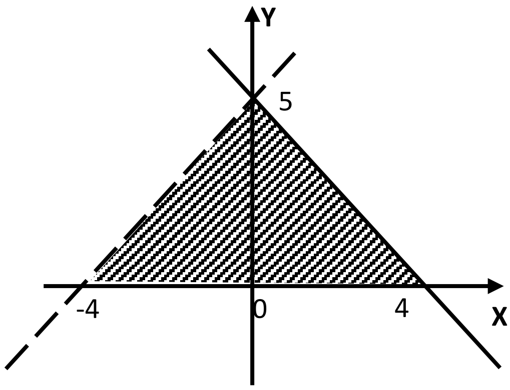
5) Graph x > 6.

6) Define the interval as displayed on the number line.



7) Graph the systems of inequalities y < - 3x + 9, x > 0 and y > 0.

8) Derive the systems of inequalities that is illustrated by the region below.



9) A SL **M** is perpendicular to SL **N** at the point (-1,1). If **M**: 2y = 2x – 5, derive equation of SL **N**.

10) Find distance between (2,7) and (-4, 15).

11.2.3 Quadratic Equations

Polynomial Equations of the form $\mathbf{ax^2 + bx + c = 0}$ are quadratic equations where $\mathbf{a \neq 0}$. When b = 0, the equation is given as $\mathbf{ax^2 + c = 0}$. when c = 0, it is expressed as $\mathbf{ax^2 + bx = 0}$.

Products of binomials or monomial and binomial such as (x + 2)(x + 5) and x(x - 6) will yield a quadratic equation or function.

11.2.3.1 Solving Quadratics Equations by Factorizing

The product of polynomials (px + r)(qx + s) is equal to $pqx^2 + psx + qrx + rs = pqx^2 + (ps + qr)x + rs$.

Suppose p = q = 1 or p and q are reciprocals of each other, then the quadratic product will be expressed as $x^2 + (r + s)x + rs$.

Suppose we equate the product to the form then we get $\mathbf{ax^2 + bx + c = x^2 + (r + s)x + rs}$. That is **a** = 1, **b** = r + s and **c** = rs. So b is the sum and c is the product of the same two numbers or integers.

So when finding factors of quadratic of the form $ax^2 + bx + c = 0$, we need to find two integers, 'r' and 's' which yield the sum 'b' and the product 'c'.

Example 1

Factorize and solve $x^2 + 9x + 18 = 0$.

Solution:

In $x^2 + 9x + 18$, r + s = 9 and rs = 18. Then one possibility is r = 3 and s = 6.

Hence $x^2 + (3 + 6)x + 3 \times 6$

$= x^2 + 3x + 6x + 18$ [group the first two terms, and group the next two terms]

$= (x^2 + 3x) + (6x + 18)$ [factor each grouping]

$= x(x + 3) + 6(x + 3)$ [regroup the common factor; take repeated factor as another factor]

$= (x + 6)(x + 3)$ Factors

When x + 6 = 0, x = -6 and when x + 3 = 0, x = - 3

The solutions are x = - 6 or -3.

Example 2

Factorize and solve $x^2 - 3x - 18 = 0$.

Solution:

In $x^2 - 3x - 18$, $r + s = -3$ and $rs = -18$. Then one possibility is $r = 3$ and $s = -6$.

Hence $x^2 + (3 + -6)x + 3 \times -6$

$= x^2 + 3x - 6x - 18$	[group the first two terms, and group the next two terms]
$= (x^2 + 3x) - (6x + 18)$	[factor each grouping]
$= x(x + 3) - 6(x + 3)$	[regroup the common factor; take repeated factor as another factor]
$= (x - 6)(x + 3)$	Factors

When $x - 6 = 0$, $x = 6$ and when $x + 3 = 0$, $x = -3$

The solutions are x = 6 or -3.

Example 3

Factorize and solve $x^2 - 9x + 14 = 0$.

Solution:

In $x^2 - 9x + 14$, $r + s = -9$ and $rs = 14$. Then one possibility is $r = -2$ and $s = -7$.

Hence $x^2 + (-2 + -7)x + -2 \times -7$

$= x^2 - 2x - 7x + 14$	[group the first two terms, and group the next two terms]
$= (x^2 - 2x) - (7x + 14)$	[factor each grouping]
$= x(x - 2) - 7(x - 2)$	[regroup the common factor; take repeated factor as another factor]
$= (x - 7)(x - 2)$	Factors

When $x - 7 = 0$, $x = 7$ and when $x - 2 = 0$, $x = 2$

The solutions are x = 2 or 7.

Example 4

Factorize and solve $x^2 + 5x - 24 = 0$.

Solution:

In $x^2 + 5x - 24$, $r + s = 5$ and $rs = -24$. Then one possibility is $r = -3$ and $s = 8$.

Hence $x^2 + (-3 + 8)x + -3 \times 8$

$= x^2 - 3x + 8x - 24$	[group the first two terms, and group the next two terms]
$= (x^2 - 3x) + (8x - 24)$	[factor each grouping]
$= x(x - 3) + 8(x - 3)$	[regroup the common factor; take repeated factor as another factor]
$= (x - 3)(x + 8)$	Factors

When $x + 8 = 0$, $x = -8$ and when $x - 3 = 0$, $x = 3$

The solutions are x = -8 or 3.

In the examples given above, **a = 1** so we focus on the sum **'b'** and the product '**c'** to identify which two integers are to be used. There is only one possible combination. Other integers may give the sum **b** but not the product **c**. Likewise, other two integers may yield the product **c** but not the sum **b**.

When a ≠1 (a > 1 or a < 1) such as $2x^2 + 7x – 4 = 0$ and $\frac{1}{4}x^2 - \frac{7}{4}x - 15 = 0$, other methods are more convenient.

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.1

30 minutes

1. Factorize the quadratic $x^2 -2x - 24 = 0$.

2. Find factors of the function $f(x) = x^2 + x - 56$.

3. Given that $x = ½$ and $x = 3$, what are the factors of the quadratic?

4. If $(2x + 5)$ is a factor of $f(x) = 2x^2 + x - 10$, what is the other factor?

5. Expand and simplify: $2x(2- 3x)$

6. Find x in $(4 - x)(2x + 5) = 0$.

7. Factorize and solve for x in $x^2 + 14x + 33 = 0$.

8. What are the roots in $y = x^2 - 4x - 21$.

9. Find the roots of the graph of $y = x^2 - 11x + 28$.

10. Find the f(x) whose roots are $x = 2$, and $x = -9$

11.2.3.2 Solving Quadratic Equations by Using Identities

Quadratics can easily be factorized if you can relate a quadratic equation to its form or type, which we call **identity**. We can use identity when the product **c** in $ax^2 + bx + c = 0$ is a square number. Quadratic identities are of the type:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

Example 1

Solve for x in $x^2 + 18x + 81 = 0$.

Solution:

The equation is of the type $(a + b)^2 = a^2 + 2ab + b^2$

Square root of 81 is 9. Thus $x^2 + 18x + 81 = (x + 9)^2$

When $(x + 9)^2 = 0$, x = - 9 or -9 [2 equal roots or solutions]

Example 2

Solve for x in $x^2 - 22x + 121 = 0$.

Solution:

The equation is of the type $(a - b)^2 = a^2 - 2ab + b^2$

Square root of 121 is 11. Thus $x^2 - 22x + 121 = (x - 11)^2$

When $(x - 11)^2 = 0$, x = 11 or 11 [2 equal roots or solutions]

Example 3

What are the x-intercepts in $x^2 - 225 = 0$.

Solution:

The equation is of the type $\mathbf{a^2 - b^2 = (a + b)(a - b)}$

Square root of 225 is 15. Thus $x^2 - 225 = (x + 15)(x - 15)$

When $(x + 15) = 0$, $x = -15$ and when $(x - 15) = 0$, $x = 15$ [2 roots are opposite integers] . The x- intercepts are $x = -15$ or 15.

Quadratic products of the form $a^2 + b^2$, such as $x^2 + 3$ and $x^2 + 4$, cannot be factorized and so has **no real roots**. The interpretation of no real roots means, the roots (solutions) are not **real numbers**.

If a number is not a member of real numbers then it is an **imaginary number**, thus the root is imaginary.

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.2

30 minutes

Use identities to factorize the following:

1. $16x^2 - 8x + 1 = 0$

2. $9x^2 - 4 = 0$

3. $2x^2 + 4x + 1 = 0$

4. $2x^2 - 12x + 9 = 0$

5. $100 - 9x^2 = 0$

6. $9x^2 + 24x + 16 = 0$

7. $4x^2 - 20x + 25 = 0$

8. $x^2 + 26x + 169 = 0$

9. $x^2 - 34x + 289 = 0$

10. $a^2 - 4a^2b + 4a^2b^2$

11.2.3.3 Solving Quadratics Equations by Square-root Method

Quadratics of the type $\mathbf{a^2 - b^2 = (a + b)(a - b)}$ can easily be solve by square root method. That is we equate a^2 to b^2 and take the square root of **both** sides, whereby the **RHS** (right-hand side) yields the two opposite integer solutions.

Example 1

What are the roots in $x^2 - 100$?

Solution:

$$x^2 - 100 = 0$$
$$x^2 = 100$$
$$\sqrt{x^2} = \sqrt{100}$$
$$X = \pm 10 \quad \text{[take both roots]}$$

Therefore the roots are -10 and 10.

Example 2

What are the roots in $x^2 - 10$?

Solution:

$$x^2 - 10 = 0$$
$$x^2 = 10$$
$$\sqrt{x^2} = \sqrt{10}$$
$$X = \pm\sqrt{10}$$ [take both roots, leave answer as surd to be **exact**]

Therefore the roots are $x = -\sqrt{10}$ and $x = \sqrt{10}$.

Example 3

Find x in $x^2 + 16$.

Solution:

When $x^2 + 16 = 0$.

$x^2 = -16$

$\sqrt{x^2} = \sqrt{-16}$ [no real number has a square of negative 16, $\sqrt{-16}$ is an imaginary number]

Therefore $x^2 + 16 = 0$, has no real roots.

Example 4

Find x in $x^2 - 4x = 45$.

Solution:

When $x^2 - 4x = 45$

$X^2 - 2x + (2)^2 = 45 + 2^2$

$(x - 2)^2 = 49$

$X - 2 = \pm 7$

$X = 2 - 7$ and $x = 2 + 7$

$X = -5$ or 9.

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.3

30 minutes

Use Square root method to solve the following quadratic equations:

1. $x^2 - 400 = 0$

2. $x^2 - 256 = 0$

3. $x^2 - 625 = 0$

4. $x^2 - 5 = 0$

5. $x^2 - 72 = 0$

6. $x^2 - 50 = 0$

7. $2x^2 - 32 = 0$

8. $3x^2 - 27 = 0$

9. $4x^2 - 9 = 0$

10. $X^2 - 6x = 0$

11.2.3.4 Discriminant

The formula $b^2 - 4ac$ is called the discriminant. It enables us to identify whether the quadratic function has **real** or **imaginary** roots.

When:

$b^2 - 4ac > 0$	**there exist two distinct real roots.**
$b^2 - 4ac = 0$	**there exist two equal real roots.**
$b^2 - 4ac < 0$	**there exist two imaginary roots [no real roots]**

Example 4

State if the quadratic function $f(x) = x^2 - 3x + 5$ has real roots.

Solution:

$f(x) = x^2 - 3x + 5$ where $a = 1$, $b = -3$ and $c = 5$.

Now $b^2 - 4ac = (-3)^2 - 4 \times 1 \times 5$

$= 9 - 20$

$= - 11$

< 0 $[- 11 < 0]$

Therefore $f(x) = x^2 - 3x + 5$ **has no real roots (solutions).**

Example 5

State if the quadratic equation $-3x^2 + 7x + 6 = 0$ has real roots.

Solution:

$-3x^2 + 7x + 6 = 0$ where $a = -3$, $b = 7$ and $c = 6$.

Now $b^2 - 4ac = (7)^2 - 4 \times -3 \times 6$

$= 49 + 72$

$= 121$

> 0 $[121 > 0]$

Therefore $-3x^2 + 7x + 6 = 0$ **has two distinct real roots (solutions).**

Example 6

Use discriminant to identify, then describe the type of roots in the graph of

$y = - 4x^2 + 20x - 25$.

Solution:

In $- 4x^2 + 20x - 25 = 0$, $a = - 4$, $b = 20$ and $c = -25$.

Now $b^2 – 4ac = (20)^2 – 4 \times -4 \times -25$

$= 400 - 400$

$= 0$ [0 = 0]

Therefore $y = - 4x^2 + 20x – 25$ **has two equal real roots (solutions). The curve goes up, touches x-axis and concaves downward.**

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.4 30 minutes

Use discriminant $\Delta = \mathbf{b^2 – 4ac}$ to identify if the quadratic has real roots:

1. $x^2 + 2x – 63$

2. $x^2 + 2x – 120$

3. $2x^2 - 7x – 9$

4. $9x^2 + 12x + 4$

5. $x^2 - 3x + 5$

6. $2x^2 + 6x - 8$

7. $6x^2 + 13x + 6$

8. $16x^2 + 24x + 9$

9. $5x^2 + 5x - 10$

10. $8x^2 - 29x - 12$

11.2.3.5 Solving Quadratics Equations by Completing the Square

Let us complete the square on the general form $\mathbf{ax^2 + bx + c = 0}$ to observe and identify changes that occur in the quantities **a**, **b** and **c** and at the same time transpose **x**. In doing so, we will derive a formula.

Now $ax^2 + bx + c = 0$ **s**ubtract **c** from both sides, yields

$ax^2 + bx = -c$ divide each term by **a** yields

$x^2 + \frac{b}{a}x = \frac{-c}{a}$ multiply $\frac{b}{a}$ by $\frac{1}{2}$ and add ITS square.

$x^2 + \frac{b}{a}.\frac{1}{2}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$ LHS is now a square of a binomial

$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$ Factorize LHS, simplify RHS

$= \frac{-4ac + b^2}{4a^2}$ Rearrange RHS

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$ take square roots of both sides

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ transpose x

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ We now have a **Quadratic Formula.**

When a quadratic function has a being greater or less than 1 (a ≠ 1), it is better to apply complete the square method to solve for x. When a = 1, use the methods given in examples above.

If we are given $2x^2 - 8x - 10$, we can take **2** as a **highest common factor** and so we get

$2(x^2 - 4x - 5)$ and then apply identity or sum and difference method to factor inside the parenthesis, then solve for **x**. There are others where we cannot simplify, that is when completing the square is more appropriate.

Example 1

Complete the square to solve for x in $2x^2 + 11x - 21 = 0$.

Solution:

$2x^2 + 11x - 21 = 0$

$2x^2 + 11x = 21$ [subtract **c**]

$x^2 + \frac{11}{2}x = \frac{21}{2}$ [divide by **a**]

$x^2 + \frac{11}{2}.\frac{1}{2}x + \left(\frac{11}{4}\right)^2 = \frac{21}{2} + \left(\frac{11}{4}\right)^2$ [multiply $\frac{b}{a}$ by $\frac{1}{2}$, add its square]

$\left(x + \frac{11}{4}\right)^2 = \frac{21}{2} + \frac{121}{16}$ [simplify RHS]

$= \frac{168 + 121}{16}$

$= \frac{289}{16}$

$x + \frac{11}{4} = \sqrt{\frac{289}{16}}$ [take square root of both sides]

$x + \frac{11}{4} = \pm\frac{17}{4}$ [transpose **x**]

$x = -\frac{11}{4} \pm \frac{17}{4}$ [simplify RHS to solve **x**]

$$x = -\frac{11}{4} \pm \frac{17}{4}$$

$$x = -\frac{11}{4} - \frac{17}{4} \text{ or } -\frac{11}{4} + \frac{17}{4}$$

$$x = -\frac{28}{4} \text{ or } \frac{6}{4}$$

$$x = -7 \text{ or } \frac{3}{2}$$ [roots]

Example 2

Complete the square to solve for x in $3x^2 - 10x - 8 = 0$.

Solution:

$3x^2 - 10x - 8 = 0$

$3x^2 - 10x = 8$

$$x^2 - \frac{10}{3}x = \frac{8}{3}$$ [divide by c]

$$x^2 - \frac{10}{3}.\frac{1}{2}x + \left(\frac{10}{6}\right)^2 = \frac{8}{3} + \frac{100}{36}$$ [$\left(\frac{b}{a}.\frac{1}{2}\right)$, add its square to complete the square]

$$\left(x - \frac{10}{6}\right)^2 = \frac{96 + 100}{36}$$ [factorize LHS. simplify RHS]

$$\left(x - \frac{10}{6}\right)^2 = \frac{196}{36}$$

$$\left(x - \frac{10}{6}\right) = \sqrt{\frac{196}{36}}$$ [take the square roots of both sides]

$\left(x - \frac{10}{6}\right) = \pm\frac{14}{6}$ [transpose x]

$x = \frac{10}{6} \pm \frac{14}{6}$ [simplify RHS, evaluating x values]

$\text{x} = \frac{10 + 14}{6} \text{ or } \frac{10 - 14}{6}$

$\text{x} = \frac{24}{6} \text{ or } \frac{-4}{6}$

$\text{x} = 8 \text{ or } -\frac{2}{3}$

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.5

30 minutes

For the quadratic equations below, complete the square to solve x:

1. Complete the square on $2x^2 + 9x – 19$ and solve for x.

2. Complete the square on $5x^2 + 7x – 6$ and solve for x.

3. Complete the square to find factors of $3x^2 -8x – 16 = 0$.

4. Complete the square on $f(x) = 2x^2 – 14x + 20$ and determine the roots.

5. Express $f(x) = 3 – 5x – 2x^2$ in the form $a(x - h)^2 + k$

11.2.3.6 Solving Quadratics Equations by Quadratic Formula

The quadratic formula $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to solve x.

The formula is derived by completing the square on the general quadratic for $ax^2 + bx + c = 0$. Like 'completing the square method', the formula is useful when **factor method** and using **identities** deem difficult to employ to solve **x**.

Example 1

Solve for x in $4x^2 + 13x - 12 = 0$ using the quadratic formula.

Solution:

In $4x^2 + 13x - 12 = 0$, a = 4, b = 13 and c = -12

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4.4.-12}}{2.4}$$

$$x = \frac{-13 \pm \sqrt{169 + 192}}{8}$$

$$x = \frac{-13 \pm \sqrt{361}}{8}$$

$$x = \frac{-13 \pm 19}{8}$$

$$x = \frac{-13 - 19}{8} \text{ or } \frac{-13 + 19}{8}$$

$$x = \frac{-32}{8} \text{ or } \frac{6}{8}$$

$$x = -4 \text{ or } \frac{3}{4}$$

Example 2

Show that the quadratic has real roots, then solve for x in $2x^2$ - 3x – 14 = 0 using the quadratic formula.

Solution:

In $2x^2$ - 7x – 14 = 0, a = 2, b = - 3 and c = -14

b^2 – 4ac = (-3)2 – 4.2.-14

= 9 + 112

= 121

> 0, therefore has 2 distinct real roots.

Now $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$x = -\frac{-3}{2.2} \pm \frac{\sqrt{(-3)^2 - 4.2.-14}}{2.2}$$

$$x = -\frac{-3}{4} \pm \frac{\sqrt{9+112}}{4}$$

$$x = -\frac{-3}{4} \pm \frac{\sqrt{121}}{4}$$

$$x = -\frac{-3}{4} \pm \frac{11}{4}$$

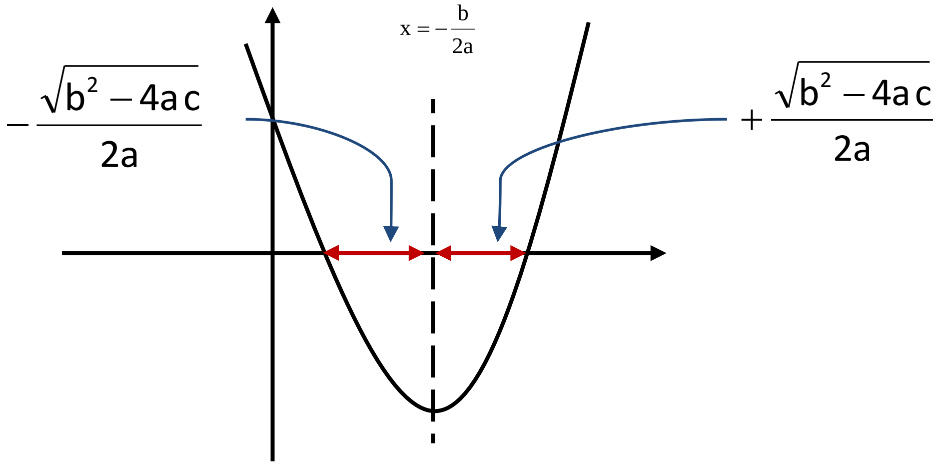
$$x = -\frac{-3-11}{4} \text{ or } \frac{-3+11}{4}$$

$$x = -\frac{-14}{4} \text{ or } \frac{8}{4}$$

$$x = -\frac{-7}{2} \text{ or } 2$$

Using this form of the formula is helpful, especially when you are sketching the graph of the quadratic function. In this formula, the part $x = -\frac{b}{2a}$ defines the axis of symmetry.

The part $\pm\frac{\sqrt{b^2 - 4ac}}{2a}$ is the uniform distance to the left and right from the axis of symmetry. Sum and difference from the axis of symmetry gives the point the quadratic curve intersects x-axis. These two points are the solution of x or the roots of the quadratic equation.



So if axis of symmetry is x = 3 and the roots are ±2 distance to the left and right of the axis of symmetry, then the roots are x =1 and x = 5.

But if the roots are known, the axis of symmetry is the average of the two roots. Say if the roots are x = -2 and x = 4. The axis of symmetry is:

$$X = \tfrac{1}{2}(x_1 + x_2) = \tfrac{1}{2}(-2 + 4)$$
$$= \tfrac{1}{2}(2)$$
$$= 1$$

For a quadratic function, the **domain** is all of x, and the **range** ends at the vertex. Say if the equation has a > 0 (positive), the set of range yields the minimum value. When a < 0 (negative), the set of range yields a maximum value.

For example, Range: $y \geq 4$ or $\{y: 4 \leq y \leq +\infty\}$.

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.6

30 minutes

Use the quadratic formula to solve for x in the following quadratic equations.

1. $x^2 + 13x + 30 = 0$

2. $x^2 + 2x - 48 = 0$

3. $x^2 - 16x + 55 = 0$

4. $0 = 15 + 2x - x^2$

5. $4x^2 + 12x + 9 = 0$

6. $4x^2 - 20x + 25 = 0$

7. $4x^2 - 5x - 6 = 0$

8. $4x^2 - 12x - 7 = 0$

9. $-5x^2 + 14x - 8 = 0$

10. $4x^2 + 15x - 4 = 0$

11.2.3.7 Graphs and Sketches of Quadratic Equations

Graphs of quadratic functions are of the form $f(x) = ax^2 + bx + c$ or quadratic graphs are of the form $y = ax^2 + bx + c$. The constants **a**, **b**, and **c** are real numbers and are parameters of the equation. They determine the shape of the parabola.

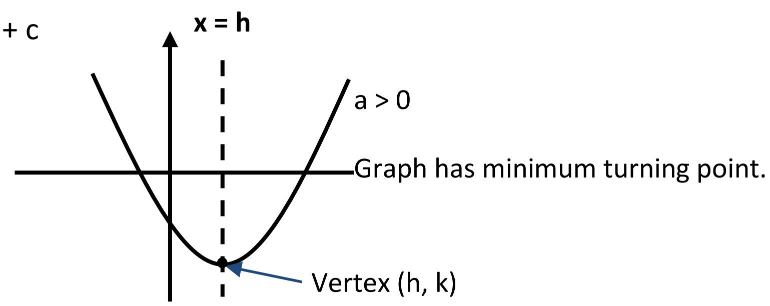
They can be related to path of a foot-ball kicked upward and comes down, transmission of mobile phone signals

Often when plotting graphs, the domain and the range are defined. For quadratic graphs, the domain is often $[-\infty, +\infty]$. The range can be either $[y \leq k]$ or $[y \geq k]$ depending on whether the parabola has a maximum or minimum turning point respectively.

When the equation is transformed to $\mathbf{y = a(x - h)^2 + k}$ form, the range can easily be identified; where **(h, k)** is the vertex and **k** is either maximum (if $a < 0$) or minimum ($a > 0$) point.

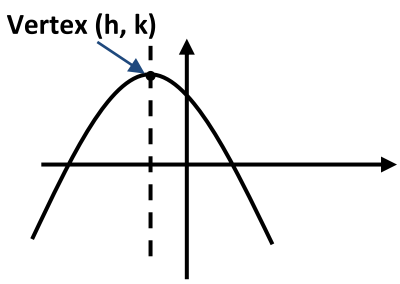
General Features of Quadratic Graphs

Graphs of the form $y = ax^2 + bx + c$



Example

Graph of $y = 2x^2 + x - 15$



a < 0

Graph has maximum turning point.

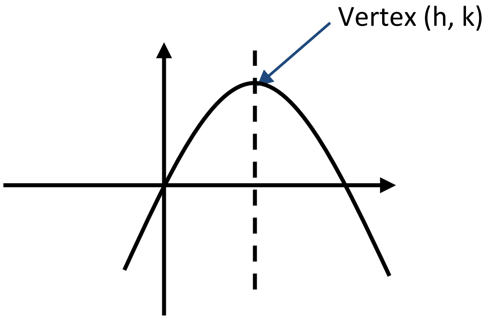
Example

Graph of $y = -2x^2 - x + 15$

Graphs of the form $y = ax^2 + bx$

where $a < 0$, has MAX

runs through ORIGIN.

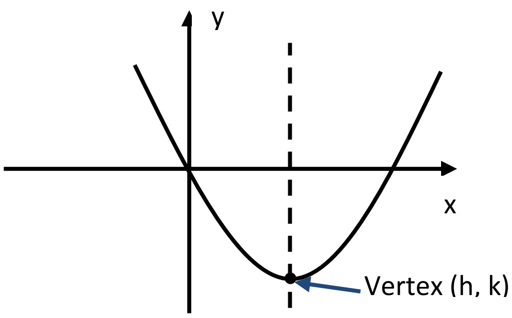


Example

Graph of $y = -x^2 + 4x$

where $a > 0$, has MIN

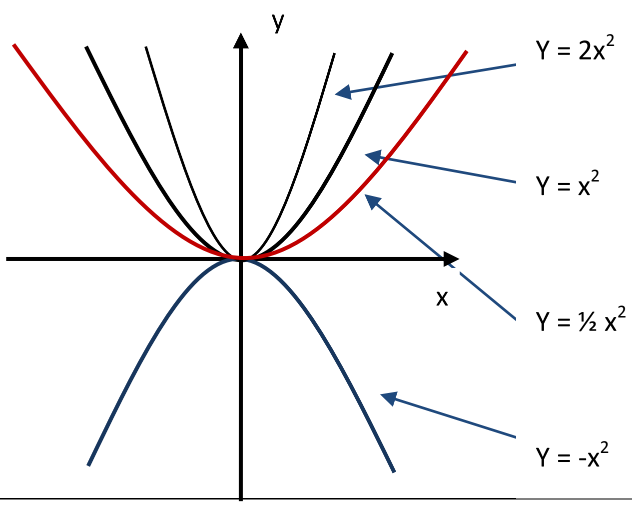
runs through ORIGIN



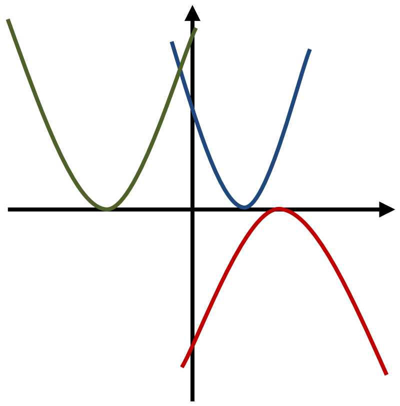
Example

Graph of $y = x^2 - 4x$

Graphs of the form $y = ax^2$ always runs through the origin. When $a > 1$, the parabola becomes narrower. When $a < 1$, the parabola becomes wider.



Graphs of the form y = $(x + h)^2$ have two equal roots, hence touch the x-axis at the root.



Example

Graphs of y = x + 6x + 9, y = x2 – 4x + 4 and y = $-x^2 + 8x - 16$.

Your understanding of these key features will enable you to plot parabolic graphs with confidence.

Steps in Plotting Quadratic Graphs

1. Complete the table of values based on given domain.
2. Rule the axes and label the axes on a graph paper.
3. Mark off scales based on given domain and range.
4. Plot the points.
5. Join the points with a smooth curve.

Example

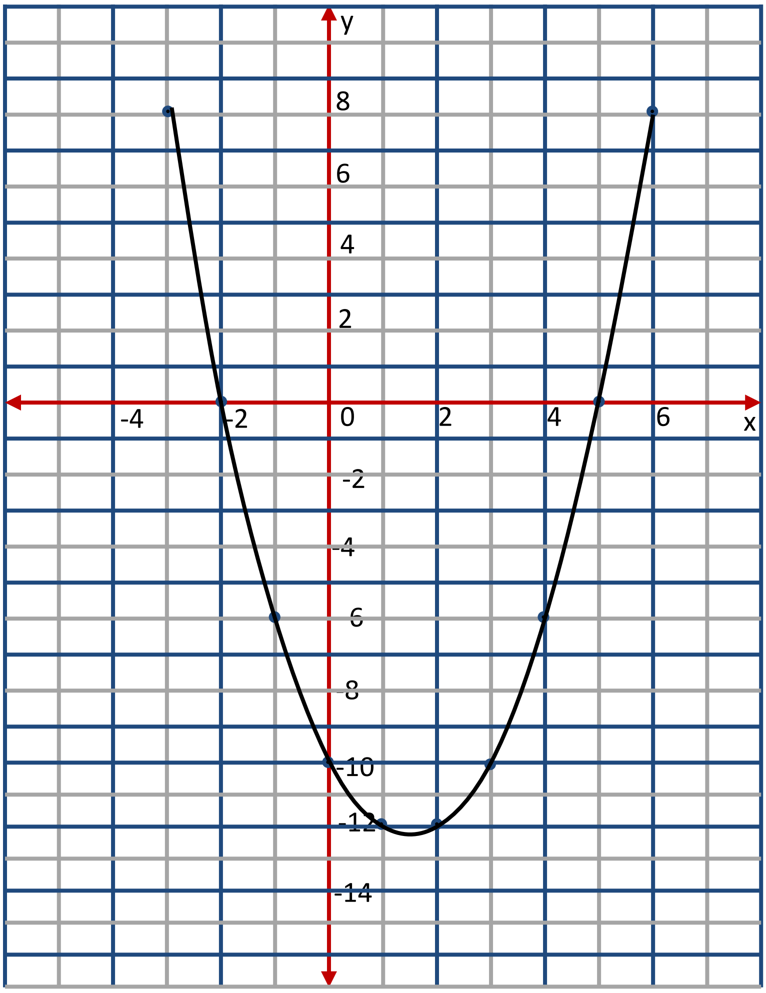
Plot the graph of $y = x^2 - 3x - 10$ for {x: - 3 ≤ x ≤ 6}.

Solution:

1. Table of Values: $y = x^2 - 3x - 10$

X	-3	-2	-1	0	1	2	3	4	5	6
y	8	0	-6	-10	-12	-12	-10	-6	0	8

2. Axes, Scale and graph.



The domain is [-∞,+∞] and the Range [y≥ -12.25]

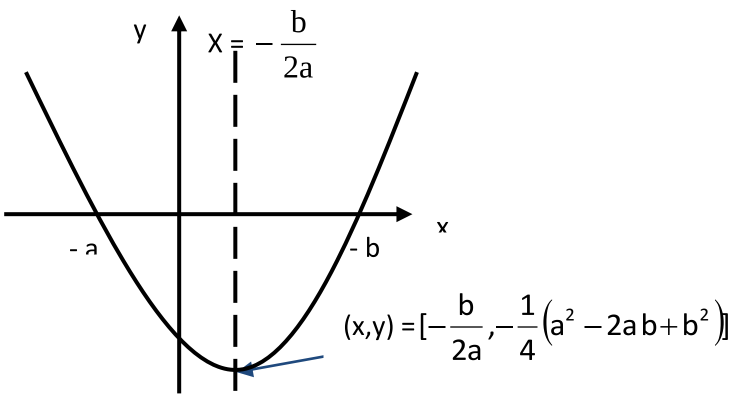
Sketch of Quadratic Graphs

When sketching a quadratic graph or a parabola, we show:

1. Where the graph crosses the y-axis
2. Where the graph crosses the x-axis
3. The axis of symmetry of the parabola
4. The vertex or turning point (TP) of the parabola.

If we sketch parabola of the form **y = (x + a)(x + b)** then:

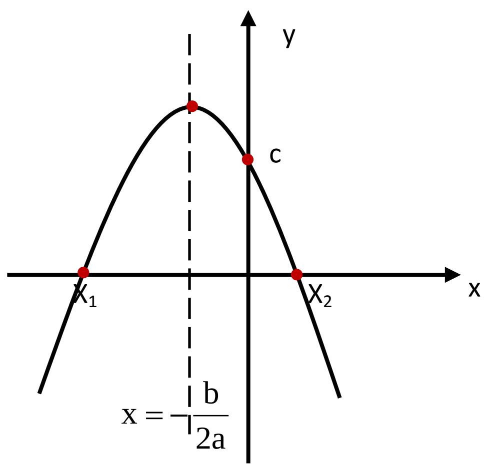
- **- a** and -**b** are roots of the parabola [(x + a)(x + b) = 0]
- **ab** is the y – intercept [$y = x^2 + (a + b)x + ab$]
- axis of symmetry x = ½ (-a + - b) or x = - ½ (a + b)
- the y-coordinate of the vertex is $y = -\frac{1}{4}(a^2 - 2ab + b^2)$



Computing for the y coordinate of the vertex seems long and challenging. But when you substitute for x with the x value of the axis of symmetry and evaluate, you will obtain the corresponding y coordinate.

If we sketch parabola of the form $y = ax^2 + bx + c$, then:

- c is the y – intercept
- x_1 and x_2 are the roots of the parabola.
- $X = -\frac{b}{2a}$ is the axis of symmetry
- Y coordinate is $y = c - \frac{b^2}{4a}$



Example 1

Sketch the parabola y = $-x^2 + 8x - 15$.

Solution:

y-intercept: c = - 15

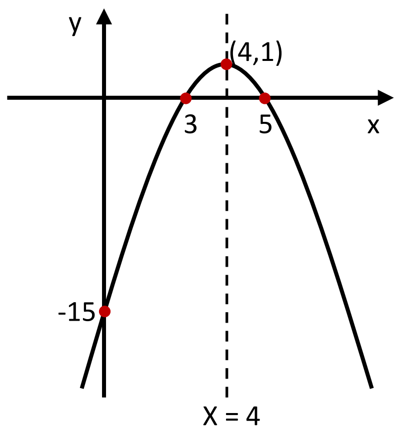
Roots: $-x^2 + 8x - 15 = 0$, $-(x^2 - 8x + 15) = 0$, $-(x-3)(x-5) = 0$

When x-3 = 0, **x = 3** and when x – 5 = 0, **x = 5**

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2.-1} = \frac{-8}{-2} = 4$

Vertex: Turning Point at **x = 4**, y = $-(4)^2 + 8(4) - 15 = -16 + 32 - 15 = 1$ TP = **(4, 1)**

Sketch:



Example 2

Sketch the graph of y = $2x^2 + x - 15$.

Solution:

y-intercept: c = - 15

Roots: $2x^2 + x - 15 = 0$, $(2x^2 + 6x - 5x + 15) = 0$, $2x(x+3) - 5x(x+3) = 0$, $(2x - 5)(x + 3) = 0$

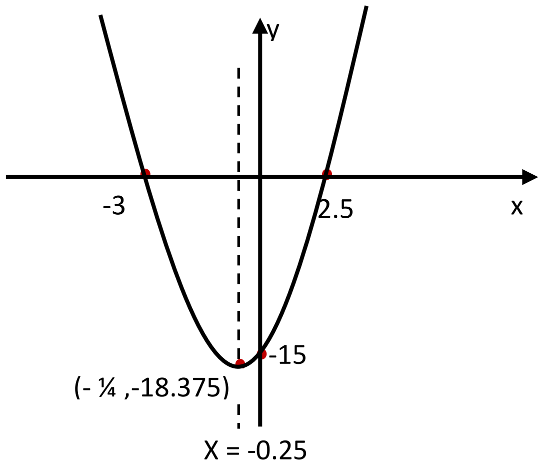
When 2x- 5 = 0, **x = 2.5** and when x + 3 = 0, **x = - 3**

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{1}{2.2} = \frac{-1}{4} = -\frac{1}{4} \text{ or } -0.25$

Vertex: Turning Point at **x = 0.25**, $y = 2(-0.25)^2 + (-0.25) - 15 = 2(0.0625) - 3.5 - 15$

$= 0.125 - 18.5 = -18.375$ TP = **(¼ , - 13 $^3/_{16}$)**

Sketch:



The quadratic sketch above shows:

1. Two roots (x- intercepts)
2. Y- intercept
3. Axis of symmetry
4. Vertex or Turning Point

Now do the learning activity.

LEARNING ACTIVITY 11.2.3.7

30 minutes

Below are questions relating to graphs of quadratic function.

1) Plot the graph of the function whose table of values are as given below.

X	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6

2) Complete table of values for $f(x) = x^2 + 5x + 4$. Then plot the points and join the points with a smooth curve to form the graph of the function.

X	-6	-5	-4	-3	-2	-1	0	1
y								

3) Express $y = 2x^2 + 5x - 12$ in the form $a(x - h)^2 + k$ and state the vertex of the graph.

4) Complete the square on $-2x^2 + 3x - 9$ to be able to find and sketch:

(a) The roots

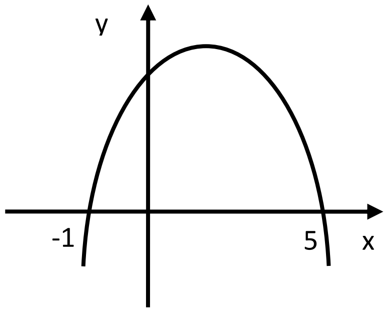
(b) The axis of symmetry

(c) The vertex

(d) Sketch.

5) Calculate all the necessary points and sketch the graph of $f(x) = x^2 + 4x - 24$.

6) Derive equation of the sketch below.

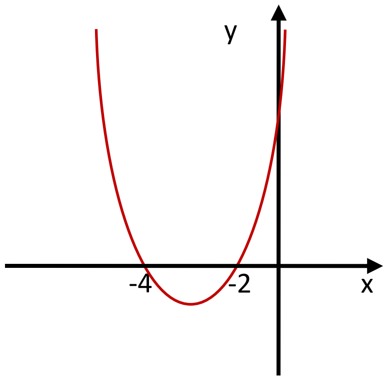


7) From the sketch find:

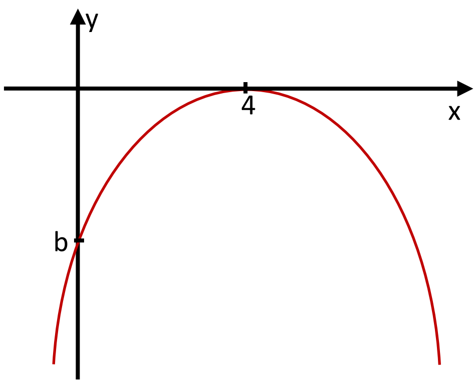
(a) Y – intercept

(b) Axis of symmetry

(c) Vertex

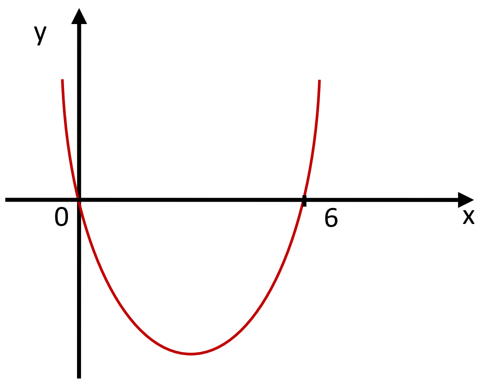


8) Derive equation of the graph , then find value of b.



9) Sketch $y = 4x^2 + 12 + 9$

10) Study the sketch and answer questions that follow:



(a) Derive equation of the graph.

(b) Show axis of symmetry on the sketch and state the axis.

(c) Mark off an state the vertex.

(d) What is the domain?

(e) What is the range?

SUMMATIVE TASK 11.2.3

30 minutes

Solve the following questions. For questions 1 to 17, apply the method specified.

1. Factorize and solve for x in x^2 -7x -18 = 0.

2. Solve by first finding factors of $x^2 – 8x + 16$.

3. What are the roots of $x^2 + 6x = 0$?

4. Find factors of $4x^2 – x – 6$ and then find values of x.

5. Use identity to factorize and then solve the quadratic $x^2 + 24x + 144$.

6. Match the quadratic equation with a quadratic identity in-order to factorize and solve $4x^2 – 12x + 9$.

7. Use quadratic identity to enable you for factorize and to solve $1- 4x + 4x^2$.

8. Use identity to factorize and solve for x in $16 – 8x + x^2$.

9. Use square-root method to solve the quadratic equation $2x^2 – 18 = 0$

10. Find square-root of x in $3x^2 – 48$.

11. Solve by taking square-root of x in $x^2 - 9x = 0$.

12. Complete the square on $3x^2 - 13x + 4 = 0$ and solve for x.

13. Solve for x in $2x^2 + 11x + 15$ by first completing the square.

14. Use completing the square method to solve the quadratic equation $4x^2 - 5x + 1 = 0$.

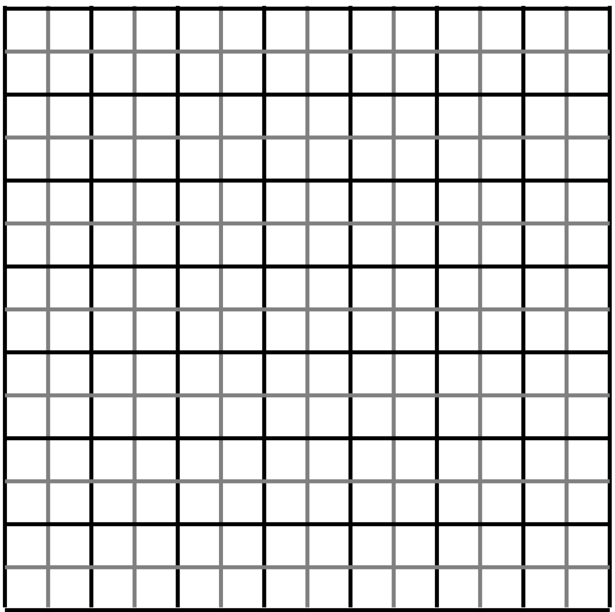
15. Use the quadratic formula to solve: $x^2 + 2x - 48 = 0$

16. Find the discriminant of $2x^2 + x - 20 = 0$, then solve by using the formula if the solution is real.

17. Find the discriminant of $4x^2 - 5x + 1 = 0$, then solve by using the formula if the solution is real.

18. Plot the graph of $y = x^2 - 8x + 12$ by first completing the table of values.

X	-1	0	1	2	3	4	5	6	7	8	9
y											



19. Plot the graph of $f(x) = x^2 – 5x$ by first completing the table of values.

X	-2	-1	0	1	2	3	4	5	6	7
y										

20. Sketch the graph of $f(x) = x^2 – 6x – 16$.

11.2.4 Graphs of Functions

In the previous topics, we discussed about linear and quadratic equations with their graphs. In this topic we will discuss other functions such as the polynomial functions, rational, exponential, logarithmic, cubic and absolute value functions.

11.2.4.1 Polynomial Functions

A **polynomial function** of degree **n** is a function of the form

$$P(x)= a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$$ where n is a nonnegative integer and $a_n \neq 0$.

The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the **coefficients** of the polynomial.

The number a_0 is the **constant coefficient** or **constant** term.

The number an, the coefficient of the highest power, is the leading coefficient, and the term a_nx^n is the **leading term**.

EXAMPLES:

P(x) = 3,

Q(x) = 4x − 7,

R(x) = x^2 + x,

S(x) = $2x^3 - 6x - 10$

Example 1

Which of the following are polynomial functions?

a) f(x) = $-x^3 + 2x + 4$

b) f(x) = (x − 2)(x − 1)(x + 4)

c) f(x) = $\dfrac{x^2+2}{x^2-2}$

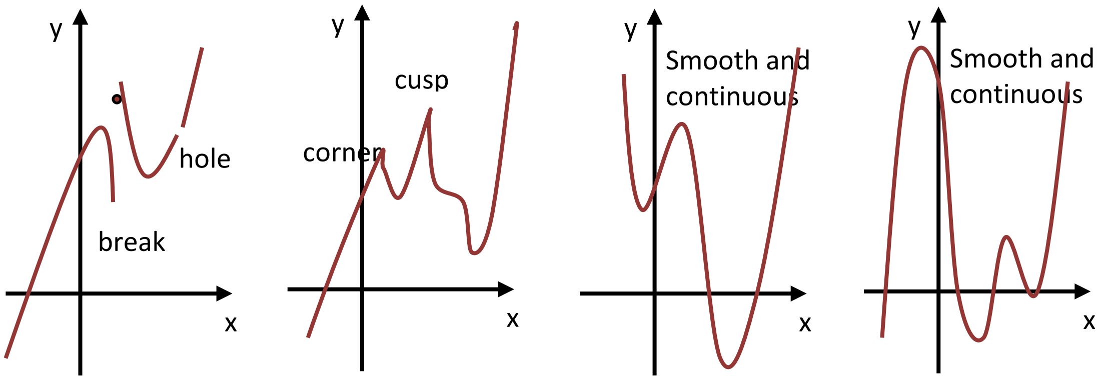
Solution:

Only (a) and (b) are polynomial functions. Option (c) is not because, it has a denominator which may be equal to zero.

Graphs of Polynomials

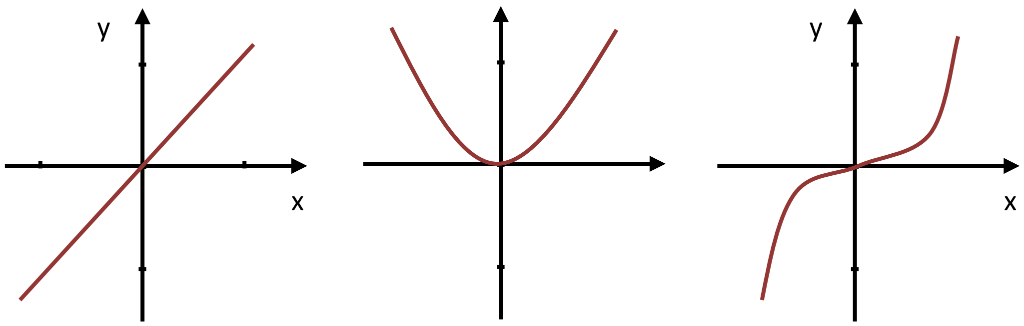
The graph of a polynomial function is always a smooth curve; that is, it has no breaks or corners and cusps.

Observe the following types of graphs:

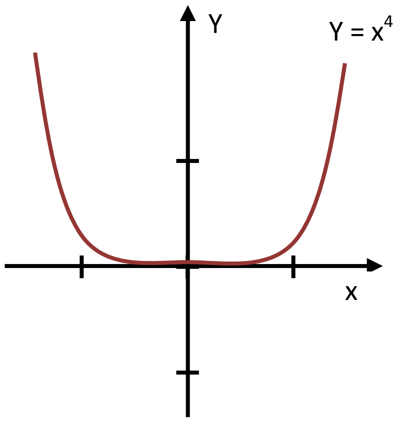


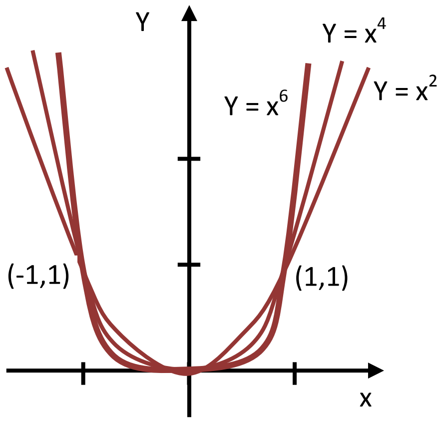
Not a graph of a polynomial function | Not a graph of a polynomial function | Graph of a polynomial function | Graph of a polynomial function

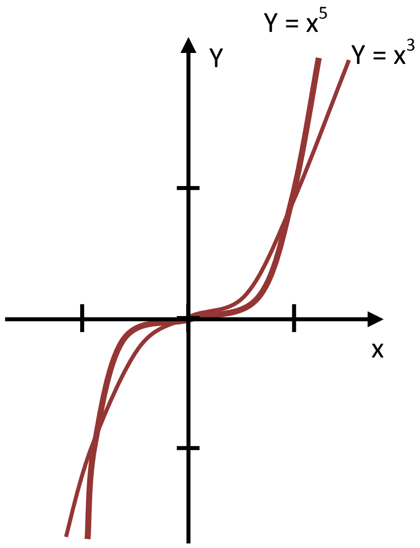
The simplest polynomial functions are the monomials $P(x) = x^n$, whose graphs are shown in the Figures below.



$y = x$ $\quad$ $y = x^2$ $\quad$ $y = x^3$







Did you notice the trend in the graphs?

Example 2

Sketch the graphs of the following functions.

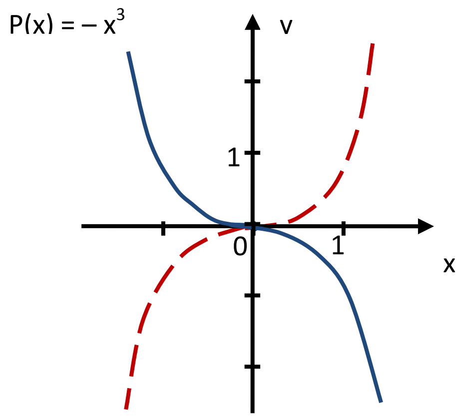
(a) $P(x) = -x^3$

(b) $Q(x) = (x - 2)^4$

(c) $R(x) = -2x^5 + 4$

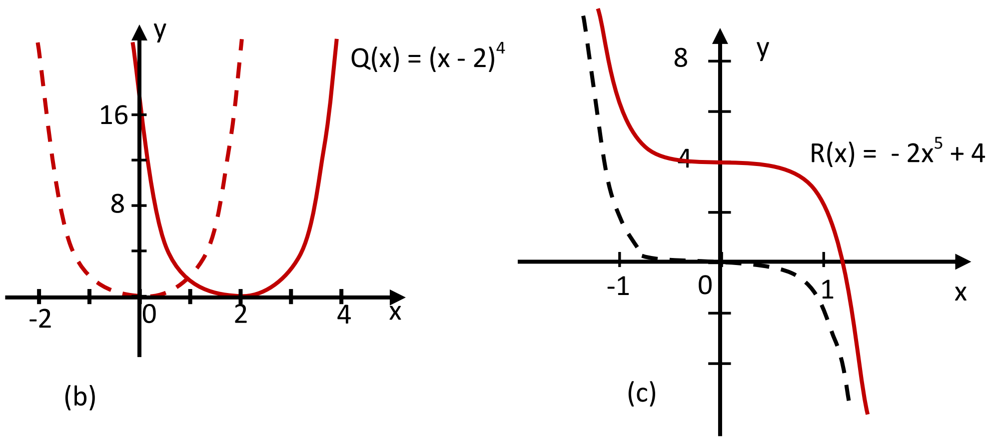
Solution:

(a) The graph of $P(x) = -x^3$ is the reflection of the graph of $y = x^3$ in the x-axis.



(b) The graph of $Q(x) = (x - 2)^4$ is the graph of y = x4 shifted to the right 2 units.

(c) We begin with the graph of $y = x^5$. The graph of $y = -2x^5$ is obtained by stretching the graph vertically and reflecting it in the x-axis. Finally, the graph of $R(x) = -2x^5 + 4$ is obtained by shifting upward 4 units.



Example 3

Sketch the graphs of the following functions:

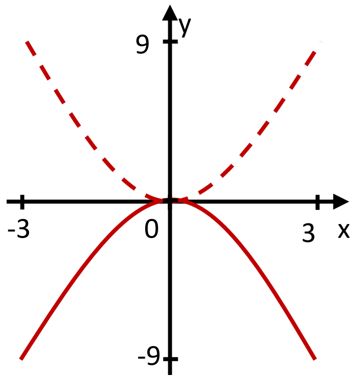
(a) $P(x) = -x^2$

(b) $Q(x) = (x + 1)^5$

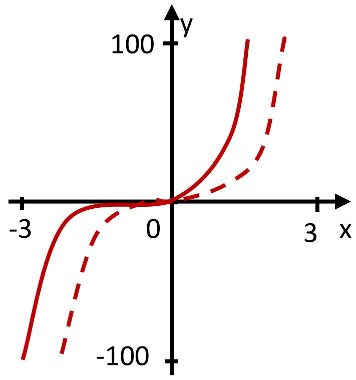
(c) (c) $R(x) = -3x^2 + 3$

Solution:

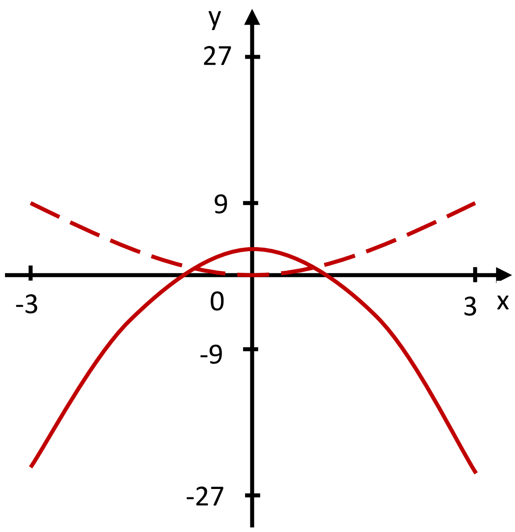
(a) The graph of $P(x) = -x^2$ is the reflection of the graph of $y = x^2$ in the x-axis.

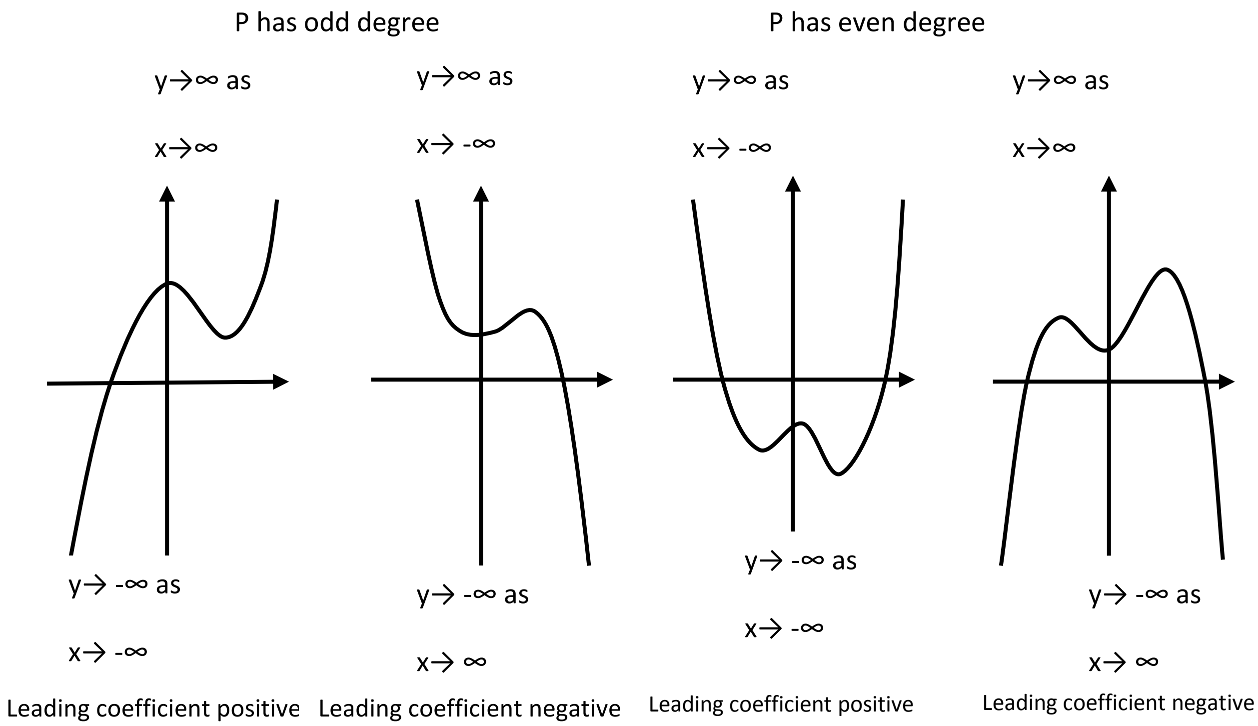


(b) The graph of Q(x) = (x + 1)5 is the graph of y = x5 shifted to the left 1 unit.



(c) We begin with the graph of $y = x^2$. The graph of $y = -3x^2$ is obtained by stretching the graph vertically and reflecting it in the x-axis. Finally, the graph of $R(x) = -3x^2 + 3$ is obtained by shifting upward 3 units.





Using Zeros to Graph Polynomials

If P is a polynomial function, then c is called a zero of P if P(c) = 0. In other words, the zeros of P are the solutions of the polynomial equation P(x) = 0. Note that if P(c) = 0, then the graph of P has an x-intercept at x = c, so the x-intercepts of the graph are the zeros of the function.

Example 4

Let $P(x) = x^3 - 2x^2 - 3x$.

(a) Find the zeros of P.

(b) Sketch the graph of P.

Solution:

(a) To find the zeros, we factor completely:

$P(x) = x^3 - 2x^2 - 3x$

$= x(x^2 - 2x - 3)$

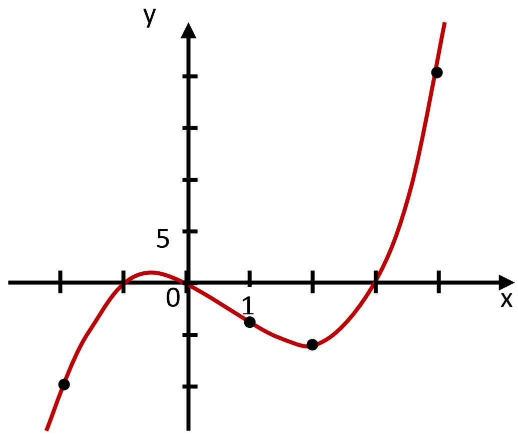
$= x(x - 3)(x + 1)$

Thus, the zeros are x = 0, x = 3, and x = −1.

(d) The x-intercepts are x = 0, x = 3, and x = −1. The y-intercept is P(0) = 0. We make a table o f values of P(x), making sure we choose test points between (and to the right and left of) successive zeros. The polynomial P has odd degree and positive leading coefficient.

We plot the points in the table and connect them by a smooth curve to complete the graph.

x	P(x)
-2	-10
-1	0
-1/2	7/8
0	0
1	-4
2	-6
3	0
4	20



Now do the learning activity.

LEARNING ACTIVITY 11.2.4.1

15 minutes

1. Let $P(x) = x^3 - 9x^2 + 20x$.

(a) Find the zeros of P.

(b) Sketch the graph of P.

2. Find zeros of $P(x) = 4x^3 – 4x^2 – 5x + 3$.

3. Is $(x - 2)$ a factor of $x^3 – 3x^2 – 4x + 12 = 0$?

4. If $P(x) = Q(x).R(x)$ and $P(x) = x^4 – x^3 – 11x^2 – 9x + 18$, find $R(x)$ when $Q(x) = x – 3$

5. Given that $P(x) = x^3 + 4x^2 + x – 6$. If $P(-3) = 0$, find the other two zeros of $P(x)$.

6. Is -2 a zero of $P(x) = x^4 – 5x^2 + 4$?

7. $P(x) = x^3 – 2x^2 – 36x + 72$. Given that $R(x) = P(x)/Q(x)$, and $Q(x) = x – 6$, find $R(x)$.

8. Expand and simplify: $(x + 2)(x^2 – 2x + 4)$.

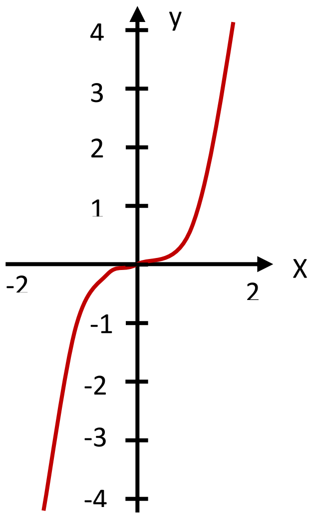
9. Factorize $x^3 + 64 = 0$.

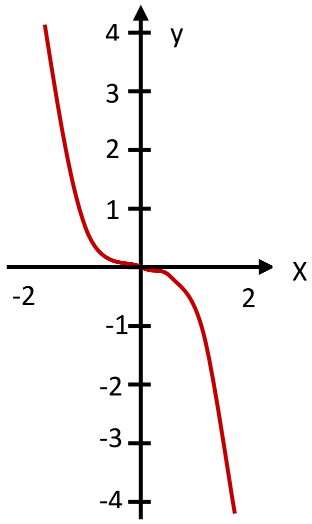
10. Find the other factors of $P(x) = x^3 + 3x^2 – 9x – 27$.

11.2.4.2 Cubic Functions

Cubic functions are functions of the degree 3.

The following are the basic cubic functions. Observe the graphs of the monomials x^3, $-x^3$.





The domain is all of x {- ∞ ≤ x ≤ +∞}, and the range is all of y{- ∞ ≤ y ≤ +∞}.

Graphs of cubic functions would either run from top left to bottom right if a < 0 or negative, or from bottom left to top right when a > 0 or positive. Examples are as provided by two graps above.

Generally, graphs of cubic functions intersects the x- axis three times if it is of the form $ax^3 + bx^2 + cx + d = 0$.

However if it is of the form $ax^3 + cx + d = 0$ or $ax^3 + bx^2 + d = 0$, it is likely to intersect twice or intersects once and touches the x-axis on the second approach. This occurs when the cubic function has a repeated root or contains a square of a binomial or sum and difference of two squares as a factor of the cubic function.

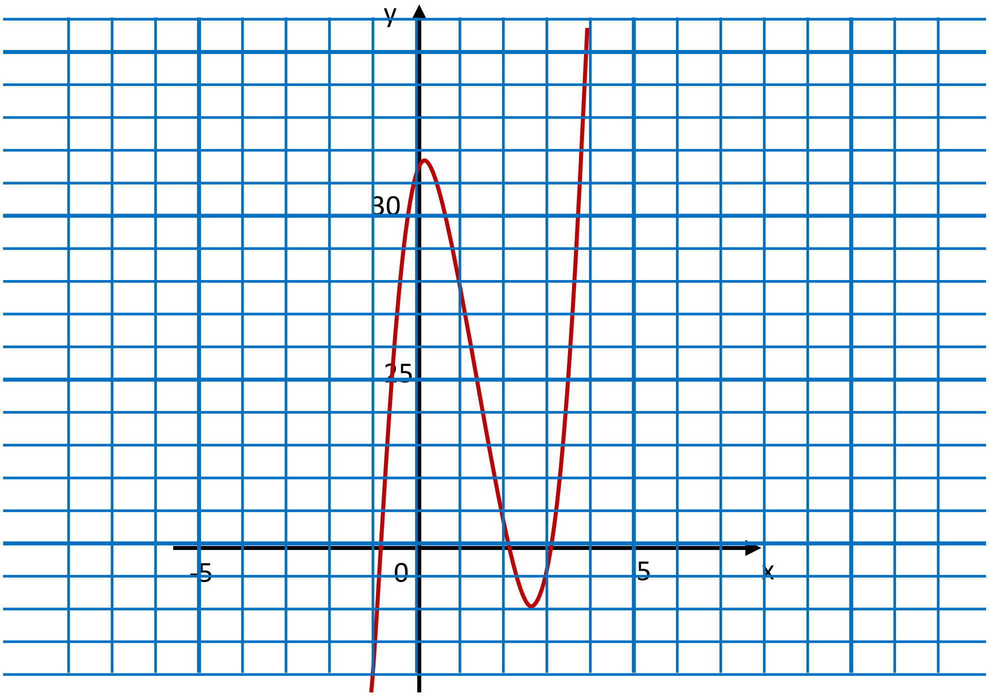
Factor Theorem or zeros of the polynomial can be applied to derive factors of the cubic function, and then solve to determine the roots or x- intercepts by solving for x.

Example 1

Graph the function $x^3 - 4x^2 + x + 6$.

Solution:

x	-3	-2	-1	0	1	2	3	4
y	-60	-20	0	6	4	0	0	10



Notice that the graph intersects the x-axis at (-1,0) , (2,0) and (3,0). You can also use the factor theorem to solve for the roots without using the table as we used above.

Example 2

Graph the function $t = -x(x-2)^2$

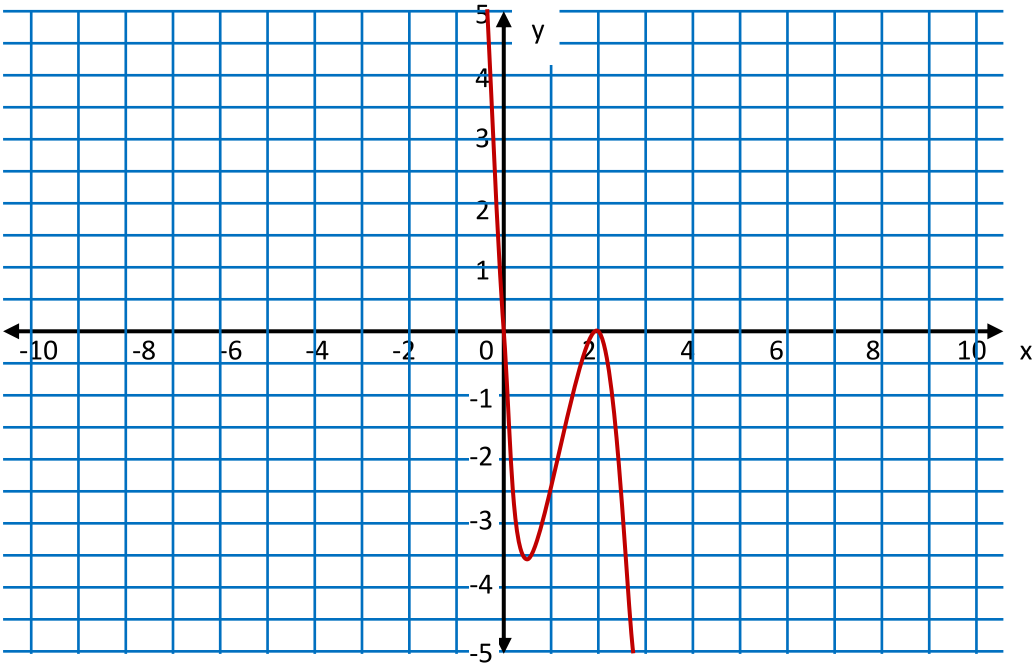
Solution:

Without the table foe values, we can easily identify the roots. By factoring the cubic function. Let y = 0

Then $-x (x – 2)^2 = 0$

When $-x = 0$ $x- 2 = 0$

$X = 0$ $X = 2$



Notice that $f(x) = -x(x-2)^2$ is like a parabola $(x - 2)^2$ connected to $-x$ in a piecewise function.

Now do the learning activity.

LEARNING ACTIVITY 11.2.4.2

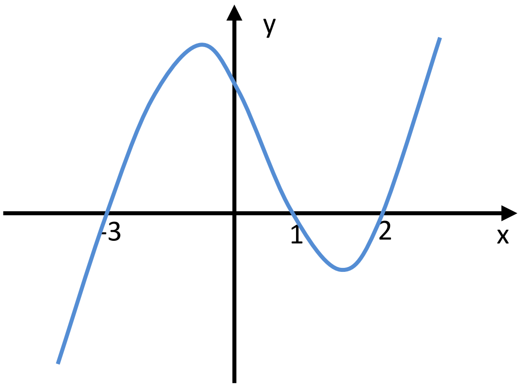
15 minutes

1) Graph the function $x^3 - 6x^2 + 9x$.

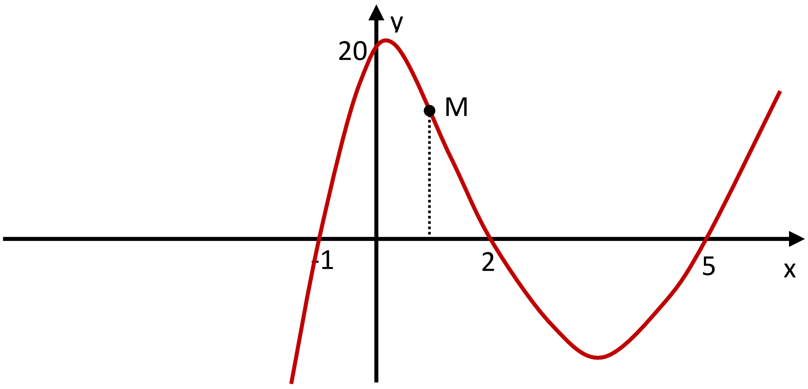
2) Graph the function $f(x) = (x + 3)(x-1)^2$

3) Find zeros of $f(x) = x^3 + x^2 -16x - 16$, and sketch the graph.

4) Derive equation of the sketch below.



5) Find coordinates of M at x = 1 in the sketch.



6) Show that (2, -10) is a point on the graph of $f(x) = x^3 – 9x$.

7) Find the point of intersection of $f(x) = x^3 – 4x^2 + 4x$ and $f(x) = 2x – 1$.

8) Which of the graphs $y = x^3 – 4x – 3x^2 + 12$ or $y = 4x + x^2 – x^3 + 12$ runs from top left to bottom right?

9) If the roots are x = -7, x = 1 and x = 7, derive the equation of the cubic function.

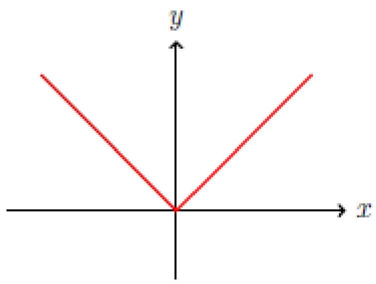
10) Factorize $x^3 – 1 = 0$.

11.2.4.3 Absolute Value Functions

Absolute Value Function The absolute value of a real number x, |x|, is

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

The graph of the absolute value function is shown below.



If the absolute value function is linear, the graph is in a form of 'V'. The line comes down to the vertex (h,k) and then goes up.

The graph will be symmetrical if the domain is equidistant either way from the vertex and maintains 'V'.

Example 1

Find the absolute value of I2I and I-2I

Solution

I2I = 2

I-2I = - (-2) = 2

Example 2

Find the interval of real numbers which contains x, if x satisfies the condition I2x – 5 I < 3.

Solution

I2x – 5 I < 3

-3 < 2x -5 < 3

2 < 2x < 8

1< X < 9

Therefore the values of x are { 2, 3, 4, 5, 6, 7 and 8}

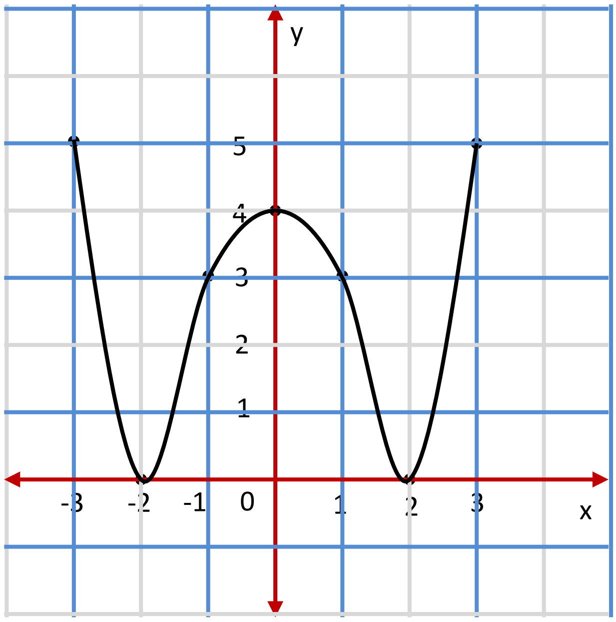
The graph of absolute value function of a quadratic forms a 'W' if the roots a real and absolute value of the entire function is required . The absolute value function of a quadratic may maintain its form if the roots are imaginary.

Example 3

Plot the graph of $y = |x^2 - 4|$ for x: $-3 \le x \le 3$.

Solution:

X	**$Y=\|x^2 - 4\|$**
-3	$\|(-3)^2 – 4\|=\|9-4\|=\|5\|= 5$
-2	$\|(-2)^2 – 4\|=\|4-4\|=\|0\|= 0$
-1	$\|(-1)^2 – 4\|=\|1-4\|=\|-3\|= 3$
0	$\|(0)^2 – 4\|=\|0-4\|=\|-4\|= 4$
1	$\|(1)^2 – 4\|=\|1-4\|=\|-3\|= 3$
2	$\|(-2)^2 – 4\|=\|4-4\|=\|0\|= 0$
3	$\|(-3)^2 – 4\|=\|9-4\|=\|5\|= 5$



The graph of absolute value function of part of a quadratic does not form a 'W' . The absolute value function of part of a quadratic may maintain its curvature but when plotted may seem it does not have real roots. The graph may seem like a translation of the original absolute value function.

Now do the learning activity.

LEARNING ACTIVITY 11.2.4.3

15 minutes

Plot the graphs of the following absolute value functions:

1. $y = |-x| + 2$

2. $y = |x| + 2$

3. $y = |x + 2|$

4. $y = -|x + 3|$

5. $y = |x^2 - 4|$

6. $y = |x^2 - 2x - 8|$

Find the interval that satisfies

7. $-7 < |3x + 1|$

8. $|2x + 3| \leq 4$

11.2.4.4 Rational, Exponential and Logarithmic Functions

The graphs of rational, exponential and logarithmic functions behave in a same way and their asymptotes can be defined.

11.2.4.4.1 Rational Function

A rational function is a function of the form $R(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials.

The **domain** of a rational function consists of all real numbers (x) except those for which the denominator is zero.

When graphing a rational function, we must pay special attention to the behavior of the graph near those x-values. We begin by graphing a very simple rational function.

Sketch a graph of the rational function $f(x) = \frac{1}{x}$ and state the domain and range.

Solution:

First note that the function $f(x) = \frac{1}{x}$ is not defined for x = 0. The function is a rectangular hyperbola .

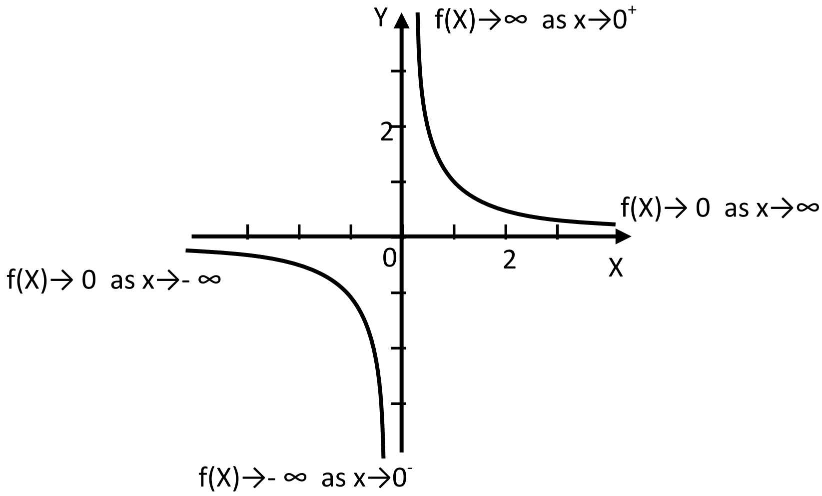
The tables below show the behavior of f near zero.

x	f(x)
- 0.1	- 10
- 0.01	- 100
- 0.000 01	- 100 000
Approaching 0^-	Approaching - ∞

x	f(x)
0.1	10
0.01	100
0.000 01	100 000
Approaching 0^-	Approaching ∞

Using the information in these tables and plotting a few additional points, we obtain the graph.

x	$f(x) = \frac{1}{x}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$



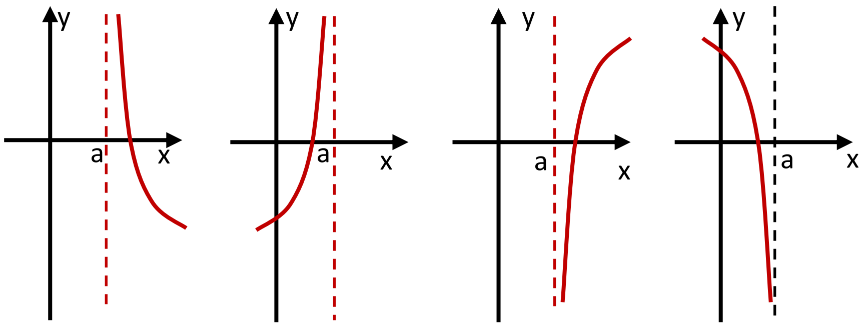
In the Example above we used the following arrow notation.

Symbol	Meaning
$x \rightarrow a^-$	X approaches a from the left
$x \rightarrow a^+$	X approaches a from the right
$x \rightarrow -\infty$	X goes to negative infinity; that is, x decreases without bound
$x \rightarrow \infty$	X goes to infinity; that is, x increases without bound

An **asymptote** of a function is a line that the graph of the function gets closer and closer to as one travels along that line.

Vertical and Horizontal Asymptotes

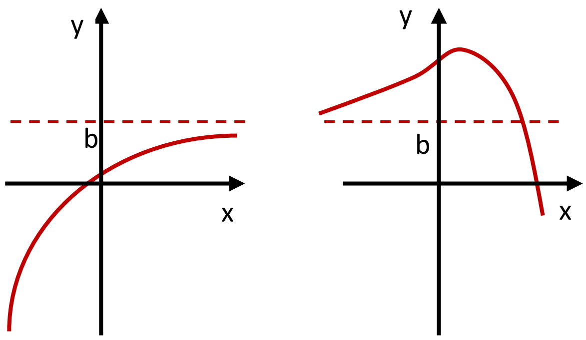
1. Line x = a is a vertical asymptote of the function y = f(x) if $y \rightarrow \pm\infty$ as $x \rightarrow a$ from left or right.



$y \rightarrow \infty$ as $x \rightarrow a^+$ $y \rightarrow \infty$ as $x \rightarrow a^-$ $y \rightarrow -\infty$ as $x \rightarrow a^+$ $y \rightarrow -\infty$ as $x \rightarrow a^-$

The line x = 0 is called a vertical asymptote of the graph of $f(x) = \frac{1}{x}$.

2. Line y = b is a horizontal asymptote of the function y = f(x) if $y \rightarrow b$ as $x \rightarrow \pm\infty$.



$y \rightarrow b$ as $x \rightarrow \infty$ $y \rightarrow b$ as $x \rightarrow -\infty$

The line y = 0 is a horizontal asymptote for $y = \frac{1}{x}$.

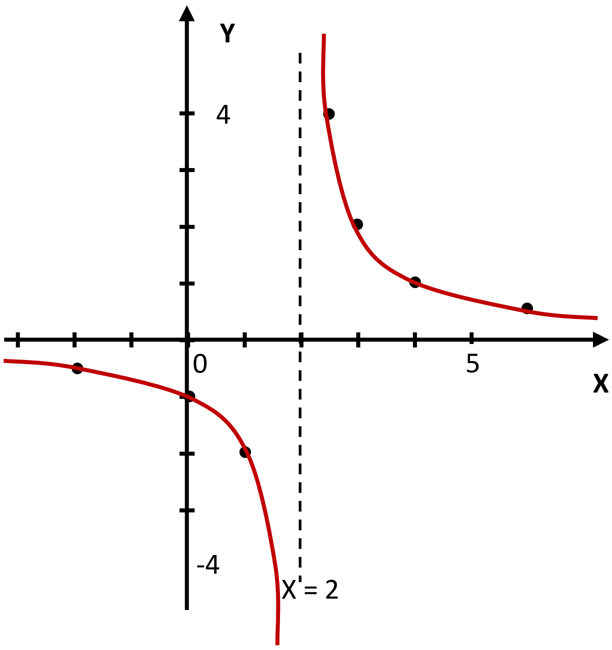
Example

Sketch the graph of y = $\frac{2}{x-2}$.

Solution:

y = $\frac{2}{x-2}$; x ≠2 (x cannot be equal to 2), thus vertical asymptote x = 2.

X	-2	0	1	1 ½	2 ½	3	4	6
Y	-½	-1	-2	-4	4	2	1	½



The function of graph of the rational function above and similar are often referred to as rectangular hyperbola. They are rational of the form **xy = k**.

The asymptote of East-West hyperbola is the y – axis and the asymptote of the North-south hyperbola is the x – axis.

11.2.4.4.2 Exponential Functions

Consider a function of the form $f(x) = a^x$, where $a > 0$. Such a function is called an exponential function. We can take three different cases, where $a = 1$, $0 < a < 1$ and $a > 1$.

If $a = 1$ then $f(x) = 1^x = 1$.

So this just gives us the constant function $f(x) = 1$.

What happens if $a > 1$? To examine this case, take a numerical example. Suppose that $a = 2$.
$f(x) = 2^x$

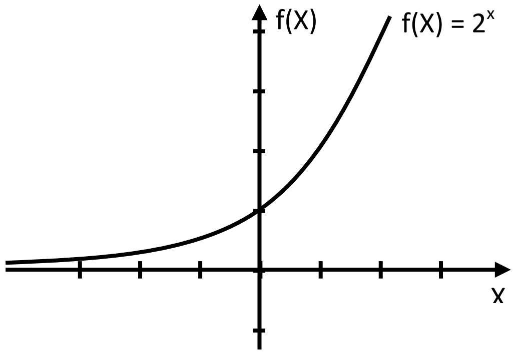
$f(0) = 2^0 = 1$

$f(1) = 2^1 = 2$ $f(−1) = 2−1 = 1/21 = 1/2$

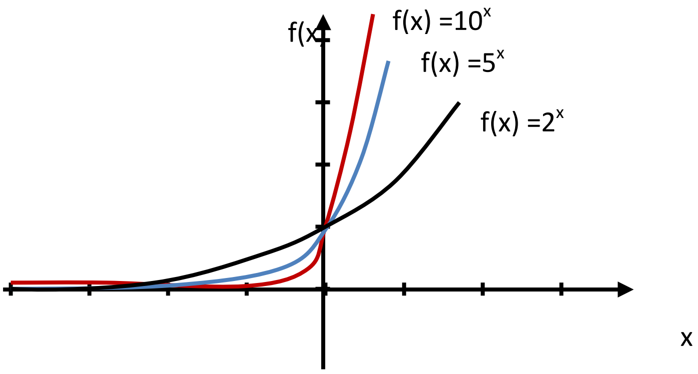
$f(2) = 2^2 = 4$ $f(−2) = 2−2 = 1/22 = 1/4$

$f(3) = 2^3 = 8$ $f(−3) = 2−3 = 1/23 = 1/8$

We can put these results into a table, and plot a graph of the function.



This example demonstrates the general shape for graphs of functions of the form $f(x) = a^x$ when $a > 1$. What is the effect of varying **a**? We can see this by looking at sketches of a few graphs of similar functions.



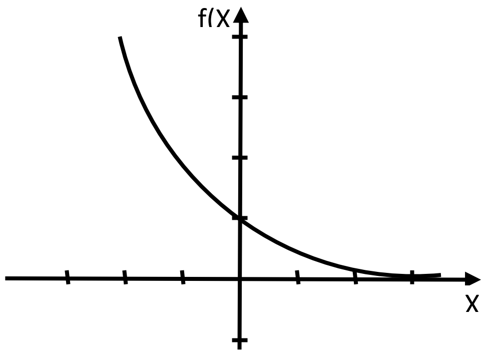
The important properties of the graphs of these types of functions are:

• $f(0) = 1$ for all values of a. This is because $a^0 = 1$ for any value of a.

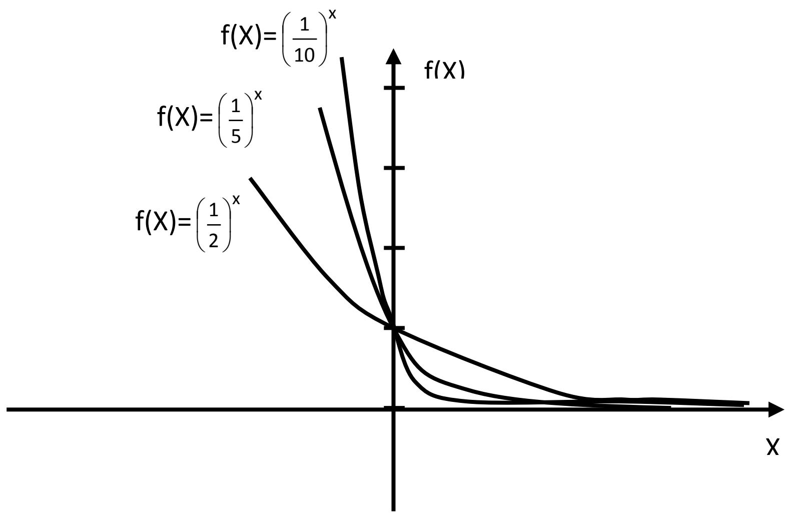
• $f(x) > 0$ for all values of a. This is because $a > 0$ implies $a^x > 0$.

What happens if $0 < a < 1$? To examine this case, take another numerical example. Suppose that $a = \frac{1}{2}$.

X	-3	-2	-1	0	1	2	3
f(x)	8	4	2	1	1/2	1/4	1/8



This example demonstrates the general shape for graphs of functions of the form $\mathbf{f(x) = a^x}$ when $\mathbf{0 < a < 1}$. What is the effect of varying **a**? Again we can see by looking at sketches of a few graphs of similar functions.



The important properties of the graphs of these types of functions are:

• $f(0) = 1$ for all values of a. This is because $a^0 = 1$ for any value of a.

• $f(x) > 0$ for all values of a. This is because $a > 0$ implies $a^x > 0$.

Note the following:

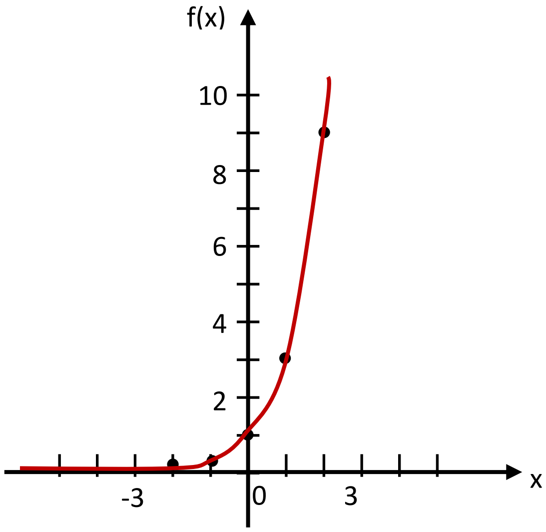
- A function of the form f(x) = ax (where a > 0) is called an exponential function.
- The function f(x) = 1x is just the constant function f(x) = 1.
- The function f(x) = ax for a > 1 has a graph which is close to the x-axis for negative x and increases rapidly for positive x.
- The function f(x) = ax for 0 < a < 1 has a graph which is close to the x-axis for positive x and increases rapidly for decreasing negative x.
- For any value of a, the graph always passes through the point (0, 1). The graph of f(x) = (1/a)x = a^{-x} is a reflection, in the vertical axis, of the graph of f(x) = a^x.
- A particularly important exponential function is f(x) = e^x, where e = 2.718 . . .. This is often called 'the' exponential function.

Example

Sketch the graph of f(x) = 3^x

Solution:

X	-2	-1	0	1	2	3
f(X)	1/9	1/3	1	3	9	27



The graph of f(x) has a horizontal asymptote of y = 0. The domain is all of x; range y > 0.

11.2.4.4.3 Logarithmic Functions

We shall now look at logarithm functions. These are functions of the form $f(x) = \log_a x$ where $a > 0$. We do not consider the case $a = 1$, as this will not give us a valid function.

What happens if $a > 1$? To examine this case, take a numerical example. Suppose that $a = 2$.

Then $f(x) = \log_2 x$ means $2^{f(x)} = x$.

An important point to note here is that, regardless of the argument, $2^{f(x)} > 0$. So we shall consider only positive arguments.

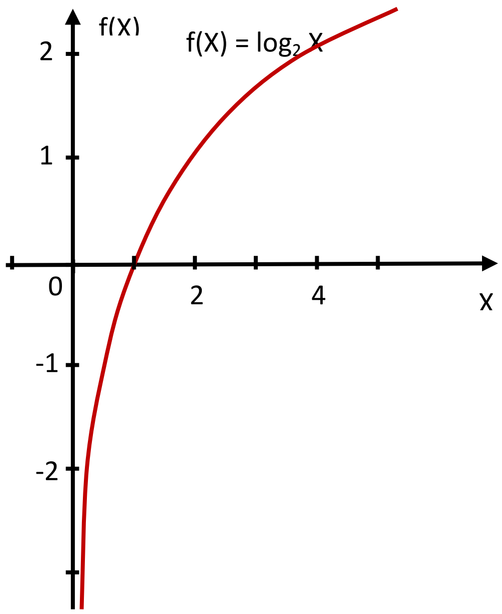
$f(1) = \log_2 1$ means $2^{f(1)} = 1$ so $f(1) = 0$

$f(2) = \log_2 2$ means $2^{f(2)} = 2$ so $f(2) = 1$

$f(4) = \log_2 4$ means $2^{f(4)} = 4$ so $f(4) = 2$

We can put these results into a table, and plot a graph of the function.

X	f(X)
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

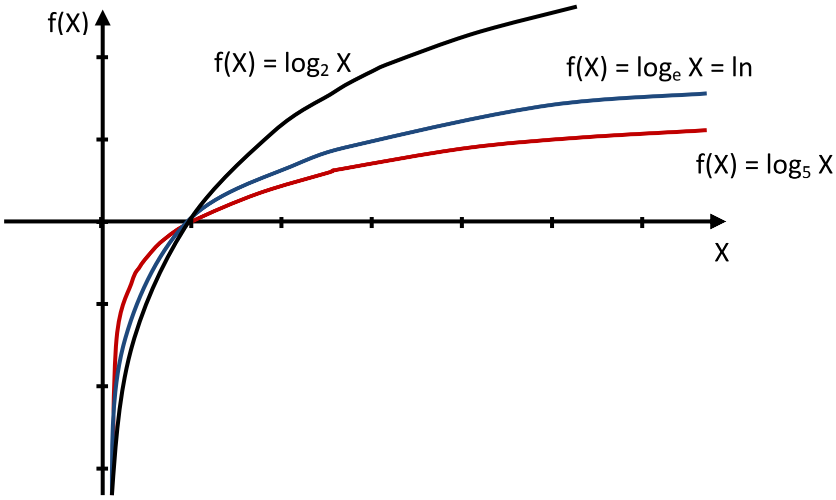


This example demonstrates the general shape for graphs of functions of the form $f(x) = \log_a x$ when $a > 1$. The asymptote is $x = 0$. The domain is $x > 0$; the range is all of y.

What is the effect of varying **a**? We can see by looking at sketches of a few graphs of similar functions. For the special case where a = e, we often write ln x instead of $\log_e x$.

The important properties of the graphs of these types of functions are:

- f(1) = 0 for all values of a;
- we must have x > 0 for all values of a.



Example

Complete the table of values for $f(X) = \log_3 X$.

Solution:

X	1/9	1/3	1	3	9
f(X)	-2	-1	0	1	2

Assign values of f(X), then use f(X) to calculate value of X. Say, f(X) = 1. Since

$3^{f(X)} = X$

$3^1 = X$

X = 3

The domain is all of x > 0; the range is all of y $\{-\infty \le y \le \infty\}$.

Now do the learning activity.

LEARNING ACTIVITY 11.2.4.4

20 minutes

1. Sketch a graph of each function, and state the domain and range.

(a) $f(x) = \dfrac{3}{x+1}$

(b) $y = 3^{x-1}$

(c) $f(x) = \log_{10} X$

(d) $f(x) = -2^x$

(e) $f(x) = \dfrac{2}{x-2}$

2) $f(x) = a^x$ for the following values of **a**, on the same axes:

(a) $a = \frac{1}{4}$

X	-3	-2	-1	0	1	2	3
f(x)							

(b) $a = \frac{1}{3}$

X	-3	-2	-1	0	1	2	3
f(x)							

(c) $a = \frac{1}{2}$

X	-3	-2	-1	0	1	2	3
f(x)							

(d) a = 2

X	-3	-2	-1	0	1	2	3
f(x)							

(e) a = 3

X	-3	-2	-1	0	1	2	3
f(x)							

(f) a = 4

X	-3	-2	-1	0	1	2	3
f(x)							

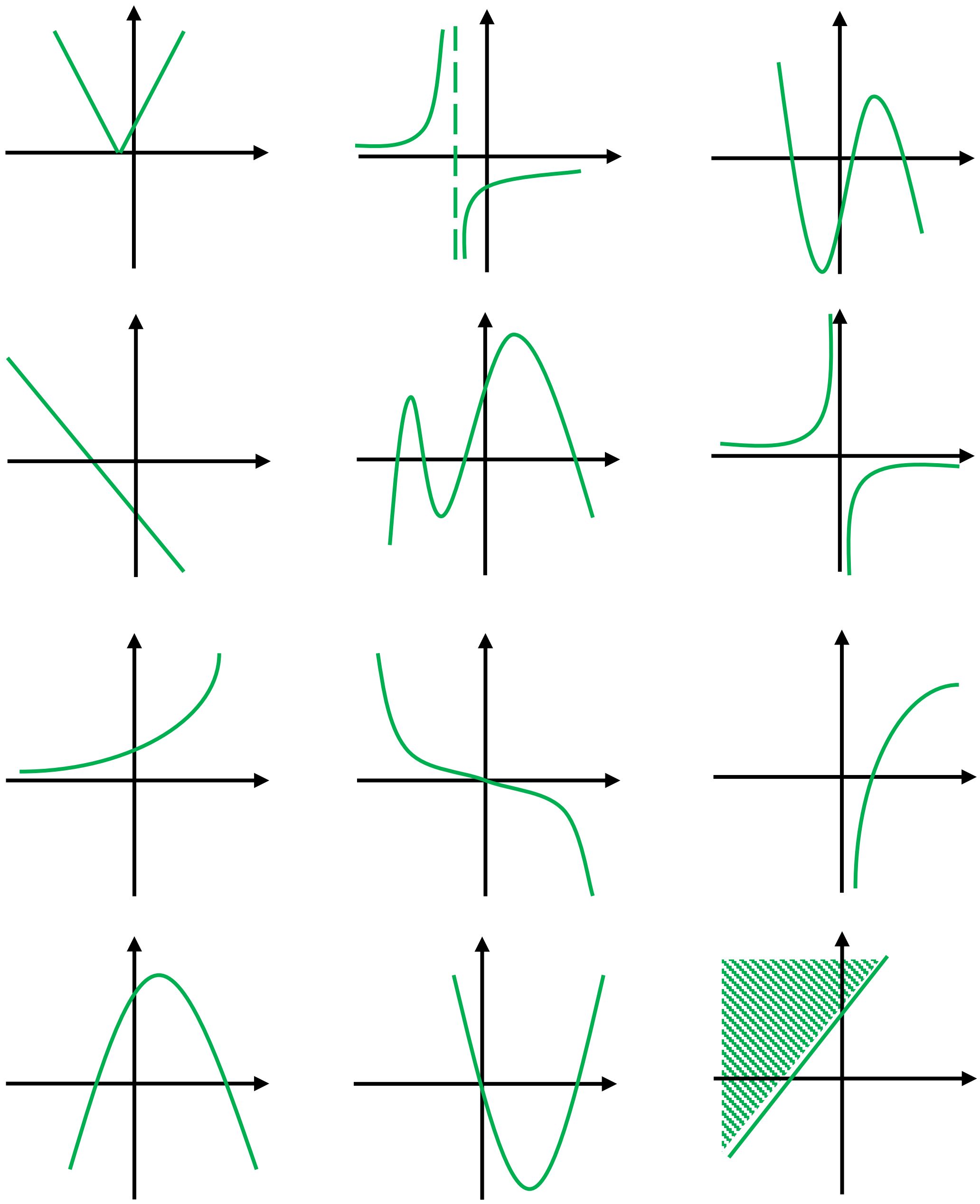
SUMMATIVE TASK 11.2.4

20 minutes

1. Match the graphs with the forms given:

Forms:

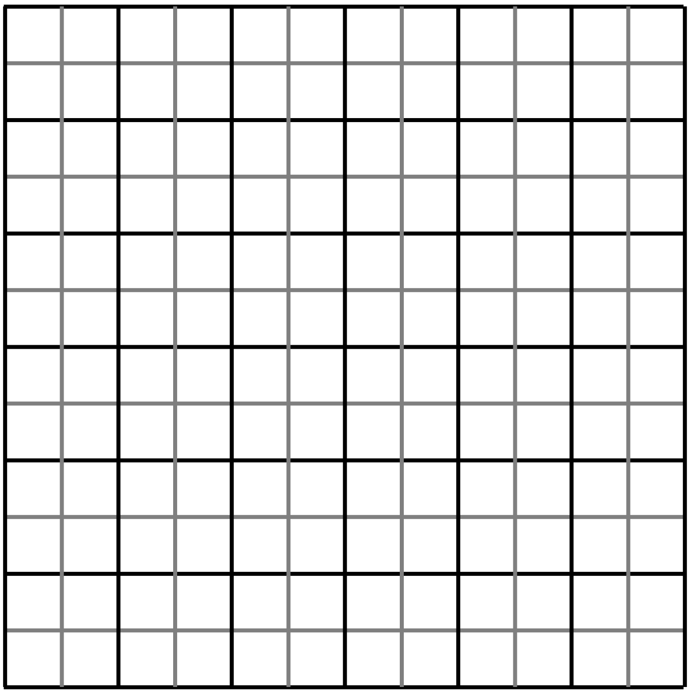
$Y = ax^2 + bx + c$
$Y = ax^3 + bx^2 + cx + d$
$Y = ax^4 + bx^3 + cx^2 + dx + e$
$Y = \log X$
$Y = mx + c$
$Y = |mx = c|$
$Y = ax^2 + bx$
$Y = a^x$
$Y = ax^3$
$XY = k$
$Y = \dfrac{k}{x + a}$



2. Plot the graph of $y = \frac{-2}{x+1}$. Show clearly the asymptote.

3. Plot the graph of $f(x) = \log_4 X$.

4. Plot the graph of the exponential function $f(x) = 3^x$.



5. Sketch the graph of $f(x) = x^3 – 7x + 6$.

UNIT SUMMARY

This summary outlines the key ideas and concepts to be remembered.

- A constant is a symbol that can take only one value. A variable is a symbol that can take one or more values from a given set of permissible values. An algebraic expression is a term used to mean a constant, a variable or a combination of variables and constants involving a finite number of indicated operations (addition, subtraction, multiplication, division, raising to powers and extraction of roots) on them.

- A polynomial is an algebraic expression involving only non-negative powers of one or more variables and containing no variable in the denominator. The degree of a polynomial in one variable is the greatest or highest exponent of its variable. The term with the highest degree is called leading term. The leading coefficient is the (numerical) coefficient of the leading term.

- Consecutive integers are integers that come right after another on the number line. Consecutive odds and consecutive evens are sequence of successive odds or even numbers which defer by 2, and are expressed as n, n + 2, n + 4, when n is the least number or n – 4, n -2 and n etc., when n is the largest.

- The Remainder Theorem. If a polynomial ƒ(x) is divided by x - k, then the remainder is r = ƒ(k).The Factor Theorem. A polynomial ƒ(x) has a factor x - k if and only if ƒ(k) = 0. Steps for Solving Exponential Equations with the Same Base
  - Step 1: Determine if the numbers can be written using the same base.
  - Step 2: Rewrite the problem using the same base.
  - Step 3: Use the properties of exponents to simplify the problem.
  - Step 4: Once the bases are the same, drop the bases and set the exponents equal to each other.
  - Step 5: Finish solving the problem by isolating the variable.

- A linear equation can be defined as an equation in which the highest exponent of the equation variable is one. A linear function is a function of the form f(x) = ax + b. It is also expressed in general form as ax + by+ c = 0 and standard form as y = mx + c. Given two points on the line, their mid-point is found by ½ $(x_1 + x_2, y_1 + y_2)$ and the distance between them is found by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$. Equations of the form x = a are vertical lines with m = ∞ (undefined). Equations of the form y = a are horizontal lines with m = 0. Two lines are said to be parallel when $m_1 = m_2$ and are perpendicular when $m_1m_2 = -1$.

- Equations of the form $ax^2 + bx + c = 0$, $ax^2 + bx = 0$ and $y = ax^2$ are quadratic. Transformation of quadratic to the form $a(x – h)^2 + k$ yields axis of symmetry x = h and the vertex (h,k). The vertex or turning point (TP) is the minimum when a > 0 and is the maximum when a < 0. A quadratic equation can be solved by use of factor method, identities, square-root method, completing the square and by quadratic formula.

- Functions can easily be identified with advanced skills in sketching graphs of functions. Graphs that do not pass a vertical line test are not functions but are just relations.

  Graphs of functions may require defining **domain** and **range** of a specific function. Rational relations such as East-west and North-South hyperbolas are not functions. Rectangular hyperbola f(x) = k/x, exponential $f(x) = a^x$ and logarithmic $f(x) = \log_a X$ are relations which are functions.

  Graphs of absolute value functions are similar to piece-wise functions and translations. Graphs of linear absolute value function form a 'V'. Graphs of quadratic absolute value function often form a 'W'.

  Sketch and graphs of Cubic function other forms may not display all the roots, thus may not intersect x-axis trice as would cubic equation of the form $ax^3 + bx^2 + cx + d = 0$.

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ANSWERS TO LEARNING ACTIVITIES

LEARNING ACTIVITY 11.2.1.1

1. a) 3 NS
   b) 2 S
   c) 4 S
   d) 2 NS
   e) 2 ns
2.

Polynomial	Degree	Leading coefficient
$x + 12$	1	1
$6xy + 4$	2	NA
$3x^3 + 7x^2 + 3x – 2$	3	3
$X^4y^2z – x^2yz + 4x^2y^5z^5$	12	NA
$2 – y^8$	8	-1
$-5y^9 + 8y^2 – 5y^8$	9	-5

LEARNING ACTIVITY 11.2.1.2

1) -39 v
2) 9 – 12k
3) 55 + 103 v
4) $8x^3 - 12x^2 + 6x – 1$
5) $x^3 + x^2 - 4x – 4$
6) $\frac{(3x+1)}{(x+6)}$
7) $9x^2 – 4x + 24$
8) $\frac{1}{3}$
9) $x^3 – 27$
10) $\frac{x+6}{x-6}$

LEARNING ACTIVITY 11.2.1.3

1.a. 15 – n

b. n + 17

c. n + 12

d. n – 12

e. n + 8

f. n – 15

g. b – 4

h. r + 6

i. 11/t

j. 3y / 5

k. 11z

l. x – 6

m. 9 – 2n

n. 2(9 - n)

o. 2(n - 9)

p. 15 + x

2. a) -14 b)12 c) 40, 41 and 42 d) 9, 11, 13

e) 22 and 24 f) K3 000 g) 58 cm

LEARNING ACTIVITY 11.2.1.4

1. (a) $3x^2 - 5x - 2 + \frac{8}{x+1}$

(b) $2x^2 - x - 2$

2. a) r = 5

   b) r = 0

   c) r = -5

   d) r = -29

3. a) (x+1)(x-2)(x+3)

   b) (x+1)(x-1)(2x+1)

   c) (2x + 1)(x + 2)(x - 1)(x - 3)

   d) (3x + 2)(3x - 2)(x - 1)

   e) (x + 3)(x + 1)(x - 1)(x - 3)

LEARNING ACTIVITY 11.2.1.5

1. (a) 1 ½
   (b) 9
   (c) 16
   (d) 125
   (e) 12

2. (a) x = -3
   (b) x = 5
   (c) x = 3
   (d) x = 4
   (e) x = ½

3. (a) $5^x = 125$
   (b) $125^x = 5$
   (c) $2^x = 16$
   (d) $3^5 = x$
   (e) $x^3 = 8$

4. (a) 0
   (b) 1
   (c) ¼
   (d) x = 3
   (e) x = 32

5. (a) $\frac{\log 12}{\log 2}$
   (b) $\log_3 7 + \log_3 3 - \log_3 5$
   (c) $3\log_2 3 + 3\log_2 2$
   (d) $\log_3 5 + \log_3 1$
   (e) $\log_8 3 - \log_8 2$

6. (a) $2\log_3 2 - \log_3 7 + 1$
   (b) $\log_4 5 + 2\log_4 2$
   (c) $3\log_6 3 - 2\log_6 2$
   (d) $\log_5 3 + \log_5 2 + 1$
   (e) $\log_2 7 + \log_2 5 + \log_2 3 + \log_2 2$

7. (a) log 18
   (b) $\log \frac{8}{5}$
   (c) $\log_2 105$
   (d) $\log_3 \frac{5}{14}$
   (e) $\log_2 7$

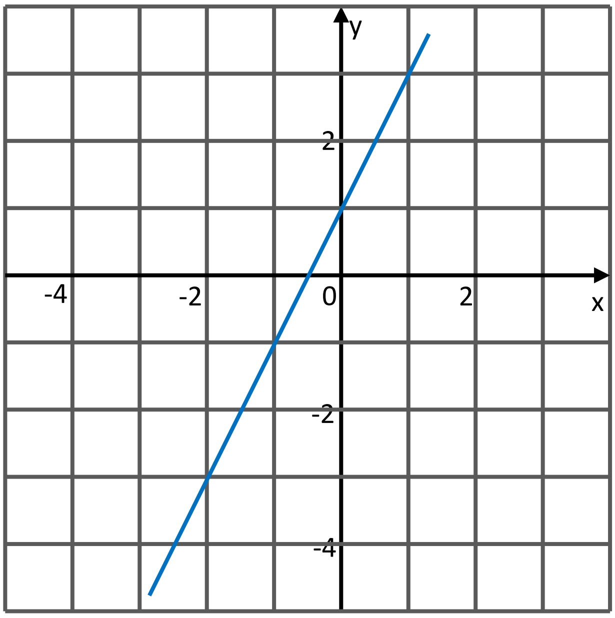
SUMMATIVE TASK 11.2.1

1. 13/n - 5
2. 5/8y – 3 = 7, y = 16
3. n + (n + 4) = 2(n + 2), 14,16,18
4. -4x/x+1
5. $2x^2 + 6x^2y$
6. r = 14
7. (x- 6)(x + 6)(x + 2)(x + 1)
8. x = 14
9. $\log_3 10 - 1$
10. $4^x = 84$

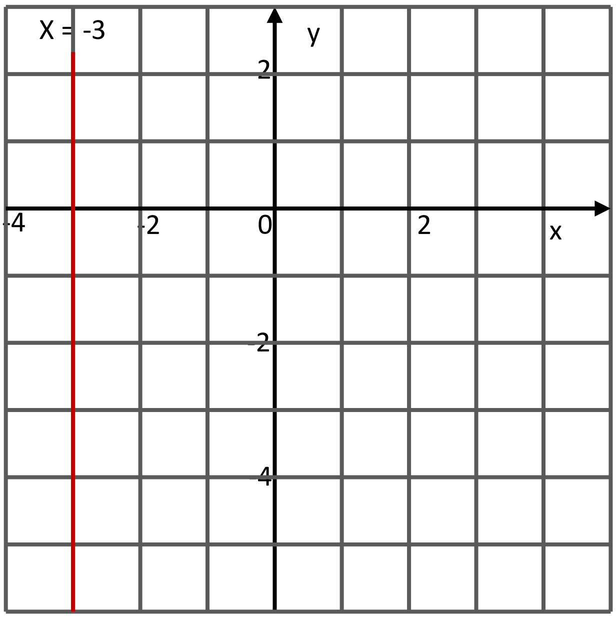
LEARNING ACTIVITY 11.2.2.1

1. a) false e) true
   b) false f) false
   c) true g) true
   d) true h) false

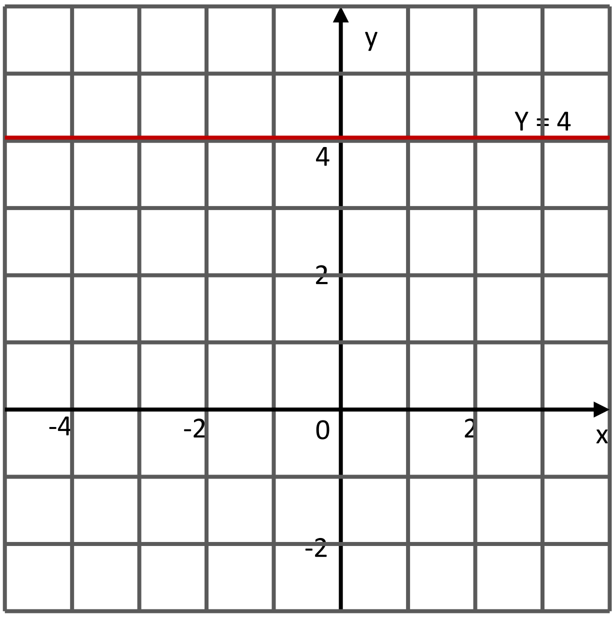
2. a) y = 2x + 1



b) x = -3



c) y = 4



e) d) Line **P:** y = 2x + 2.

f) Line **Q**: y = $\frac{1}{3}$x + 11.

g) Line **N**: y = -2x + 13.

h) m = $\frac{5}{2}$

i) M.P. of **LM** = (1, -1)

j) Y = -x + 4

k) d = 5 units

LEARNING ACTIVITY 11.2.2.2

1) x =2 y= 3

2) x = 3, y = −2

3) x = 2, y = 0

4) x = -3, y = 5

5) x = ½ , y = 3

6. x = 2, y = 3/5

7. x = 3, y = -3

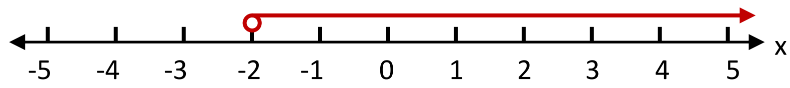
8. x = -4, y = 2

9. x = - ½ , y = 8

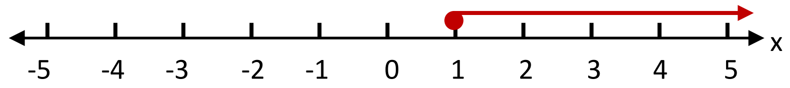
10. x = 4/5, y = 2/3

LEARNING ACTIVITY 11.2.2.3

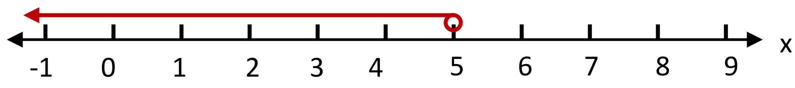
1. X > -2



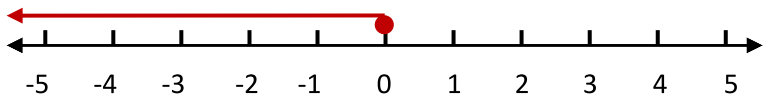
2. X ≥ 1



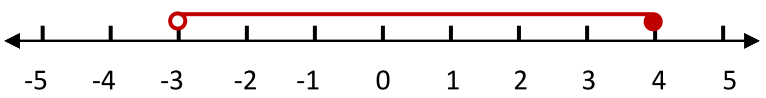
3. X < 5



4. X ≤ 0



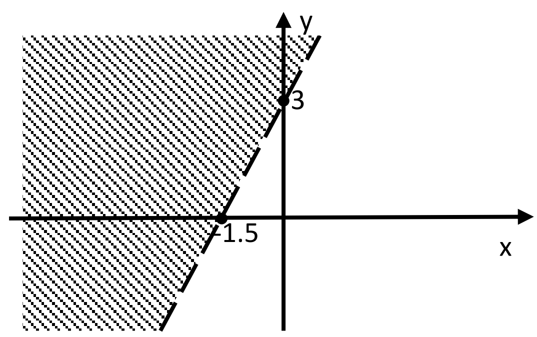
5. -3 < x ≤ 4



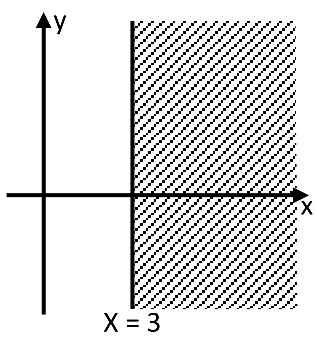
6. x < 2
7. -4 ≤ x
8. 4< x <16
9. -4 ≤x < 2
10. -6 ≤ x ≤ 2

LEARNING ACTIVITY 11.2.2.4

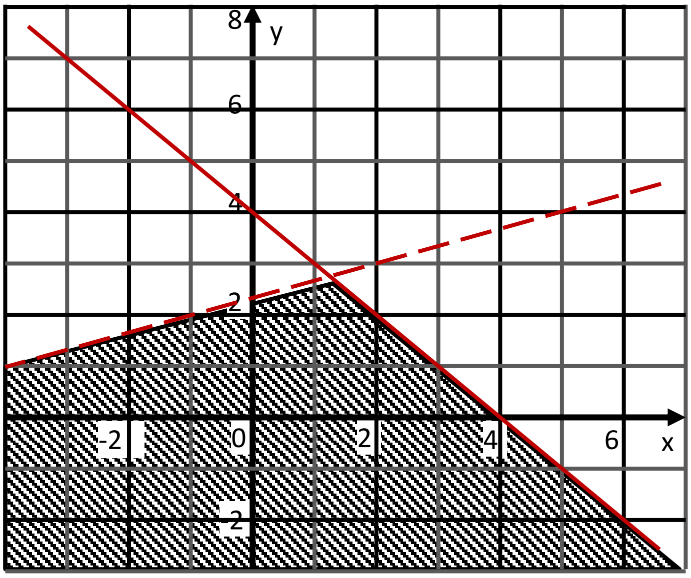
1. Region y > 2x + 3.



2. x ≥ 3



3. y + x ≤ 4 and 3y – x < 7.



4. (a) Line y = $\frac{-3}{4}$ x + 8 , Region 4y + 3x ≥ 32

(b) Line y = -x – 3, Region y + x < - 3

(c) Lines y = -x + 3 and y = x + 3, Region y + x ≥ 3 and y – x ≤ 3

(d) Lines y = $-\frac{5}{4}$ x + 5, x = 0, y = 0, Region 4y + 5x ≤ 20, x ≥ 0 and y ≥ 0.

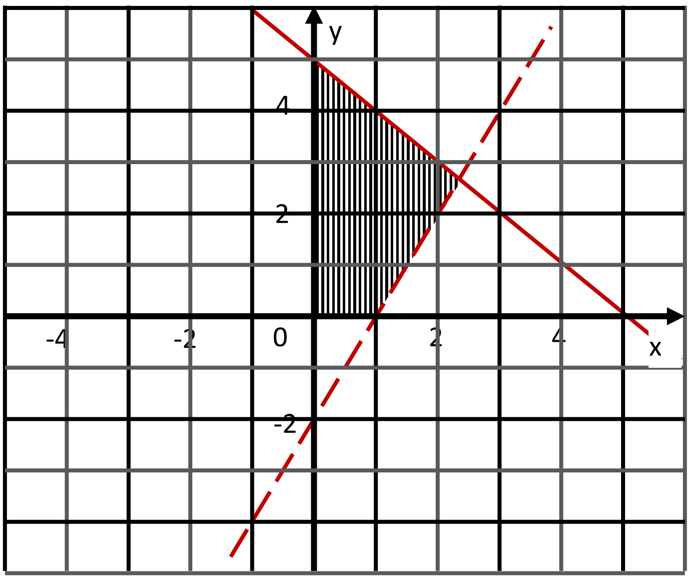
5. Region

Y > 2x – 2

Y≤ - x + 5

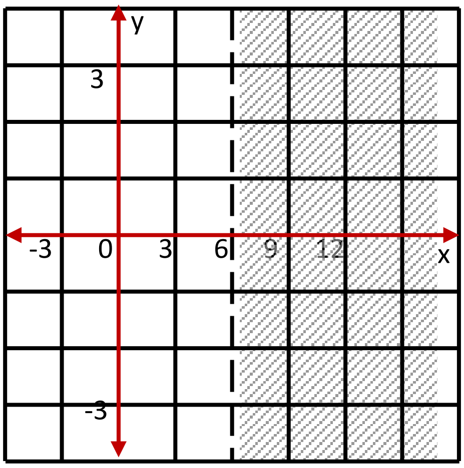
Y > 0

X > 0

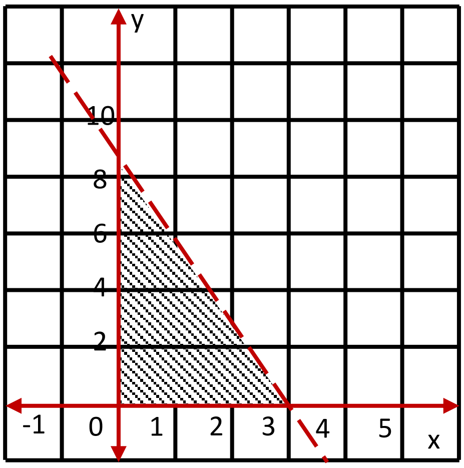


SUMMATIVE TASK 11.2.2

1) (1, 2)
2) (-4, -15)
3) line **P:** y = -2x + 6
4) line **Q:** 4y = 5x - 5
5) Graph x > 6.
6) -6 < x ≤ 6



7) Region: y < - 3x + 9, x > 0 and y > 0.



8) 4Y < 5x + 20, 4y < 20 -5x, y > 0
9) Line **N**: 2y + 5x = 0.
10) 10 units

LEARNING ACTIVITY 11.2.3.1

1. (x- 6)(x + 4)
2. (x - 7)(x + 8)
3. (2x -1)(x - 3)
4. (**x - 2)(**2x + 5)
5. $- 6x^2 + 4x = 0$
6. X = 4, x = - 2.5
7. X = -3, x = - 11
8. x = 7, x = - 3
9. X = 4, x = 7
10. $f(x) = x^2 + 7x - 19$

LEARNING ACTIVITY 11.2.3.2

1. $(4x – 1)^2$
2. $(3x - 2)(3x + 2)$
3. $(2x+1)^2$
4. $(2x - 3)^2$
5. $(10 - 3x)(10 + 3x)$
6. $(3x + 4)^2$
7. $(2x - 5)^2$
8. $(x + 13)^2$
9. $(x - 17)^2$
10. $(a – 2ab)^2$

LEARNING ACTIVITY 11.2.3.3

1. $x = \pm 20$
2. $x = \pm 16$
3. $x = \pm 25$
4. $x = \pm\sqrt{5}$
5. $x = \pm 6\sqrt{2}$
6. $x = \pm 5\sqrt{2}$
7. $x = \pm 4$
8. $x = \pm 3$
9. $x = \pm \frac{3}{2}$
10. $x = 0$ or 6

LEARNING ACTIVITY 11.2.3.4

1. has 2 distinct solutions
2. has 2 distinct solutions
3. has 2 distinct solutions
4. has 2 equal solutions
5. has NO real solutions
6. has NO real solutions
7. has 2 distinct solutions
8. has 2 equal solutions
9. has 2 distinct solutions
10. has 2 distinct solutions

LEARNING ACTIVITY 11.2.3.5

1. $2(x + {}^{9}/_{4})^2 - {}^{207}/_{16} = 0$ — $X = -6$, $x = 3/2$
2. $5(x + {}^{7}/_{10})^2 – {}^{169}/_{100} = 0$ — $x = -2$, $x = 3/5$
3. $(x - {}^{4}/_{3})^2 – {}^{64}/_{9} = 0$ — $x = -4/3$ or $x = 4$
4. $2(x – {}^{7}/_{4})^2 – 16\ {}^{15}/_{16}$ — $x = 2$ or $x = 5$
5. $(x + {}^{5}/_{4})^2 – 6\ {}^{1}/_{8}$ — $X = -3$ or $x = ½$

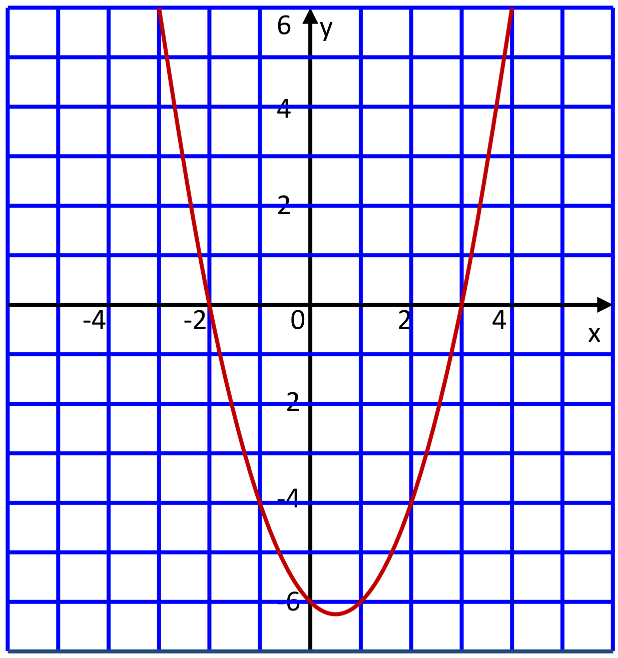
LEARNING ACTIVITY 11.2.3.6

1. x = -10, x = -3
2. x = -8, x = 6
3. x = 5, x = 11
4. x = -3, x = 5
5. x = -3/2, x = -3/2 [or x = -1.5, x = -1.5]
6. x = 5/2, x = 5/2 [or x = 2.5, x = 2.5]
7. x = -3/4, x = 2 [or x = -0.75, x = 2]
8. x = - ½ , x = 3 ½ [or x = -0.5, x = 3.5]
9. x = 4/5, x = 2 [or x = 0.8, x = 2]
10. x = -1/4 , x = -4 [or x = -0.25, x = -4]

LEARNING ACTIVITY 11.2.3.7

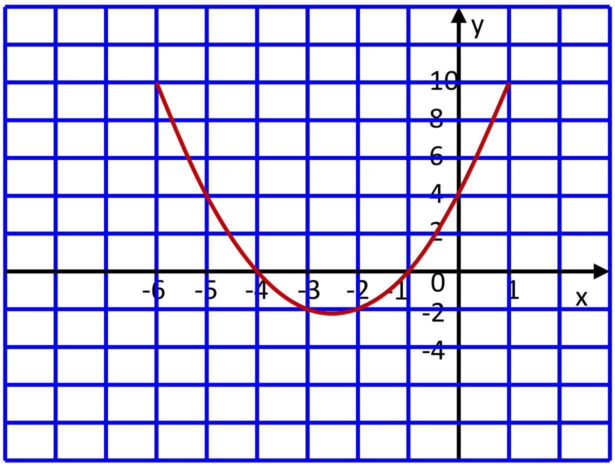
1)

X	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6



2) $f(x) = x^2 + 5x + 4$.

X	-6	-5	-4	-3	-2	-1	0	1
y	10	4	0	-2	-2	0	4	10



3) $2\left(x + \frac{5}{4}\right)^2 - \frac{121}{8}$

4) $-2x^2 + 7x - 3$

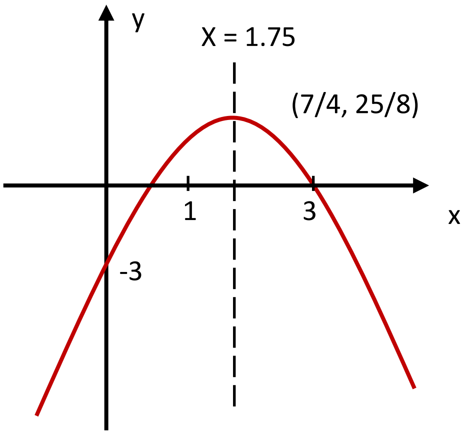
$\left(x - \frac{7}{4}\right)^2 = \frac{25}{8}$

(a) The roots: $x = \frac{1}{2}$, $x = 3$

(b) The axis of symmetry: $x = \frac{7}{4}$

(c) The vertex: $(\frac{7}{4}, \frac{25}{8})$

(d) Sketch.



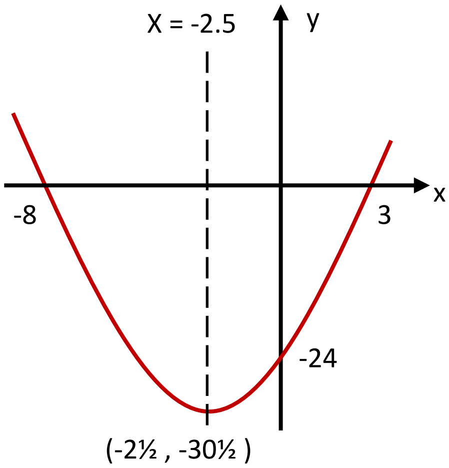
5) $f(x) = x^2 + 5x - 24$.

(i) c = - 24

(ii) roots x = -8 or x = 3

(iii) axis of sym. X = - 2 ½

(iv) TP or Vertex (-2 ½ , -30 ¼)

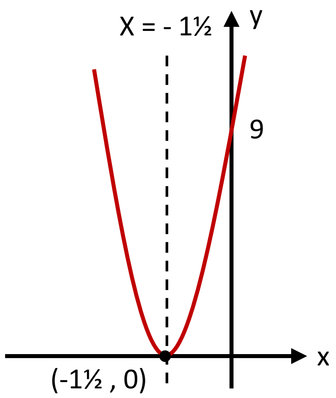


6) $Y = -x^2 + 4x + 5$

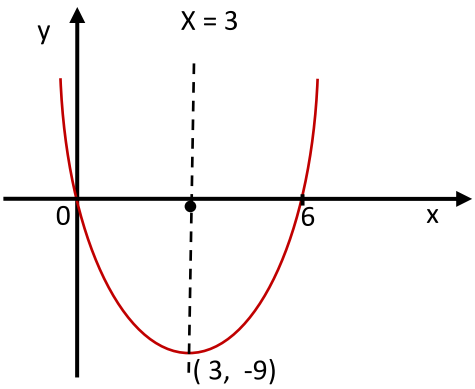
7) $f(x) = x^2 + 6x + 8$
(a) c = 8
(b) x = -3
(c) TP = (-3, -1)

8) $Y = -(x – 4)^2$; b = y – intercept = -16

9) $y = 4x^2 + 12 + 9$
C = 9
Roots: x = -1.5, x = x= - 1.5 (- 1 ½ or – 3/2)
TP = (-1 ½ , 0)



10)



(a) $y = x^2 – 6x$

(d) domain is all of x.
{-∞ ≤ x ≤ +∞}

(e) range: y ≥ -9
{-9 ≤ y ≤ ∞}

SUMMATIVE TASK 11.2.3

1) (x + 2)(x - 9) = 0 x = -2 or x = 9
2) (x - 4)(x - 4) = 0 x = 4 or x = 4
3) X = - 6 or x = 0
4) (2x + 3)(x - 2) = 0 x = - 1½ or x = 2.
5) $(x + 12)^2 = 0$ x = -12 or x = -12
6) $(2x- 3)^2 = 0$ x = 1 ½ or x = 1 ½
7) $(1 – 2x)^2$ x = ½ or x = ½
8) $(4 - x)^2 = 0$ x = 4 or x = 4
9) X = - 3 or x = 3
10) X = - 4 or x = 4
11) X = 0 or x = 9
12) (3x -1)(x - 4) = 0 x = 1/3 or x = 4

13) $(2x + 5)(x + 3) = 0$ $x = -2\frac{1}{2}$ or $x = -3$

14) $X = \frac{1}{4}$ or $x = 1$

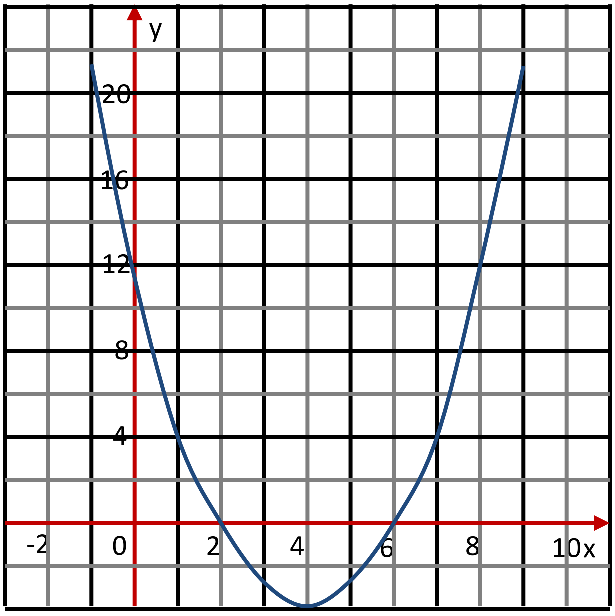
15) $X = -8$ or $x = 6$

16) $b^2 - 4ac > 0$, has 2 distinct real roots. $X = -5/2$ or $x = 2$

17) $b^2 - 4ac > 0$, has 2 distinct real roots .$x = \frac{1}{4}$ or $x = 1$

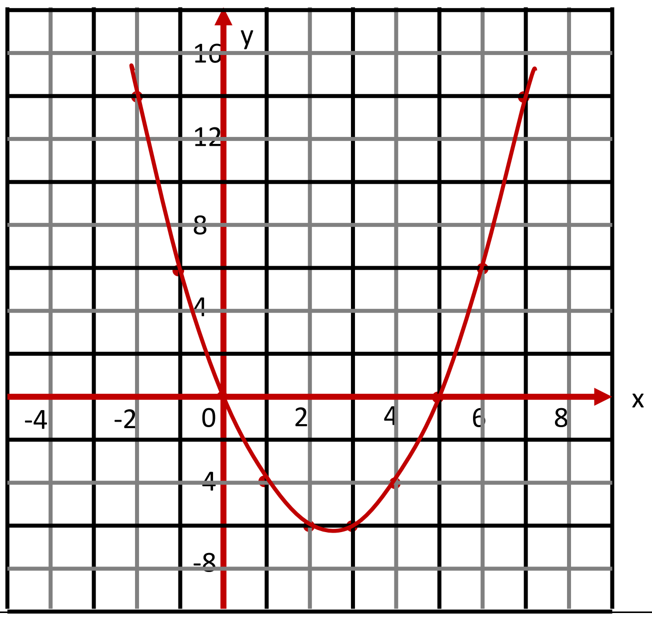
18) $y = x^2 - 8x + 12$

X	-1	0	1	2	3	4	5	6	7	8	9
y	21	12	4	0	-3	-4	-3	0	4	12	21



19) $f(x) = x^2 - 5x$

X	-2	-1	0	1	2	3	4	5	6	7
y	14	6	0	-4	-6	-6	-4	0	6	14

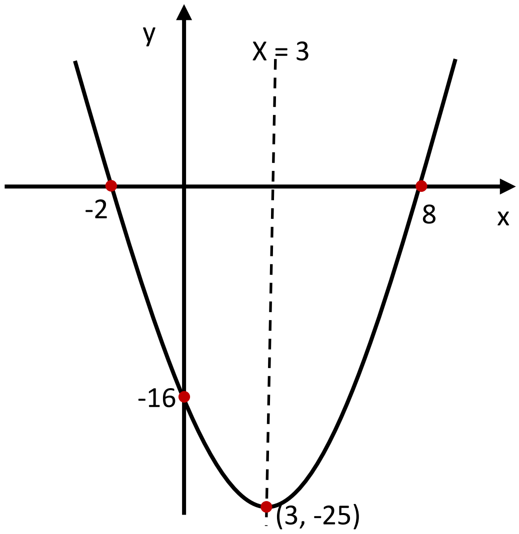


20) $f(x) = x^2 - 6x - 16$.

$(x + 2)(x - 8) = 0$ so $x = -2$ or $x = 8$

Y – intercept $c = -16$

AOS $x = 3$, TP = (3, -25)

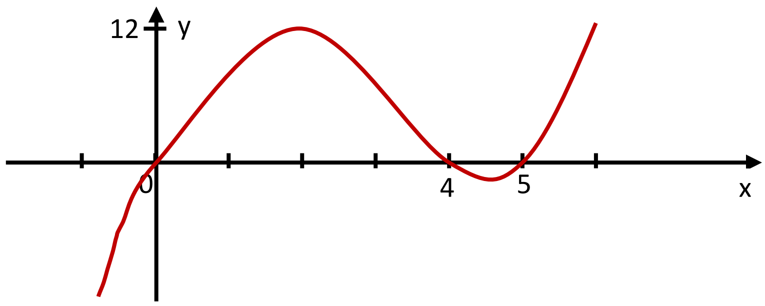


LEARNING ACTIVITY 11.2.4.1

1. $P(x) = x^3 - 9x^2 + 20x$.

(a) P (0), P(4) and P(5) or x = 0, x = 4 and x = 5.

(b) Sketch the graph of P.



2. $P(x) = 4x^3 - 4x^2 - 5x + 3$.

   P(-1) = 0, P(1/2) = 0, P (1 ½) = 0 or $(x + 1)(2x - 1)(2x - 3) = 0$

3. Since f(2) = 0, therefore (x - 2) a is a factor of $x^3 - 3x^2 - 4x + 12 = 0$.

4. $R(x) = x^3 + 2x^2 - 5x - 24$ R - 54

5. P(-3) = 0, P(-2) = 0 and P (1) = 0

6. Yes; since $P(2) = 2^3 - 5 \times 2^2 + 4 = 0$

7. $R(x) = x^2 - 4x + 12$

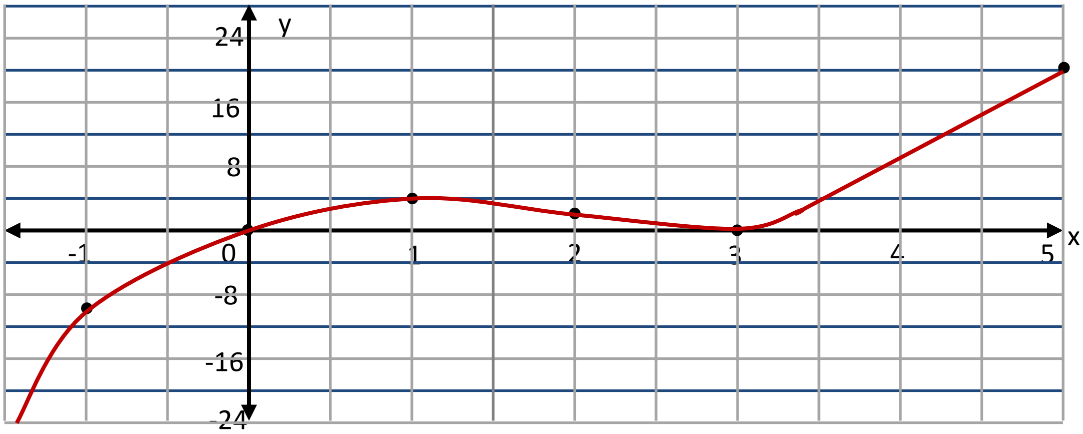
8. $x^3 + 8$

9. $(x + 4)(x^2 + 4x - 16)$

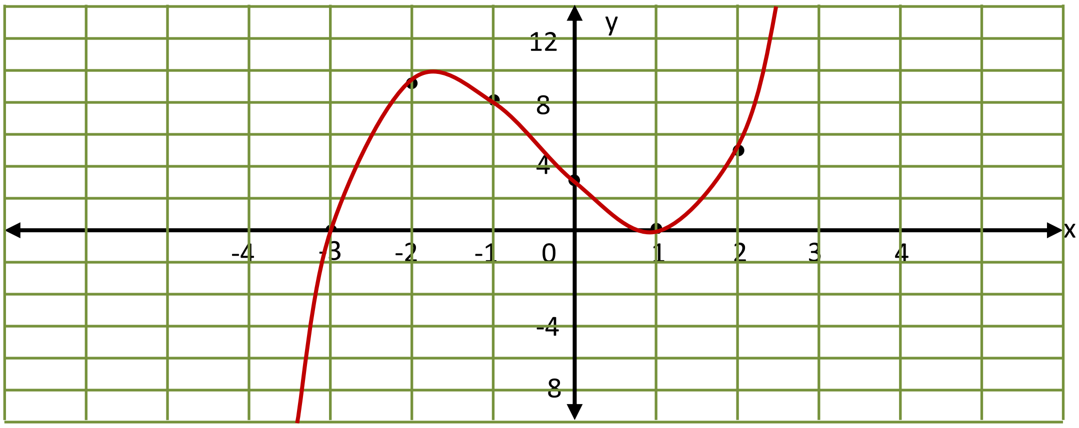
10. $(x + 3)(x - 3)^2$

LEARNING ACTIVITY 11.2.4.2

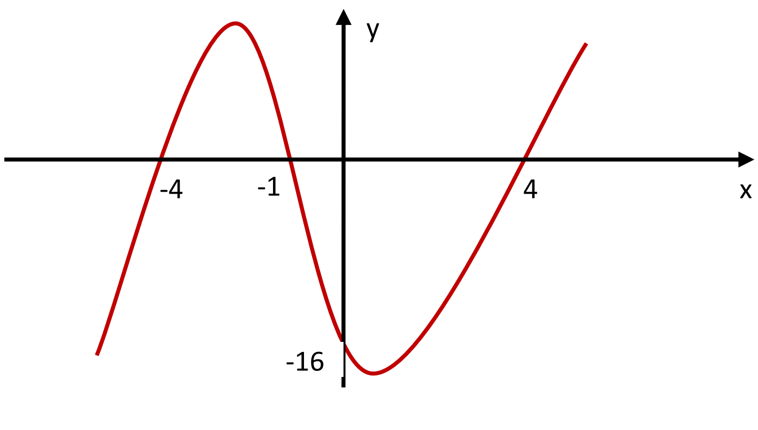
1. Graph the function $x^3 - 6x^2 + 9x$.



2) Graph of $f(x) = (x + 3)(x-1)^2$



3) f(- 4) = 0, f(-1) = 0 and f(4) = 0.



4) $Y = (x + 3)(x - 1)(x - 2) = x^2 – 7x + 6$

5) $Y = (x + 1)(x - 2)(x - 5) = x^3 – 6x^2 + 3x + 10$, M = (1, 8)

6) $f(2) = 2^3 – 9x2 = 8 – 18 = - 10$.

7) POI (1, 1)

8) Graph of $y = 4x + x^2 – x^3 + 12$ runs from top left to bottom right.

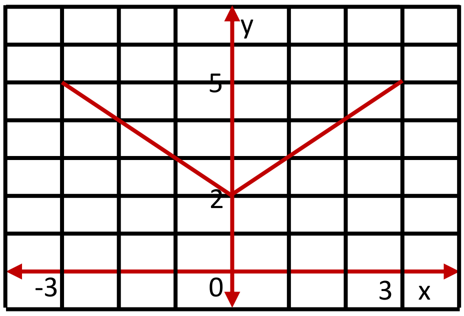
9) $f(x) = x^3 –x^2 – 49x + 49$

10) $x^3 – 1 = (x + 1)(x^2 – x + 1)$

LEARNING ACTIVITY 11.2.4.3

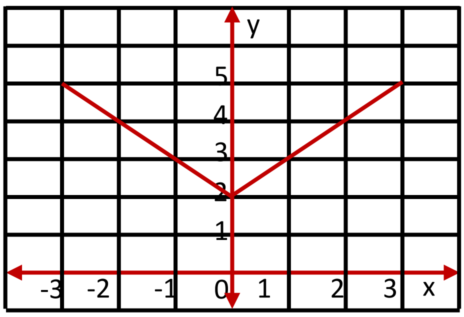
1. y = |-x| + 2

x	Y = |-x|+2
-3	|--3|+ 2 = |3| + 2 = 3 + 2 = 5
-2	|--2|+ 2 = |2| + 2 = 2 = 2 = 4
-1	|--1|+2 = |1|+2 = 1+2 = 3
0	|-0|+2 = |0| + 2 = 0 +2 = 2
1	|-1| + 2 = 1 + 2 = 3
2	|-2| + 2 = 2 + 2 = 4
3	|-3| + 2 = 3 + 2 = 5



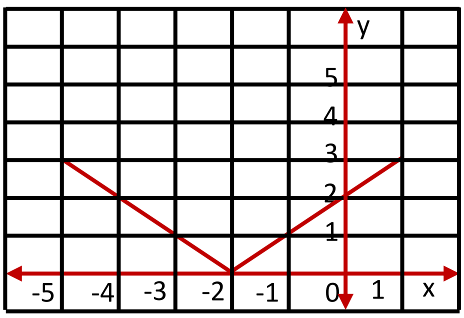
2. y = |x| + 2

x	Y = |x|+2
-3	|3|+ 2 = 3 + 2 = 5
-2	|2|+ 2 = 2 + 2 = 4
-1	|1|+ 2 = 1+2 = 3
0	|0|+ 2 = 0 + 2 = 2
1	|-1| + 2 = 1 + 2 = 3
2	|-2| + 2 = 2 + 2 = 4
3	|-3| + 2 = 3 + 2 = 5



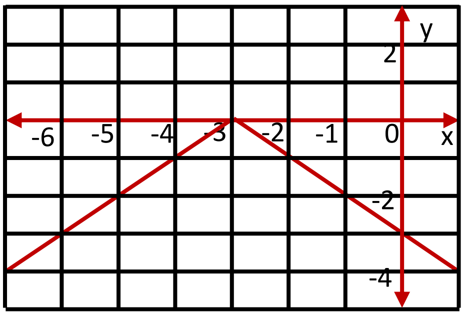
3. y = |x + 2|

x	Y = |x + 2|
-5	|-5 + 2| = |-3| = 3
-4	|-4 + 2| = |-2| = 2
-3	|-3 + 2| = |-1| = 1
-2	|-2 + 2| = |0| = 0
-1	|-1 + 2| = |1| = 1
0	|0 + 2| = |2| = 2
1	|1 + 2| = |3| = 3



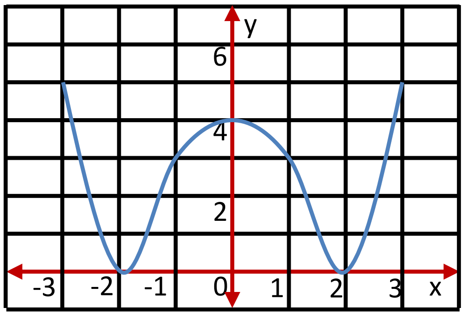
4. $y = -\lvert x + 3 \rvert$

x	$Y = -\lvert x + 3 \rvert$
-7	$-\lvert -7 + 3 \rvert = -\lvert -4 \rvert = -4$
-6	$-\lvert -6 + 3 \rvert = -\lvert -3 \rvert = -3$
-5	$-\lvert -5 + 3 \rvert = -\lvert -2 \rvert = -2$
-4	$-\lvert -4 + 3 \rvert = -\lvert -1 \rvert = -1$
-3	$-\lvert -3 + 3 \rvert = -\lvert 0 \rvert = 0$
-2	$-\lvert -2 + 3 \rvert = -\lvert 1 \rvert = -1$
-1	$-\lvert -1 + 3 \rvert = -\lvert 2 \rvert = -2$
0	$-\lvert 0 + 3 \rvert = -\lvert 3 \rvert = -3$
1	$-\lvert 1 + 3 \rvert = -\lvert 4 \rvert = -4$



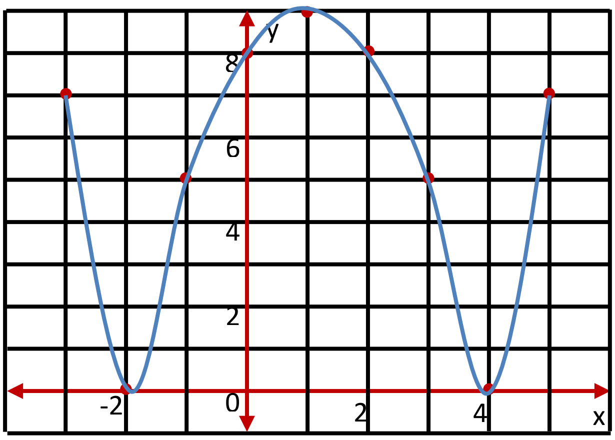
5. $y = \lvert x^2 - 4 \rvert$

x	$Y = \lvert x^2 - 4 \rvert$
-3	$\lvert -3^2 - 4 \rvert = \lvert 9 - 4 \rvert = \lvert 5 \rvert = 5$
-2	$\lvert -2^2 - 4 \rvert = \lvert 4 - 4 \rvert = \lvert 0 \rvert = 0$
-1	$\lvert -1^2 - 4 \rvert = \lvert 1 - 4 \rvert = \lvert -3 \rvert = 3$
0	$\lvert 0^2 - 4 \rvert = \lvert 0 - 4 \rvert = \lvert -4 \rvert = 4$
1	$\lvert 1^2 - 4 \rvert = \lvert 1 - 4 \rvert = \lvert -3 \rvert = 3$
2	$\lvert 2^2 - 4 \rvert = \lvert 4 - 4 \rvert = \lvert 0 \rvert = 0$
3	$\lvert -3^2 - 4 \rvert = \lvert 9 - 4 \rvert = \lvert 5 \rvert = 5$



6. $y = \lvert x^2 - 2x - 8 \rvert$

x	$Y = \lvert x^2 - 2x - 8 \rvert$
-3	$\lvert -3^2 - 2.-3 - 8 \rvert = \lvert 9+6-8 \rvert = \lvert 7 \rvert = 7$
-2	$\lvert -2^2 - 2.-2 - 8 \rvert = \lvert 4+4-8 \rvert = \lvert 0 \rvert = 0$
-1	$\lvert -1^2 - 2.-1 - 8 \rvert = \lvert 1+2-8 \rvert = \lvert -5 \rvert = 5$
0	$\lvert 0^2 - 2.0 - 8 \rvert = \lvert 0+0-8 \rvert = \lvert -8 \rvert = 8$
1	$\lvert 1^2 - 2.1 - 8 \rvert = \lvert 1-2-8 \rvert = \lvert -9 \rvert = 9$
2	$\lvert 2^2 - 2.2 - 8 \rvert = \lvert 4-4-8 \rvert = \lvert -8 \rvert = 8$
3	$\lvert 3^2 - 2.3 - 8 \rvert = \lvert 9-6-8 \rvert = \lvert -5 \rvert = 5$
4	$\lvert 4^2 - 2.4 - 8 \rvert = \lvert 16-8-8 \rvert = \lvert 0 \rvert = 0$
5	$\lvert 5^2 - 2.5 - 8 \rvert = \lvert 25-10-8 \rvert = \lvert -7 \rvert = 7$



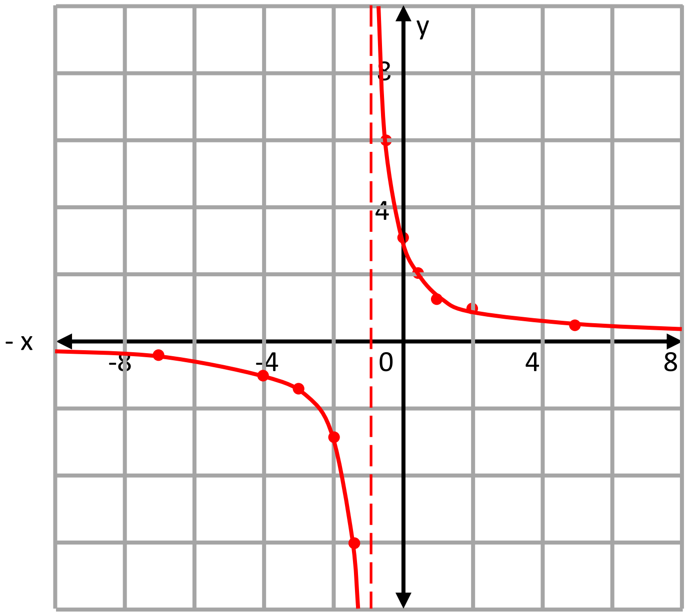
7. $-\frac{8}{3} < x < 2$

8. $-\frac{7}{2} \leq x \leq \frac{1}{2}$

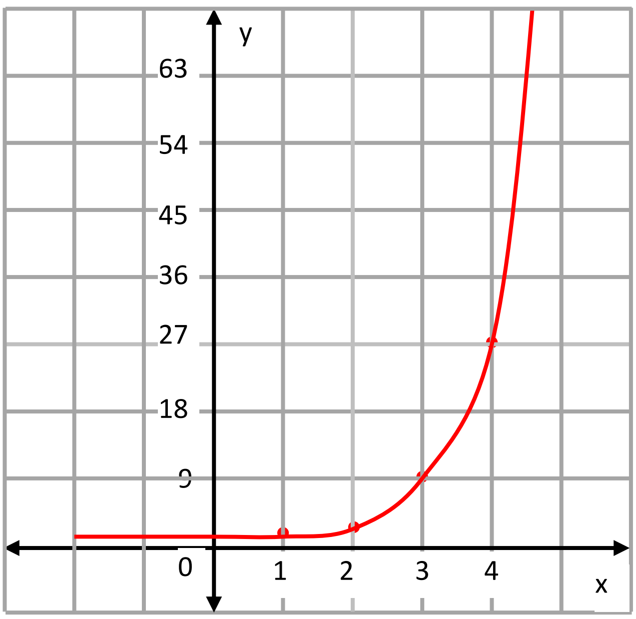
LEARNING ACTIVITY 11.2.4.4

1.(a) $f(x) = \dfrac{3}{x+1}$, $x \neq -1$

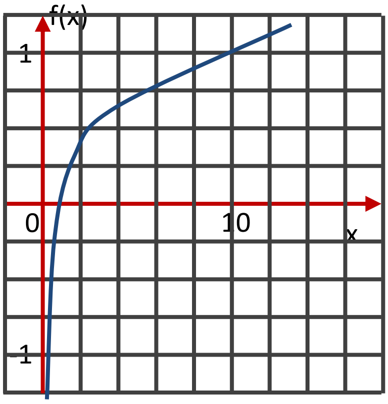
X	-7	-4	-3	-2	-1 ½	- ½	0	½	1	2	5
y	- ½	-1	-1 ½	-3	-6	6	3	2	1 ½	1	½



(b) $y = 3^{x-1}$

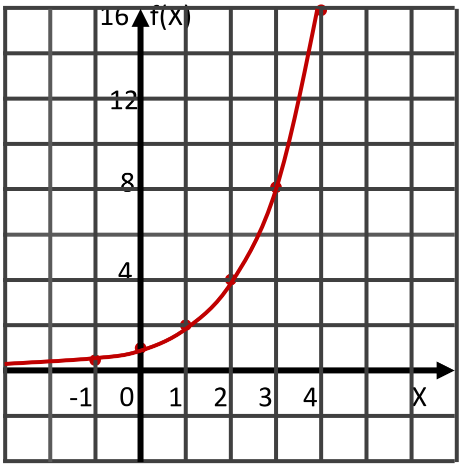


(c) $f(x) = \log_{10} X$



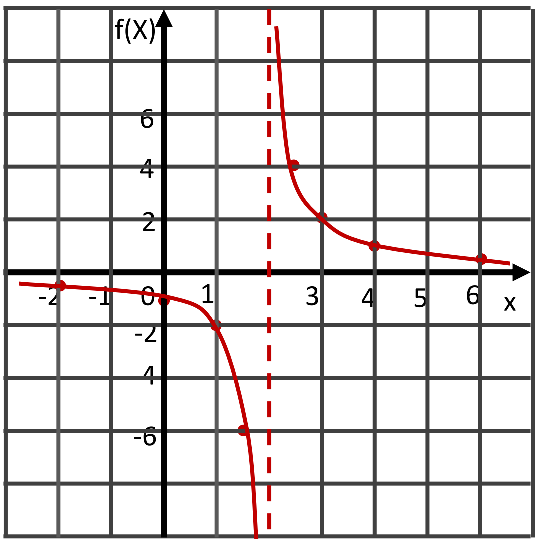
(d) $f(x) = 2^x$

X	-1	0	1	2	3	4
f(X)	½	1	2	4	8	16



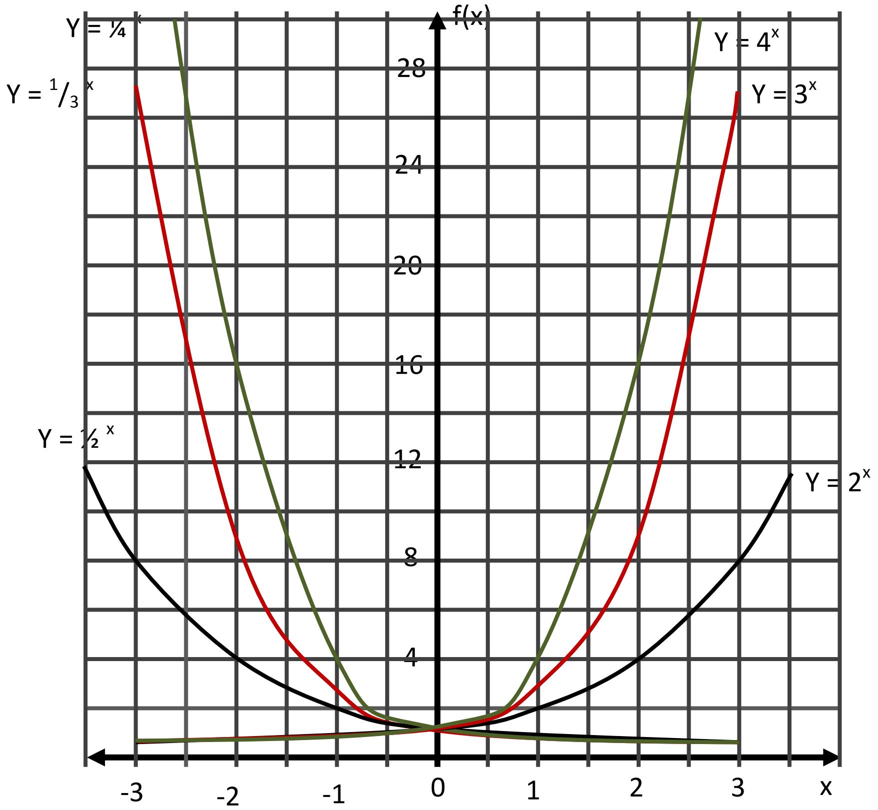
(e) $f(x) = \frac{2}{x-2}$

X	-2	0	1	1 ½	2 ½	3	4	6
f(X)	- ½	-1	-2	-6	4	2	1	½



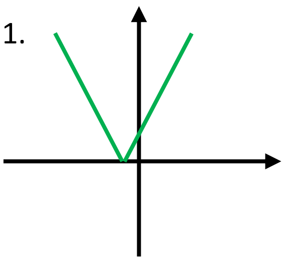
2) $f(x) = a^x$

(a) $a = \frac{1}{4}$ (b) $a = \frac{1}{3}$ (c) $a = \frac{1}{2}$ (d) a = 2 (e) a = 3 (f) a = 4

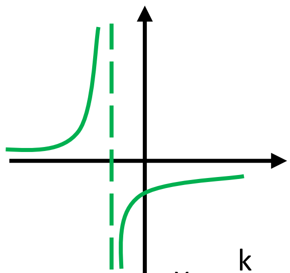


SUMMATIVE TASK 11.2.4

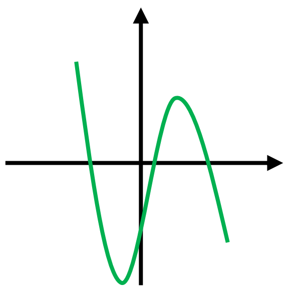
1.



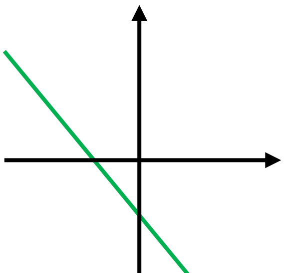
Y = |mx = c|



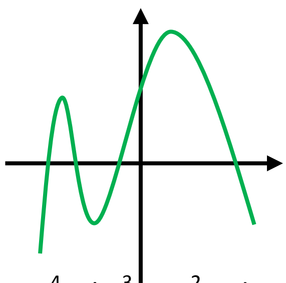
$Y = \frac{k}{x+a}$



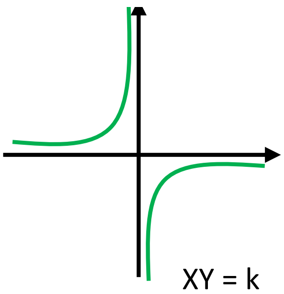
$Y = ax^3 + bx^2 + cx + d$



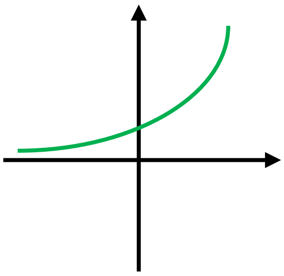
Y = mx + c



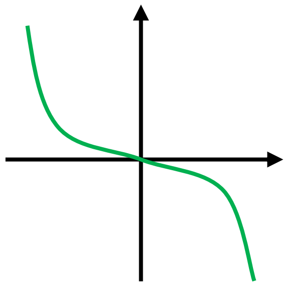
$Y = ax^4 + bx^3 + cx^2 + dx + e$



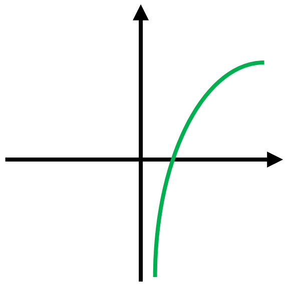
XY = k



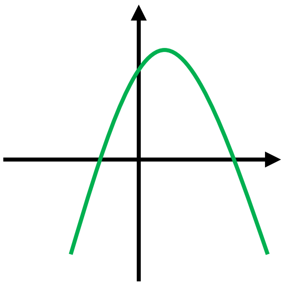
$Y = a^x$



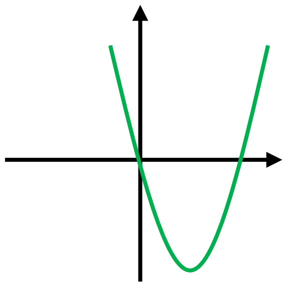
$Y = ax^3$



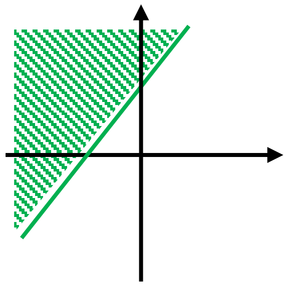
Y = log X



$Y = ax^2 + bx + c$



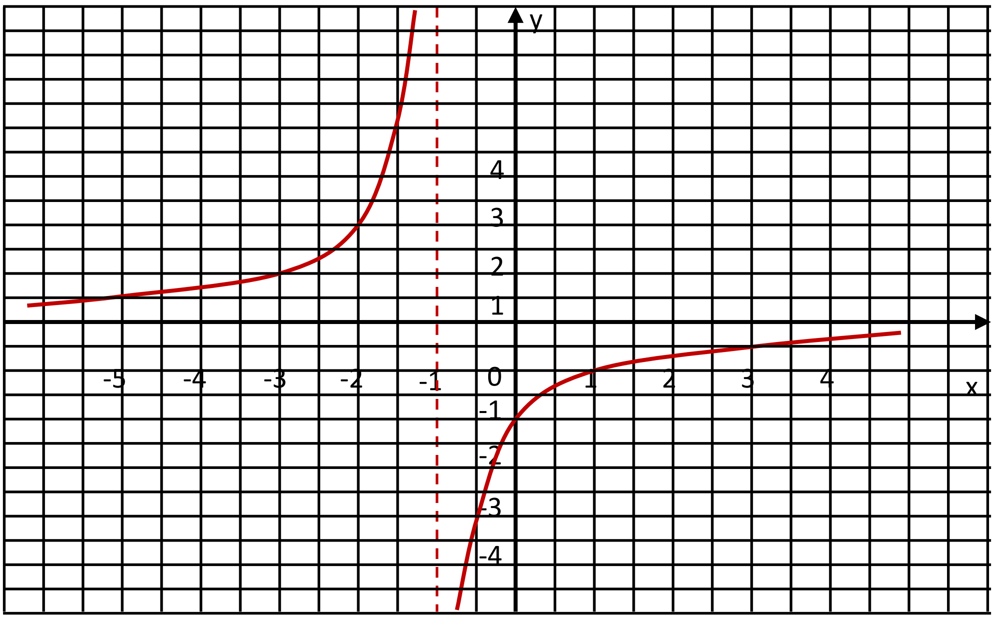
$Y = ax^2 + bx$



None

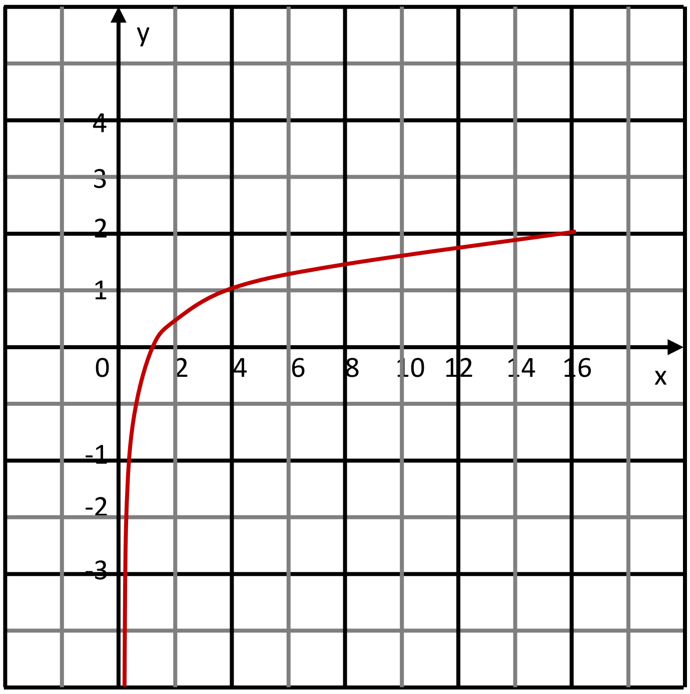
2. $y = \dfrac{-2}{x+1}$

X	-5	-3	- 2	- 1 ½	- ½	0	1	3
y	½	1	2	4	-4	-2	-1	- ½



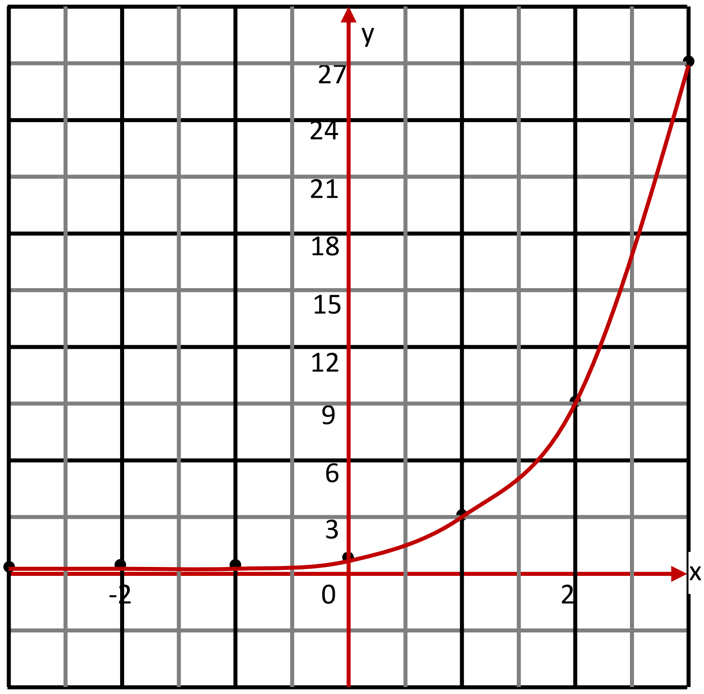
3. $f(x) = \log_4 X$.

X	f(X)
½	- ½
1	0
2	½
4	1
16	2



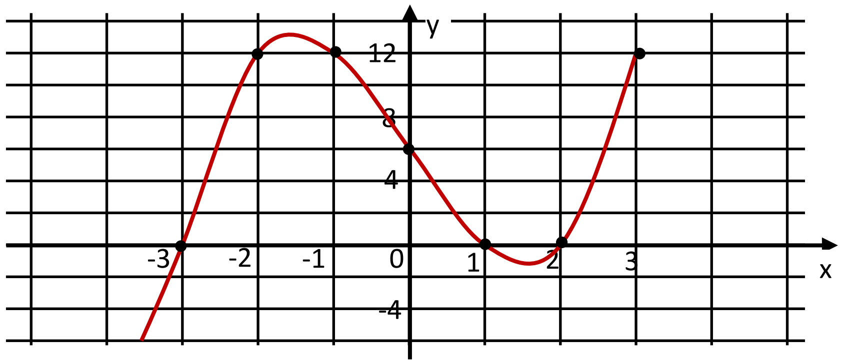
4. $f(x) = 3^x$.

x	f(x)
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27



5. $f(x) = x^3 - 7x + 6$.

X	-4	-3	-2	-1	0	1	2	3
y	-30	0	12	12	6	0	0	12



(your sketch does not have to be placed on a grid)

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